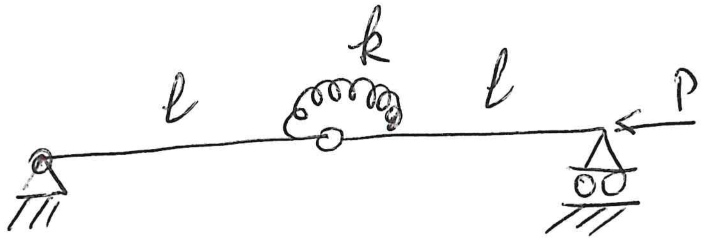
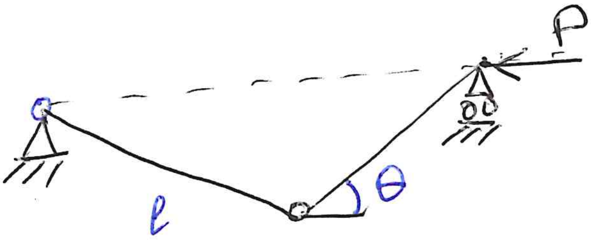
Example 1 : Static criterium

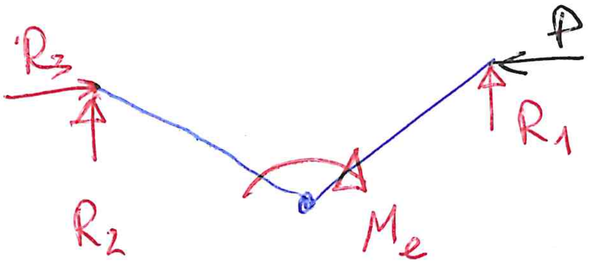




$$\begin{cases} \sum_i F_i = 0 \\ \sum_i M_{Oi} = 0 \end{cases}$$

with respect to point O

Free body diagram

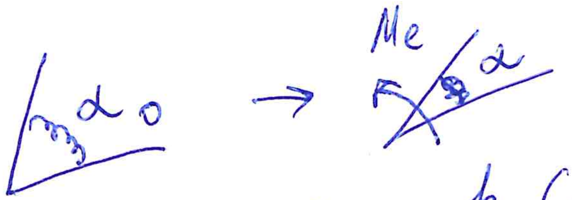


$\uparrow$: $R_1 + R_2 = 0$

$\rightarrow$: $R_3 = P$

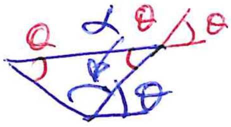
$\Rightarrow R_1 = R_2 = 0$

M_e - ?



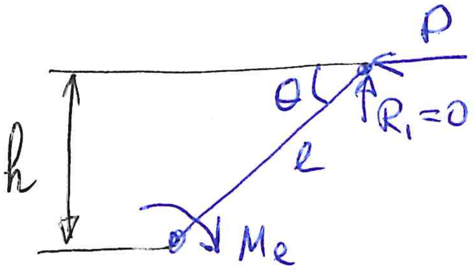
$M_e \sim k(\alpha - \alpha_0)$

$\alpha_0 = \pi$



$\alpha = \pi - 2\theta$

$\alpha - \alpha_0 = -2\theta$



$\Rightarrow M_e = M_p$

$M_p = Ph = Pl\sin\theta$

$M_e = k\,2\theta \quad , \quad M_p = Pl\sin\theta$

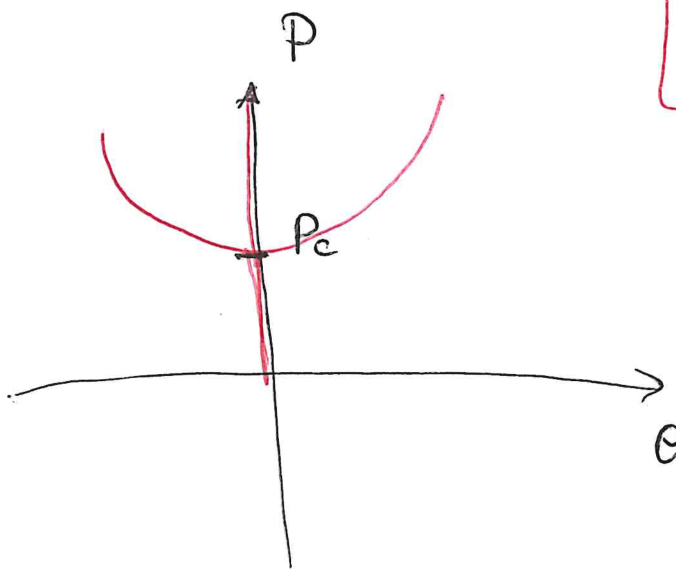
$k 2\theta = P l \sin\theta$ — equilibrium equation

