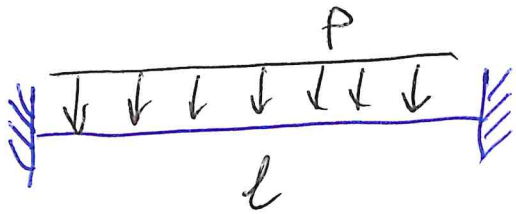


il collasso plastico



$$EI, M_0$$

$$\underline{M(x) - ?}$$

$$|M| \leq M_0$$

l'equilibrio:

$$EI v^{(4)} = P \quad \underline{v(x) - ?} \quad \downarrow v$$

$$EI v^{(3)} = Px + C_1$$

$$EI v'' = P \frac{x^2}{2} + C_1 x + C_2$$

$$EI v' = P \frac{x^3}{6} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EI v = P \frac{x^4}{24} + \frac{C_1 x^3}{6} + C_2 \frac{x^2}{2} + C_3 x + C_4$$

le condizioni al contorno:

$$v(0) = 0, v(l) = 0$$

$$v'(0) = 0, v'(l) = 0$$

$$v(0) = 0 \Rightarrow \boxed{C_4 = 0}$$

$$v'(0) = 0 \Rightarrow \boxed{C_3 = 0}$$

$$EIv = p \frac{x^4}{24} + c_1 \frac{x^3}{6} + c_2 \frac{x^2}{2}$$

$$EIv' = p \frac{x^3}{6} + c_1 \frac{x^2}{2} + c_2 x$$

$$v(l) = 0 \Rightarrow \left. \begin{aligned} \frac{pl^4}{24} + \underline{c_1} \frac{l^3}{6} + \underline{c_2} \frac{l^2}{2} = 0 \end{aligned} \right\}$$

$$v'(l) = 0 \Rightarrow \left. \begin{aligned} \frac{pl^3}{6} + \underline{c_1} \frac{l^2}{2} + \underline{c_2} l = 0 \end{aligned} \right\}$$

$$\begin{cases} \frac{pl^2}{12} + c_1 \frac{l}{3} + c_2 = 0 \rightarrow c_2 = -c_1 \frac{l}{3} - p \frac{l^2}{12} \\ \frac{pl^2}{6} + c_1 \frac{l}{2} + c_2 = 0 \end{cases}$$

$$\downarrow$$
$$\frac{pl^2}{6} + c_1 \frac{l}{2} - c_1 \frac{l}{3} - p \frac{l^2}{12} = 0$$

$$c_1 \left(\frac{1}{2} - \frac{1}{3} \right) + pl \left(\frac{1}{6} - \frac{1}{12} \right) = 0$$

$$c_1 \frac{1}{6} = -\frac{1}{12} pl \Rightarrow \boxed{c_1 = -\frac{1}{2} pl}$$

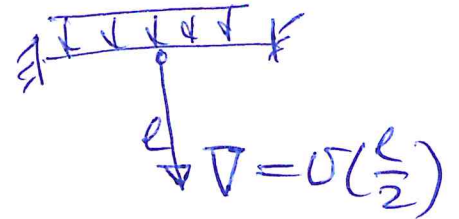
$$c_2 = -\frac{l}{3} \left(-\frac{1}{2} pl \right) - p \frac{l^2}{12} = \frac{pl^2}{6} - \frac{pl^2}{12}$$

$$\boxed{c_2 = \frac{1}{12} pl^2}$$

$$EI v(x) = \frac{px^4}{24} - \frac{1}{2}pl \cdot \frac{x^3}{6} + \frac{1}{12}pl^2 \frac{x^2}{2}$$

$$EI v(x) = \frac{px^4}{24} - \frac{pl}{12}x^3 + \frac{pl^2}{24}x^2$$

$$V = v\left(\frac{l}{2}\right)$$



$$EIV = p \frac{l^4}{24 \cdot 24} - \frac{pl l^3}{12 \cdot 8} + \frac{pl^2 l^2}{24 \cdot 4}$$

$$EIV = pl^4 \left[\frac{1}{16 \cdot 24} - \frac{1}{12 \cdot 8} + \frac{1}{24 \cdot 4} \right] =$$

$$= \frac{pl^4}{8} \left[\frac{1}{24 \cdot 2} - \frac{1}{12} + \frac{1}{3 \cdot 4} \right] =$$

$$= \frac{pl^4}{8 \cdot 24 \cdot 2} = \frac{pl^4}{384}$$

$$V \equiv v\left(\frac{l}{2}\right) = \frac{pl^4}{384 EI}$$

per la curva \$p-v\$.

