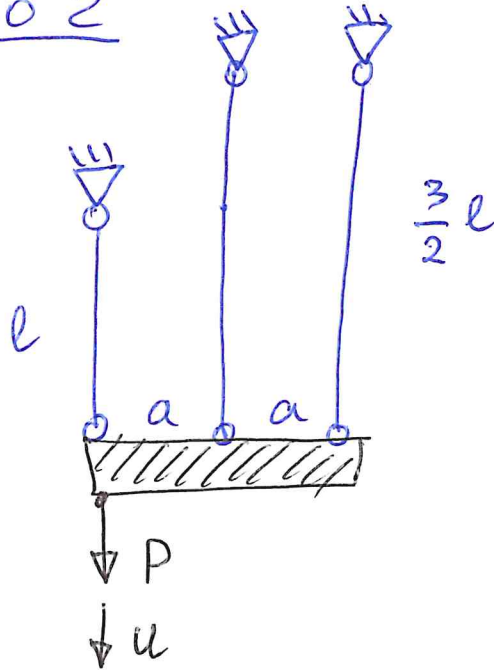
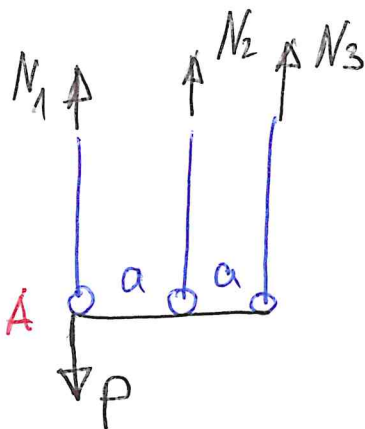


Esempio 2

EA
 σ_s



$P-u$?



d'equilibrio:

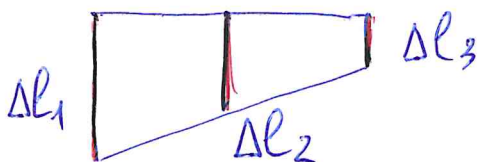
$$P = N_1 + N_2 + N_3$$

$$N_2 a + N_3 2a = 0$$

$$\Rightarrow N_2 + 2N_3 = 0$$

$$\Rightarrow \begin{cases} N_1 + N_2 + N_3 = P \\ N_2 + 2N_3 = 0 \end{cases}$$

$\Delta l_1, \Delta l_2, \Delta l_3$



$$\Rightarrow \Delta l_2 = \frac{1}{2} (\Delta l_1 + \Delta l_3)$$

$$\underline{\underline{\Delta l_1 - 2\Delta l_2 + \Delta l_3 = 0}}$$

$$\begin{cases} N_1 + N_2 + N_3 = P \\ N_2 + 2N_3 = 0 \\ \Delta l_1 - 2\Delta l_2 + \Delta l_3 = 0 \end{cases}$$

$$\Delta l_{1,2,3} \rightarrow N_{1,2,3} \quad ?$$

$$\sigma = E\varepsilon, \quad \varepsilon = \frac{\Delta l}{l} \Rightarrow \Delta l = \frac{\sigma l}{E}$$

$$N = \sigma A \Rightarrow \Delta l = \frac{Nl}{EA}$$

$$\Delta l_1 = \frac{N_1 l}{EA},$$

$$\Delta l_2 = \frac{N_2 \cdot \frac{3}{2} l}{EA} = \frac{3}{2} \frac{N_2 l}{EA}$$

$$\Delta l_3 = \frac{3}{2} \frac{N_3 l}{EA}$$

$$\Delta l_1 - 2\Delta l_2 + \Delta l_3 = 0$$

$$\frac{N_1 l}{EA} - 2 \cdot \frac{3}{2} \frac{N_2 l}{EA} + \frac{3}{2} \frac{N_3 l}{EA} = 0$$

$$\underline{N_1 - 3N_2 + \frac{3}{2}N_3 = 0}$$

$$\begin{cases} N_1 + N_2 + N_3 = P \\ N_2 + 2N_3 = 0 \\ N_1 - 3N_2 + \frac{3}{2}N_3 = 0 \end{cases}$$