$2k\theta - P l \sin\theta = 0$

1) $\theta = 0$ — trivial solution

2) $\theta \neq 0$ $\quad P = \dfrac{2k\theta}{l\sin\theta}$

$$\boxed{P_c = \frac{2k}{l}}$$

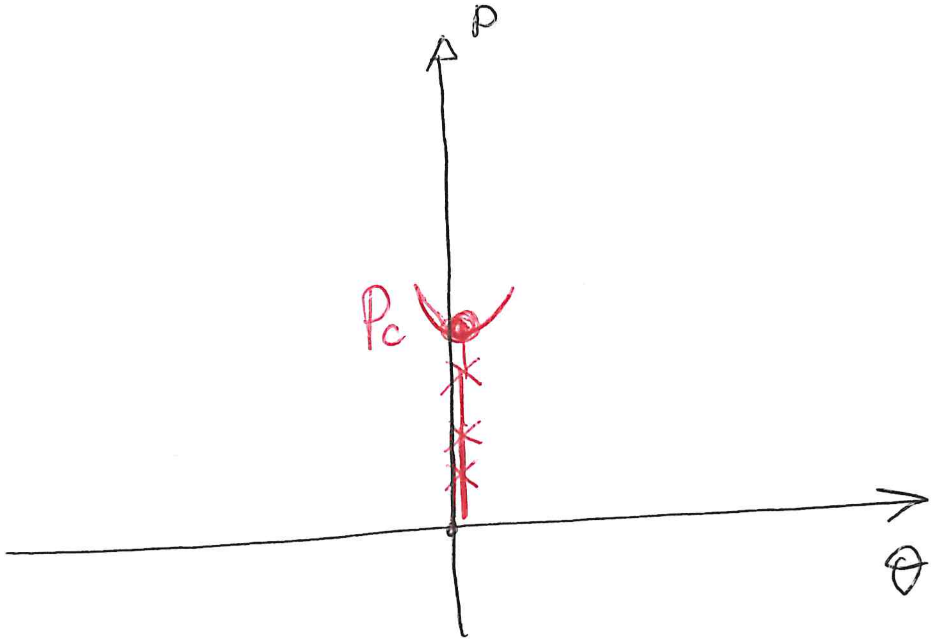


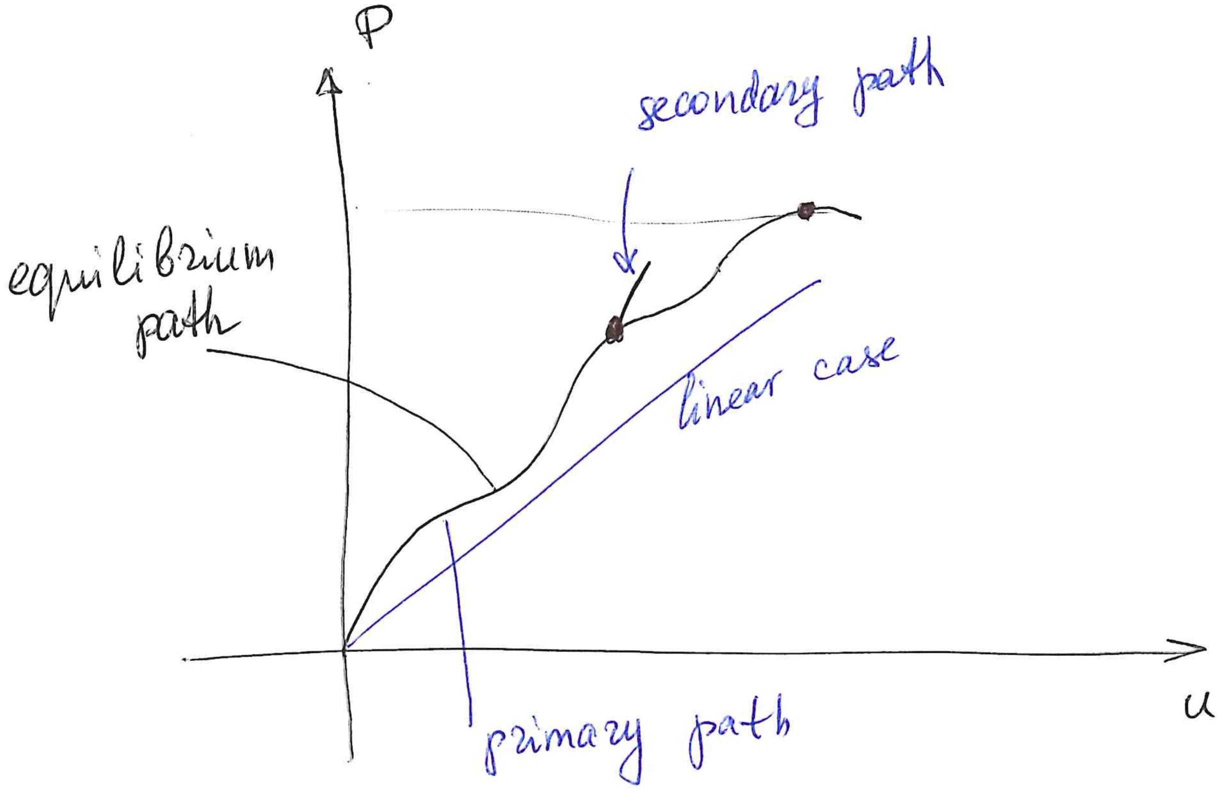
Linearization: $\theta = 0$

$\theta \approx 0$, $\sin\theta = \theta$