$M(x) = ?$

$$M(x) = -EI v''(x)$$

$$EI v(x) = \frac{px^4}{24} - \frac{pl}{12}x^3 + \frac{pl^2}{24}x^2$$

$$EI v'(x) = \frac{px^3}{6} - \frac{pl}{4}x^2 + \frac{pl^2}{12}x$$

$$\underline{EI v''(x)} = \frac{px^2}{2} - \frac{1}{2}plx + \frac{pl^2}{12}$$

$M(0), M(l), M(l/2)$

$$M(0) = -EI v''(0) = -\frac{pl^2}{12}$$

$$M(l) = -EI v''(l) = -\frac{pl^2}{2} + \frac{1}{2}pl^2 - \frac{pl^2}{12}$$

$$M(l) = -\frac{pl^2}{12}$$

$$M\left(\frac{l}{2}\right) = -EI v''\left(\frac{l}{2}\right) =$$

$$= -\frac{pl^2}{2 \cdot 4} + \frac{1}{2}pl \cdot \frac{l}{2} - \frac{pl^2}{12} =$$

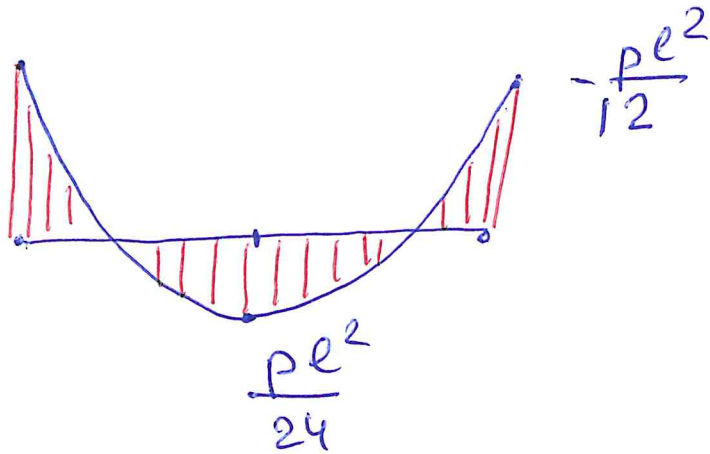
$$= -\frac{pl^2}{8} + \frac{pl^2}{4} - \frac{pl^2}{12} =$$

$$= pl^2 \left[\underbrace{-\frac{1}{8} + \frac{1}{4} - \frac{1}{12}}_{1/24} \right] = \frac{pl^2}{24}$$

(9)

Diagramma dei momenti

$M(x)$



$$M_{\max} = \frac{pe^2}{12}$$

$$|M(x)| \leq M_{\max} \leq M_0 \quad (\text{il momento limite})$$

$P_e = ?$ $0 < P < P_e$ lo stato elastico

$$M_0 = \frac{pe^2}{12} \Rightarrow \boxed{P_e = \frac{12 M_0}{e^2}}$$

$$V_e = \frac{pe^4}{384 EI} = \frac{12 M_0}{e^2} \frac{e^4}{384 EI} = \frac{1}{32} \frac{M_0 e^2}{EI}$$