$$N_2 = -2N_3$$

$$N_1 = 3N_2 - \frac{3}{2}N_3 = -6N_3 - \frac{3}{2}N_3 = -\frac{15}{2}N_3$$

$$-\frac{15}{2}N_3 - 2N_3 + N_3 = P$$

$$-\left(\frac{15}{2} + 1\right)N_3 = P \Rightarrow N_3 = -\frac{2}{17}P$$

$$N_2 = \frac{4}{17}P$$

$$N_1 = \frac{15}{17}P$$

$$|N_1| \leq \sigma_s A, |N_2| \leq \sigma_s A, |N_3| \leq \sigma_s A$$

$$N_1 = N_{1c} = \sigma_s A$$

$$\Delta l_1 = u$$

$$\Delta l_1 = \frac{N_1 l}{EA} = \frac{15}{17} \frac{Pl}{EA}$$

$$\Delta l_2 = \frac{3}{2} \frac{N_2 l}{EA} = \frac{6}{17} \frac{Pl}{EA}$$

$$\Delta l_3 = \frac{3}{2} \frac{N_3 l}{EA} = -\frac{3}{17} \frac{Pl}{EA}$$

(3)

Stato 1 : limite elastica.

$$N_1 = \underline{N_{1c}} = \sigma_s A$$

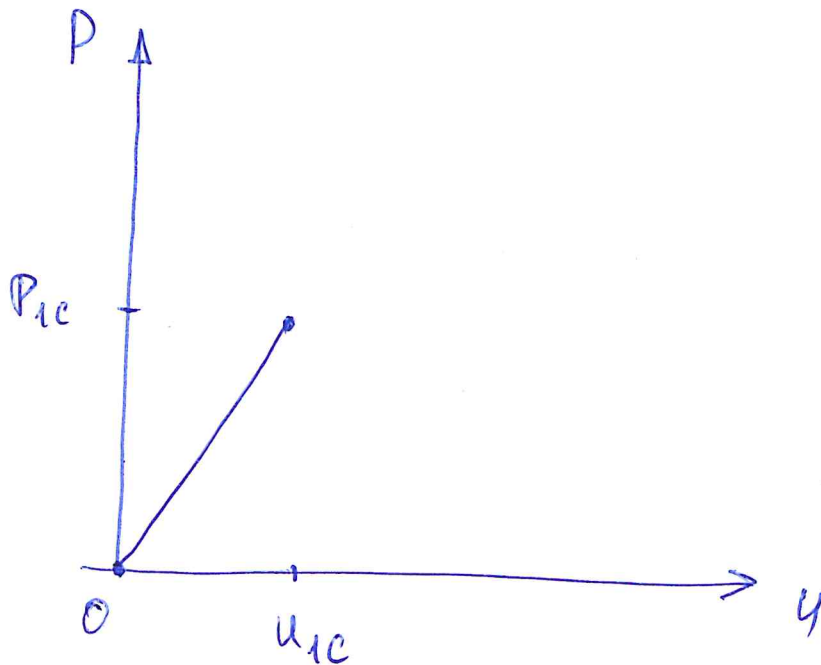
$$N_1 = \frac{15}{17} P$$

$$\Delta l_1 = \frac{15}{17} \frac{P l}{EA}$$

$$\Delta l_1 = \frac{N_1 l}{EA}$$

$$P_{1c} = \frac{17}{15} N_{1c} = \frac{17}{15} \sigma_s A$$

$$u_{1c} = \Delta l_1 = \frac{15}{17} \frac{P_{1c} l}{EA} = \frac{\sigma_s A l}{EA} = \frac{\sigma_s l}{E}$$



Stato 2: risposta elasto-plastica

$$P > P_{1c}$$

$$N_1 = N_{1c} = \sigma_s A.$$

$$N_1 + N_2 + N_3 = P$$

$$N_2 + 2N_3 = 0$$

$$N_2 = -2N_3$$

$$N_1 - 2N_3 + N_3 = P$$

$$N_1 - N_3 = P$$

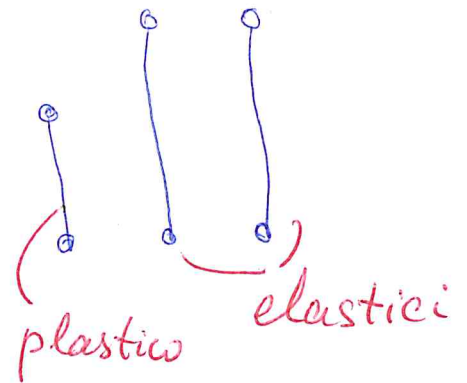
$$N_3 = N_1 - P$$

$$N_2 = -2N_1 + P \cdot 2$$

$$\begin{cases} N_3 = N_{1c} - P \\ N_2 = 2P - 2N_{1c} \\ N_1 = N_{1c} = \sigma_s A \end{cases}$$