$\rightarrow \quad 2k\theta - Pl\theta = \underbrace{(2k - Pl)}_{=0}\theta = 0$

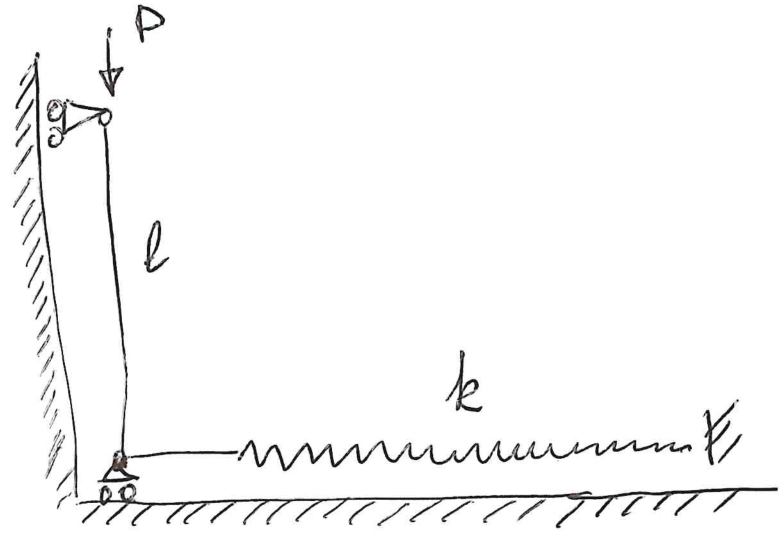
$2k - Pl = 0 \Rightarrow P = P_c = \dfrac{2k}{l}$

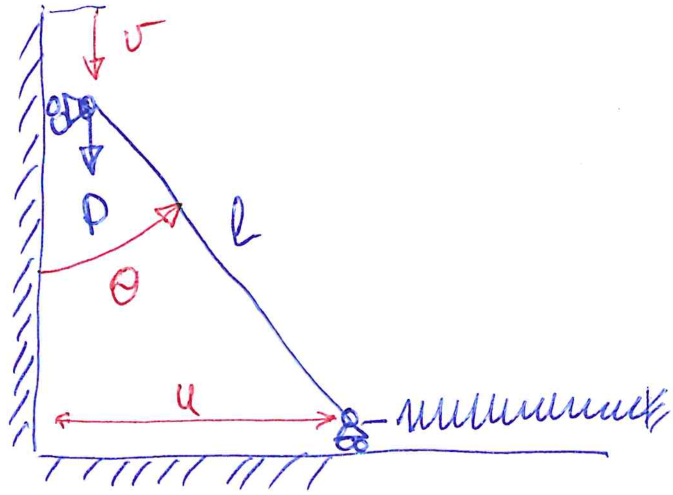




$$F(\theta, P) = 0 \quad : \quad \frac{\partial F}{\partial \theta} = 0 \rightarrow P_c$$