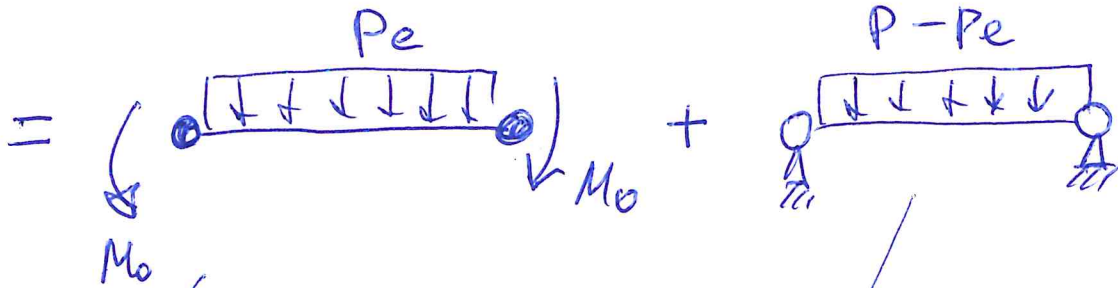
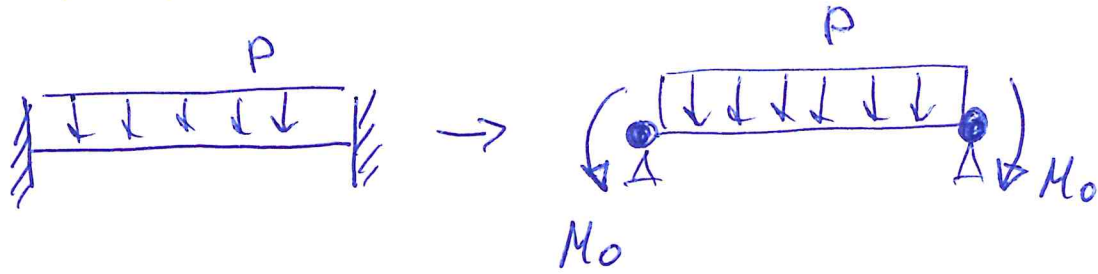
$$\boxed{P_e = 12 \frac{M_0}{e^2}, \quad V_e = \frac{1}{32} \frac{M_0 e^2}{EI}}$$

Stato elastico ↑

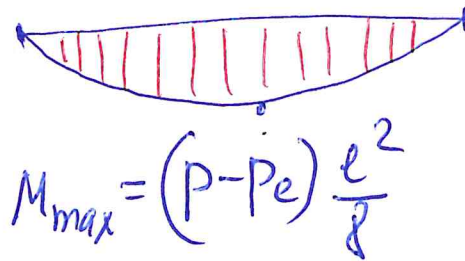
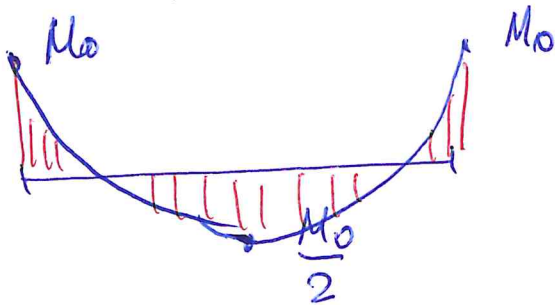
⑤

$$P > P_e$$

Stato elasto-plastico

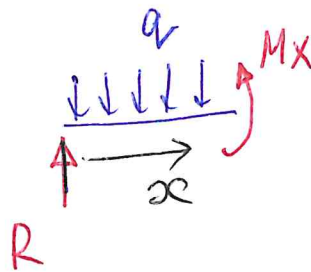
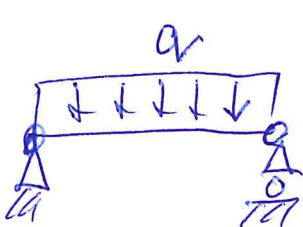


M(x) - ?



