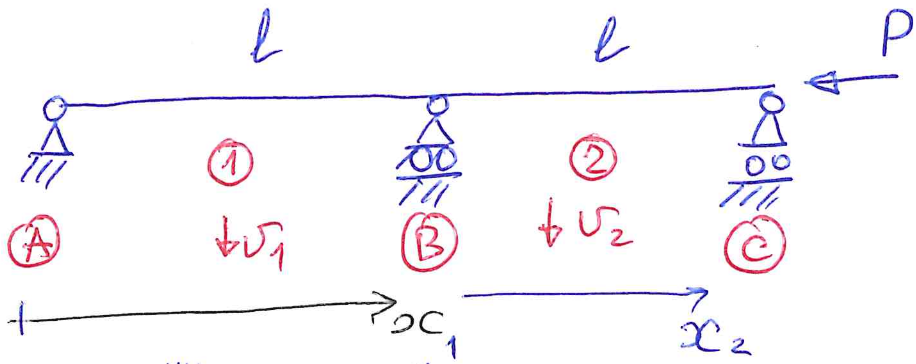
Problema 2



$\boxed{P_c}$ – ?

$$EI v'''' + P v'' = 0$$

A $\quad v = 0, \quad v'' = 0$

C $\quad v = 0, \quad v'' = 0$

B $\quad \underbrace{v_1 = v_2}_{= 0}, \quad v_1' = v_2', \quad v_1'' = v_2''$

$$EI v'''' + P v'' = 0$$

$$v(x) = c_1 \cos \beta x + c_2 \sin \beta x + c_3 x + c_4$$

dove $\beta = \sqrt{\dfrac{P}{EI}}$

$$\begin{cases} v_1(x) = c_1 \cos \beta x_1 + c_2 \sin \beta x_1 + c_3 x_1 + c_4 \\ v_2(x) = c_5 \cos \beta x_2 + c_6 \sin \beta x_2 + c_7 x_2 + c_8 \end{cases}$$

$v_1(0) = 0 \Rightarrow \quad \underline{c_1 + c_4 = 0} \qquad \underline{c_4 = 0}$

$v_1''(0) = 0 \Rightarrow \quad \underline{-c_1\beta^2 = 0} \qquad \underline{c_1 = 0}$

$v_1' = -c_1\beta\sin\beta x_1 + c_2\beta\cos\beta x_1 + c_3$

$v_1'' = -c_1\beta^2\cos\beta x_1 - c_2\beta^2\sin x_1$

$v_1(l) = 0 \Rightarrow \boxed{c_2\sin\beta l + c_3 l = 0}$

$\underline{v_1(x_1) = c_2\sin\beta x_1 + c_3 x_1}$

$v_2(x) = c_5\cos\beta x_2 + c_6\sin\beta x_2 + c_7 x_2 + c_8$

$v_2'(x) = -c_5\beta\sin\beta x_2 + c_6\beta\cos\beta x_2 + c_7$

$v_2''(x) = -c_5\beta^2\cos\beta x_2 - c_6\beta^2\sin\beta x_2$

$v_2(0) = 0 \qquad \underline{c_5 + c_8 = 0}$

$v_2(l) = 0 \qquad \underline{c_5\cos\beta l + c_6\sin\beta l + c_7 l + c_8 = 0}$

$v_2''(l) = 0 \qquad \underline{-c_5\beta^2\cos\beta l - c_6\beta^2\sin\beta l = 0}$