Example 2





$$\begin{cases} v = l - l\cos\theta \\ u = l\sin\theta \end{cases}$$

I. Energetic approach

1) $\Pi = U + V$, $\quad U = \frac{1}{2}ku^2 = \frac{1}{2}l^2k\sin^2\theta$

$$V = -Pv = -Pl(1-\cos\theta)$$

$$\Rightarrow \quad \Pi = \frac{1}{2}kl^2\sin^2\theta - Pl(1-\cos\theta)$$

$$\boxed{0 \le \theta \le \frac{\pi}{2}}$$

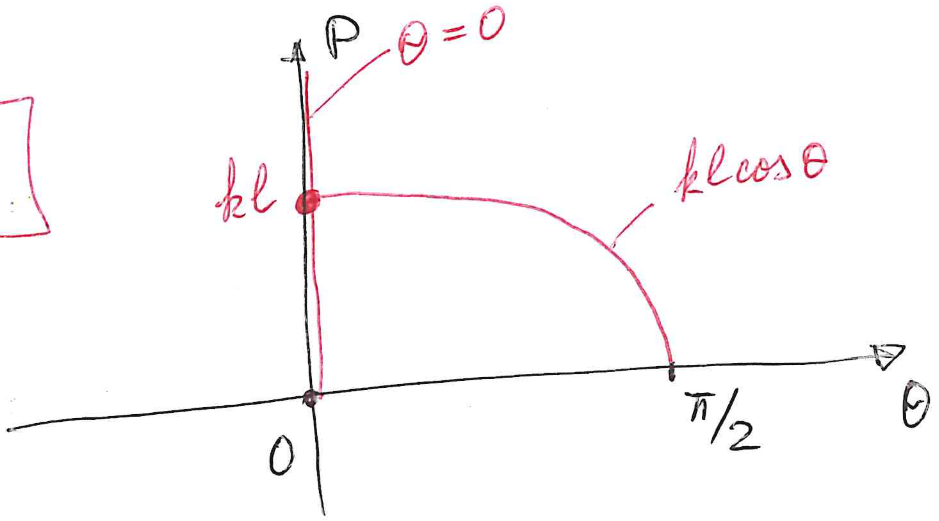
2) $\dfrac{d\Pi}{d\theta} = 0$

$$\frac{d\Pi}{d\theta} = k\ell^2 \sin\theta \cos\theta - P\ell \sin\theta = 0$$

$$\sin\theta \left[k\ell^2 \cos\theta - P\ell \right] = 0$$

$$\Rightarrow \begin{cases} \sin\theta = 0 & \rightarrow \theta = 0 \\ k\ell^2\cos\theta - P\ell = 0 & \rightarrow P = k\ell\cos\theta \end{cases}$$

$\boxed{P_c = k\ell}$



3) $$\frac{d^2\Pi}{d\theta^2} = \frac{d}{d\theta}\left[k\ell^2 \underbrace{\sin\theta\cos\theta}_{\frac{1}{2}\sin 2\theta} - P\ell\sin\theta \right] =$$

$$= \frac{d}{d\theta}\left[\frac{1}{2}k\ell^2 \sin 2\theta - P\ell\sin\theta \right] =$$

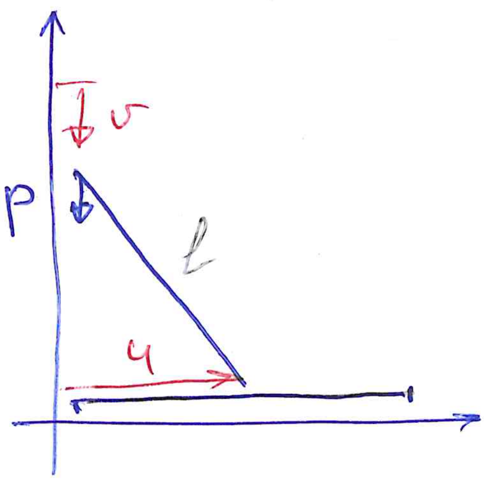
$$= k\ell^2\cos 2\theta - P\ell\cos\theta$$

$\theta = \frac{\pi}{2}$:

$\Pi = U + V$

$V = -P v$

$U = \frac{1}{2} k u^2$



$\theta = \frac{\pi}{2}$: $v = l$, $u = l$

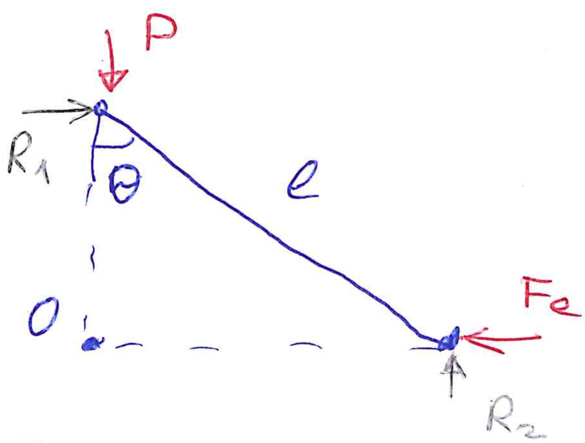
$$\Pi = \frac{1}{2} k l^2 - P l = \text{const.}$$

$$\frac{d\Pi}{d\theta} = \frac{d^2\Pi}{d\theta^2} = ?(0) \quad \forall P$$

Static criterium

$\Sigma F_i = 0$

$\Sigma M_{0i} = 0$



$\rightarrow \quad R_1 = F_e$

$\uparrow \quad P = R_2$

$\curvearrowright \quad R_1 \, l\cos\theta - R_2 \, l\sin\theta = 0$

$F_e \, l\cos\theta - P l \sin\theta = 0$

$F_e = k u = k l \sin\theta$

$\Rightarrow \quad \underline{k l^2 \sin\theta\cos\theta - P l \sin\theta = 0}$ Equilibrium

$$kl \sin\theta \cos\theta - P \sin\theta = 0$$

$$\Rightarrow \begin{cases} \sin\theta = 0 \\ kl\cos\theta - P = 0 \end{cases}$$

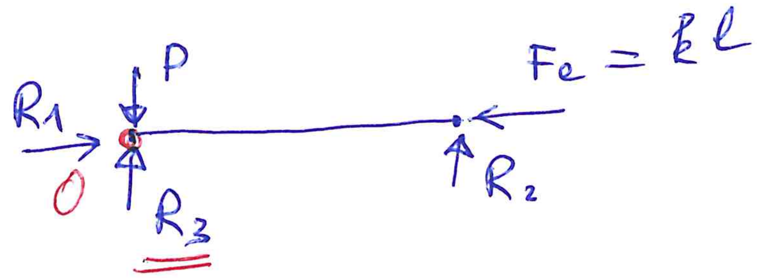
Linearization : $\sim \theta \approx 0$

$$\sin\theta \sim \theta \; , \; \cos\theta = 1$$

$$\rightarrow kl\,\theta \cdot 1 - P\theta = 0$$

$$(kl - P)\,\theta = 0$$

$$\Rightarrow \boxed{P = P_c = kl}$$



$\rightarrow$ $R_1 = F_e$, $\rightarrow$ $R_1 = kl$

$\uparrow$ $R_2 + R_3 = P$ $\rightarrow$ $R_3 = P$

$\curvearrowleft$ $R_2\, l = 0$ $\rightarrow$ $R_2 = 0$

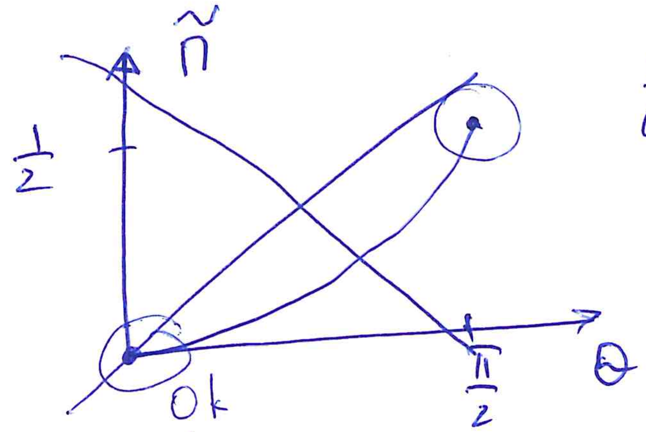
$$\Pi = \frac{1}{2} k l^2 \sin^2\theta - Pl(1-\cos\theta)$$