$$M_{max} = (P - P_e) \frac{l^2}{8}$$

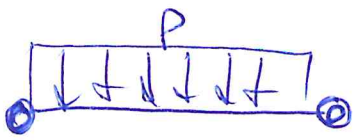
$$P = P_e$$



$$R = \frac{ql}{2}$$

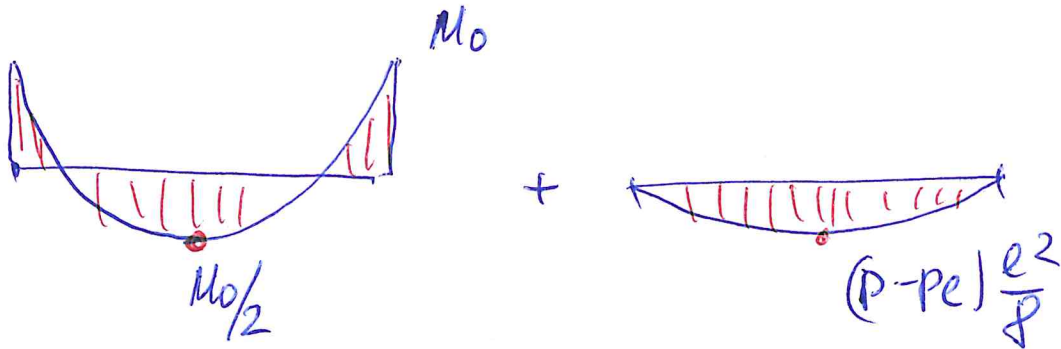
$$M(x) = Rx - q \frac{x^2}{2} = x \frac{ql}{2} - q \frac{x^2}{2}$$

$$M(x) = q \frac{x}{2} (l - x), \quad M_{max} = M\left(\frac{l}{2}\right) = \frac{pl^2}{8}$$



$$\underline{P > P_e}$$

$M(x)$:



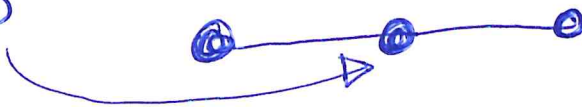
$$\underline{\Delta P = P - P_e}$$

$$\frac{M_0}{2} + \Delta P \frac{l^2}{8} = M_0$$

$$\Delta P \frac{l^2}{8} = \frac{M_0}{2} \Rightarrow$$

$$\Delta P = \frac{4 M_0}{l^2}$$

per $P = P_e + \Delta P$



$$P_e = 12 \frac{M_0}{l^2}$$

$$P_c = 12 \frac{M_0}{l^2} + 4 \frac{M_0}{l^2},$$

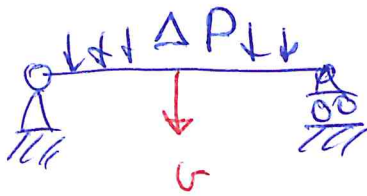
$$\boxed{P_c = 16 \frac{M_0}{l^2}}$$

Il collasso plastico



$P > P_c \rightarrow$ collasso

Per la curva $p-V$, v -?



$$\underline{M(x) = \Delta P \frac{x}{2} (l-x)}$$

$$EI v'' = -M(x) = \Delta P \frac{x}{2} (x-l)$$

$$v(0) = 0, v(l) = 0$$

$$EI v' = -\Delta P \frac{l}{2} \frac{x^2}{2} + \Delta P \frac{x^3}{6} + C_1$$

$$EI v = -\Delta P \frac{l}{2} \frac{x^3}{6} + \Delta P \frac{x^4}{24} + C_1 x + C_2$$

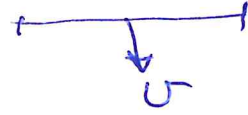
$$v(0) = 0 \Rightarrow \boxed{C_2 = 0}$$

$$v(l) = 0 \quad -\Delta P \frac{l^4}{12} + \Delta P \frac{l^4}{24} + C_1 l = 0$$

$$\boxed{C_1 = \frac{\Delta P l^3}{24}}$$

$$EI v(x) = -\Delta P \frac{e x^3}{12} + \Delta P \frac{x^4}{24} + \frac{\Delta P e^3}{24} x$$