$v_2'(0) = \cancel{0} \quad \underline{c_6\beta + c_7 \cancel{= 0}}$

$$c_2 \sin\beta l + c_3 l = 0$$

$$c_5 + c_8 = 0$$

$$c_5 \cos\beta l + c_6 \sin\beta l + c_7 l + c_8 = 0$$

$$-c_5 \beta^2 \cos\beta l - c_6 \beta^2 \sin\beta l = 0$$

$v_1'(l) = v_2'(0) \Rightarrow$

$$\beta c_2 \cos\beta l + c_3 = c_6 \beta + c_7$$

$c_2, c_3, c_5, c_6, c_7, c_8$

$v_1''(l) = v_2''(0)$

$$-\overset{=0}{c_1} \beta^2 \cos\beta l - c_2 \beta^2 \sin\beta l =$$

$$= -c_5 \beta^2$$

$$C_8 = -C_5$$

$$C_5 \cos\beta l + C_6 \sin\beta l + C_7 l - C_5 = 0$$

$$C_3 = -\, C_2 \frac{\sin\beta l}{l}$$

$$\Rightarrow \begin{cases} C_5 \cos\beta l + C_6 \sin\beta l + C_7 l - C_5 = 0 \\ -C_5 \beta^2 \cos\beta l - C_6 \beta^2 \sin\beta l = 0 \\ \beta C_2 \cos\beta l + \left(-C_2 \dfrac{\sin\beta l}{l}\right) = C_6 \beta + C_7 \\ -C_2 \beta^2 \sin\beta l = -C_5 \beta^2 \end{cases}$$

$$C_2,\; C_5,\; C_6,\; C_7$$

$$\begin{vmatrix} 0 & \cos\beta l - 1 & \sin\beta l & l \\ 0 & \beta^2 \cos\beta l & \beta^2 \sin\beta l & 0 \\ \beta\cos\beta l - \dfrac{\sin\beta l}{l} & 0 & -\beta & -1 \\ \sin\beta l & 1 & 0 & 0 \end{vmatrix} = 0$$

$$\begin{array}{cccc} C_2 & C_5 & C_6 & C_7 \end{array}$$

$$\begin{vmatrix} -\beta\cos\beta l + \dfrac{\sin\beta l}{l} & 0 & \beta & 1 \\ -\sin\beta l & 1 & 0 & 0 \\ 0 & \cos\beta l - 1 & \sin\beta l & l \\ 0 & \cos\beta l & \sin\beta l & 0 \end{vmatrix} =$$

$$= \left(-\beta\cos\beta l + \frac{\sin\beta l}{l}\right)\left(-l\sin\beta l\right) + \sin\beta l \times$$

$$\times \begin{vmatrix} 0 & \beta & 1 \\ \cos\beta l - 1 & \sin\beta l & l \\ \cos\beta l & \sin\beta l & 0 \end{vmatrix} =$$

$$= \left(\beta\cos\beta l - \frac{\sin\beta l}{l}\right) l\sin\beta l +$$

$$+ \sin\beta l \left[\beta l\cos\beta l + (\cos\beta l - 1)\sin\beta l - \sin\beta l\cos\beta l\right] =$$

$$= \left(\beta\cos\beta l - \frac{\sin\beta l}{l}\right) l\sin\beta l +$$

$$+ \sin\beta l \left[\beta l\cos\beta l - \sin\beta l\right] =$$

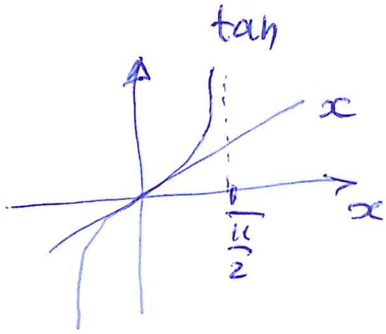
$$= (\beta l \cos\beta l - \sin\beta l)\sin\beta l + \sin\beta l(\beta l\cos\beta l - \sin\beta l) =$$

$$= 2\sin\beta l\,(\beta l\cos\beta l - \sin\beta l)$$

$$\Rightarrow \underline{2\sin\beta l\,(\beta l\cos\beta l - \sin\beta l) = 0}$$

$$\Rightarrow \sin\beta l = 0$$

$$\beta l = \tan\beta l$$



$$\beta l = \pi$$

$$\beta = \frac{\pi}{l}$$

$$\sqrt{\frac{P}{EI}} = \frac{\pi}{l} \Rightarrow \boxed{P = \frac{\pi^2 EI}{l^2}}$$

P_c