$$P_c = kl$$

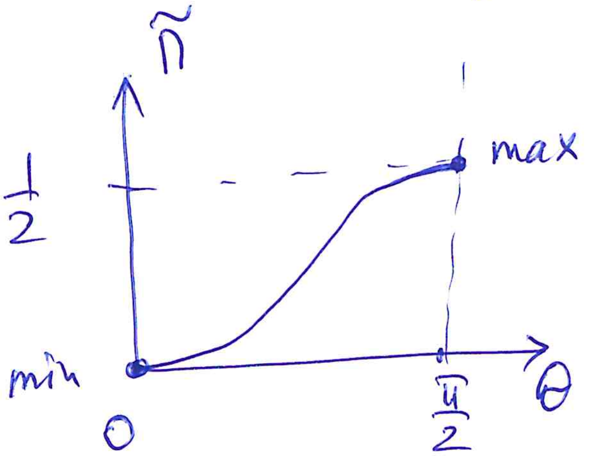
$$\tilde{\Pi} = \frac{1}{kl^2}\Pi = \frac{1}{2}\sin^2\theta - \underbrace{\frac{P}{P_c}}_{\mu}(1-\cos\theta)$$

$$\mu = \frac{P}{P_c} \qquad \mu = 1 \Leftrightarrow P = P_c$$

1) $\tilde{\Pi}$: $\mu = 0$ $\quad \tilde{\Pi} = \frac{1}{2}\sin^2\theta$, $\theta \in [0, \frac{\pi}{2}]$



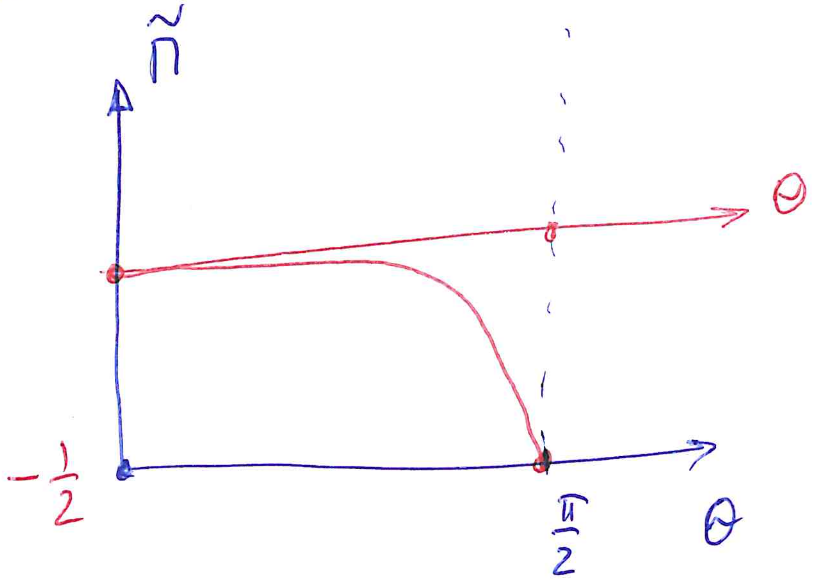
$$\left.\frac{d\tilde{\Pi}}{d\theta}\right|_{\theta=\frac{\pi}{2}} = 0$$



$\mu = 1 \qquad \tilde{\Pi} = \frac{1}{2}\sin^2\theta - 1 + \cos\theta$

$$\frac{d\tilde{\Pi}}{d\theta} = \sin\theta\cos\theta - \sin\theta$$

$$\frac{d^2\tilde{\Pi}}{d\theta^2} = \cos 2\theta - \cos\theta$$



$$\tilde{\Pi}\left(\frac{\pi}{2}\right) = \frac{1}{2} - 1 + 0 = -\frac{1}{2}$$

$$\tilde{\Pi}(0) = 0$$

$$\frac{d\tilde{\Pi}}{d\theta}(0) = 0 \qquad \frac{d\tilde{\Pi}}{d\theta}\left(\frac{\pi}{2}\right) = -1$$

$$\frac{d^2\tilde{\Pi}}{d\theta^2}(0) = 0 \qquad \frac{d^2\tilde{\Pi}}{d\theta^2}\left(\frac{\pi}{2}\right) = -1$$