$$v\left(\frac{e}{2}\right) = ?$$



$$\begin{aligned} EI v\left(\frac{e}{2}\right) &= \Delta P \left[-\frac{e}{12} \frac{e^3}{8} + \frac{e^4}{24 \cdot 16} + \frac{e^3 \cdot e}{24 \cdot 2} \right] \\ &= \Delta P e^4 \left[-\frac{1}{96} + \frac{1}{384} + \frac{1}{384} \right] = \\ &= \Delta P e^4 \left[\frac{1}{96} + \frac{1}{384} \right] = \Delta P e^4 \left[\frac{4+1}{384} \right] \end{aligned}$$

$$v\left(\frac{e}{2}\right) = \frac{\Delta P e^4}{EI} \frac{5}{384}$$

$$V_c = V_e + v\left(\frac{e}{2}\right)$$

$$v\left(\frac{e}{2}\right) = \frac{4 M_0}{e^2} \frac{e^4}{EI} \frac{5}{384}$$

$$v\left(\frac{e}{2}\right) = \frac{5}{96} \frac{M_0 e^2}{EI}$$

$$V_c = \frac{1}{32} \frac{M_0 e^2}{EI} + \frac{5}{96} \frac{M_0 e^2}{EI} = \frac{1}{12} \frac{M_0 e^2}{EI}$$

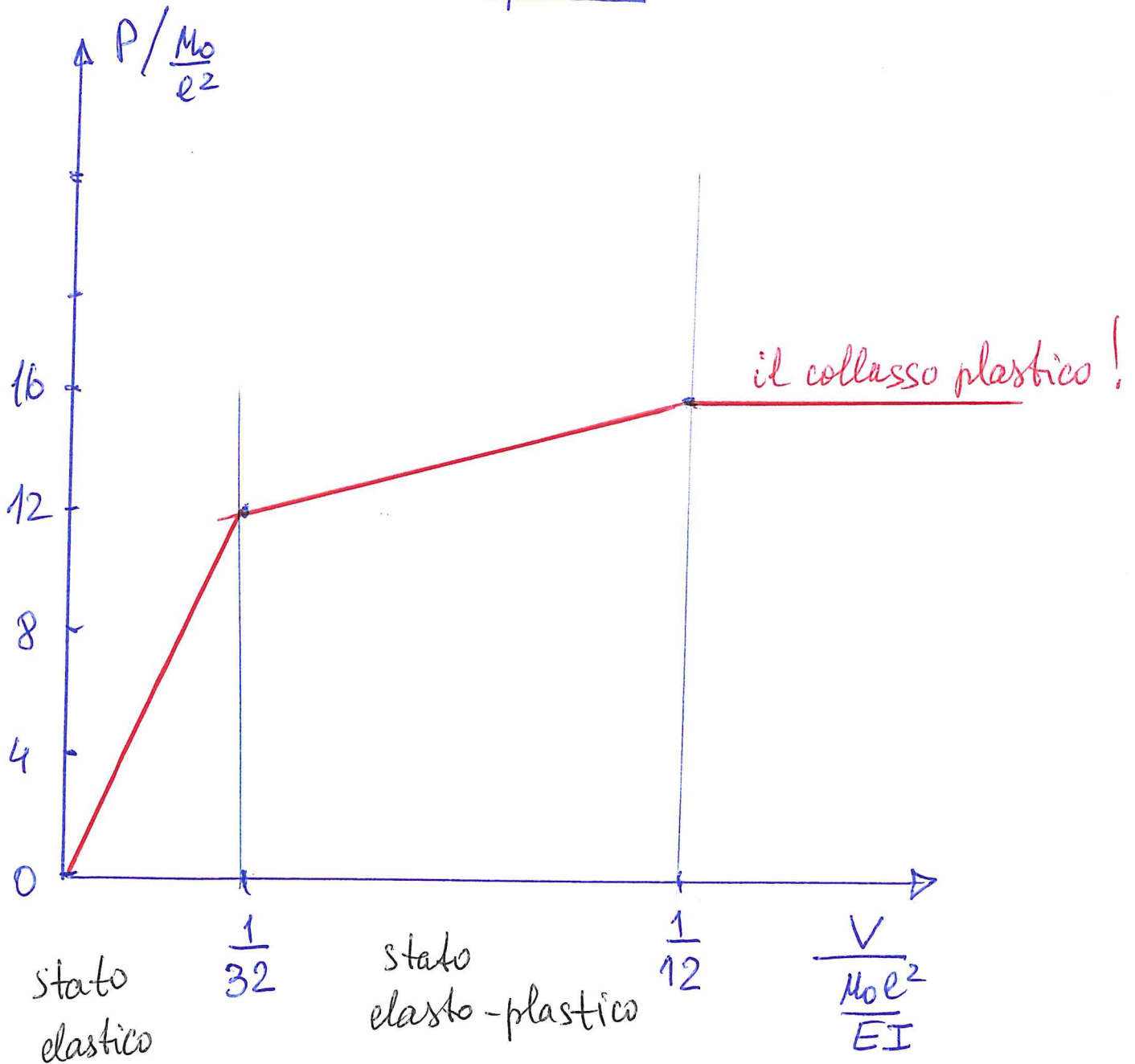
$$V_e = \frac{1}{32} \frac{M_0 e^2}{EI}$$