$\mu > 1$

$$\tilde{\Pi} = \frac{1}{2}\sin^2\theta - \mu(1-\cos\theta)$$

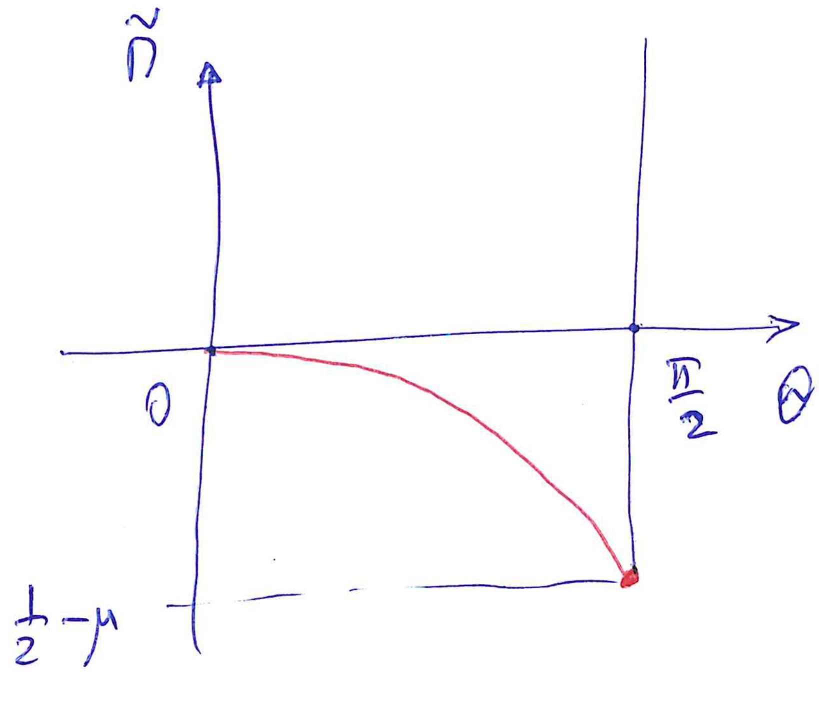
$$\frac{d\tilde{\Pi}}{d\theta} = \sin\theta\cos\theta - \mu\sin\theta$$

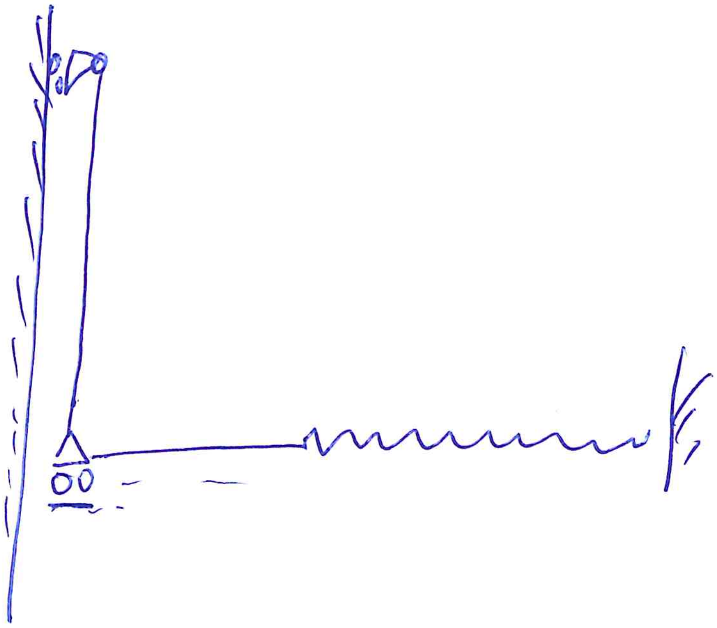
$$\frac{d^2\tilde{\Pi}}{d\theta^2} = \cos 2\theta - \mu\cos\theta$$

$\tilde{\Pi}(0) = 0$, $\tilde{\Pi}\left(\frac{\pi}{2}\right) = \frac{1}{2} - \mu < 0$

$\frac{d\tilde{\Pi}}{d\theta}(0) = 0$ $\frac{d\tilde{\Pi}}{d\theta}\left(\frac{\pi}{2}\right) = -\mu$

$\frac{d^2\tilde{\Pi}}{d\theta^2}(0) = 1-\mu < 0$ $\frac{d^2\tilde{\Pi}}{d\theta^2}\left(\frac{\pi}{2}\right) = -1 \cancel{-\mu} < 0$





Another problem

no obstacle !