$$\Delta P_c = \frac{4 M_0}{e^2}$$

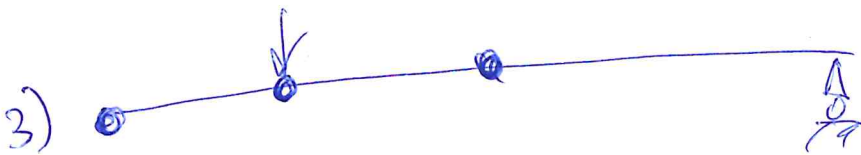
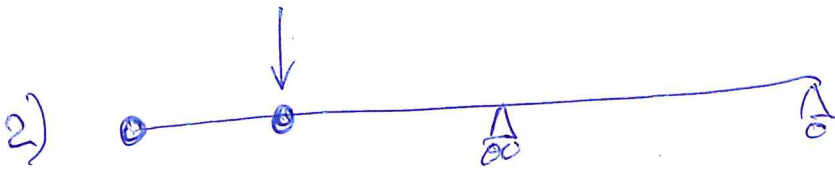
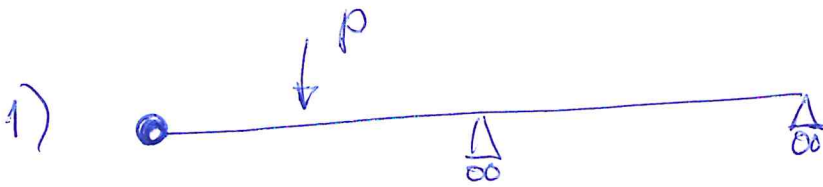
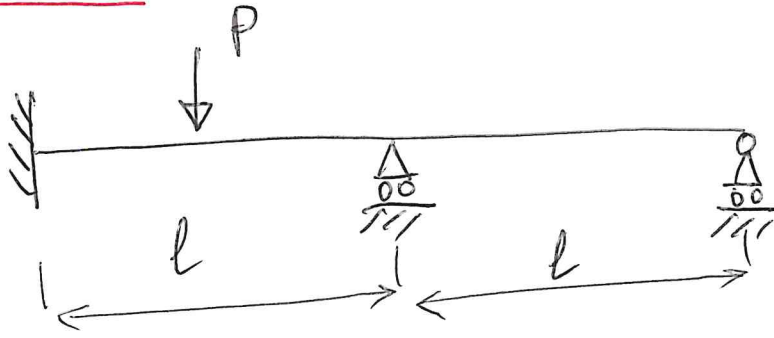
$$P_e = 12 \frac{M_0}{l^2}, \quad V_e = \frac{1}{32} \frac{M_0 l^2}{EI}$$

$$P_c = 16 \frac{M_0}{l^2}, \quad V_c = \frac{1}{12} \frac{M_0 l^2}{EI}$$

La curva p-V



Esempio



il collasso
plastico