$$\Delta l_2 = \frac{N_2}{EA} \frac{3}{2} l, \quad \Delta l_3 = \frac{N_3}{EA} \frac{3}{2} l$$

$$|N_2| < \sigma_s A, \quad |N_3| < \sigma_s A$$

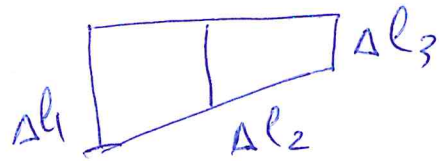


(5)

$$\underline{u = \Delta l_1}$$

$$\cancel{\Delta l_1 \sim N_1} \quad \text{non c'è}$$

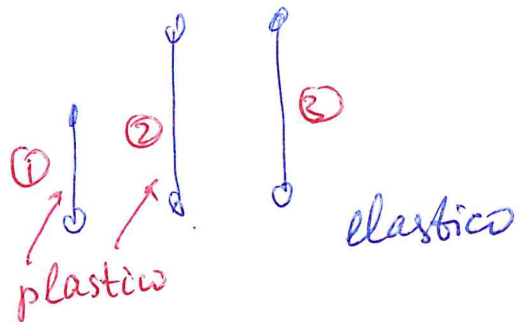
$$\underline{\Delta l_1 = 2\Delta l_2 - \Delta l_3}$$



$$\Delta l_2 = \frac{1}{2}(\Delta l_1 + \Delta l_3)$$

$$\Delta l_1 + \Delta l_3 - 2\Delta l_2 = 0$$

$$\underline{N_2 = \sigma_s A}$$



$$\Delta l_1 = 2\Delta l_2 - \Delta l_3$$

$$\Delta l_2 = \frac{3N_2 l}{2EA} = \frac{3}{2} \frac{\sigma_s A l}{EA} = \frac{3}{2} \frac{\sigma_s l}{E}$$

$$\Delta l_3 = \frac{3}{2} \frac{N_3 l}{EA} \quad ; \quad N_3 = N_{1e} - P \quad \& \quad \Delta l_3 = -\frac{3}{4} \frac{\sigma_s l}{E}$$

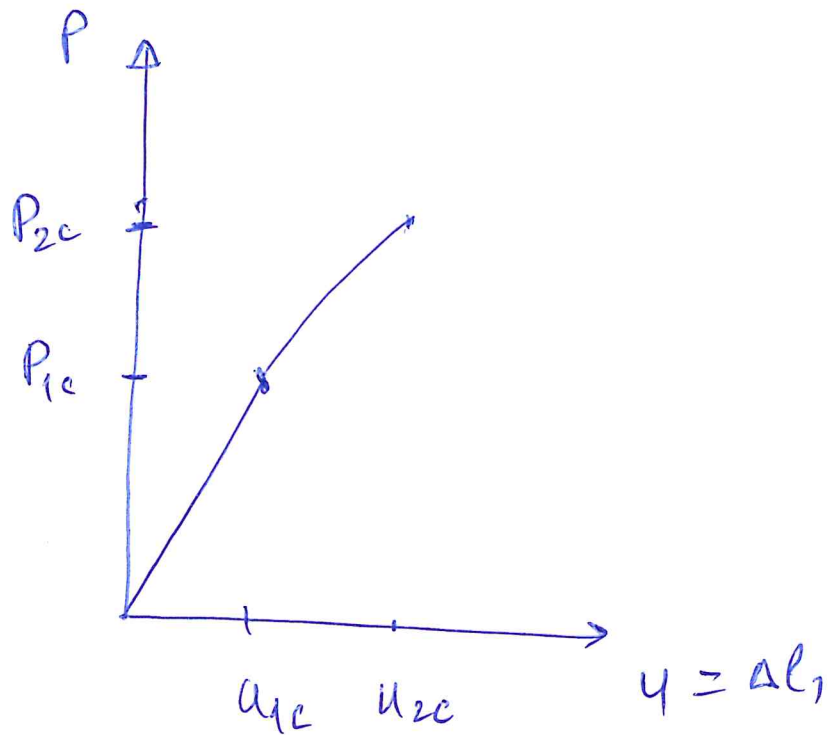
$$\boxed{\Delta l_1 = 3 \frac{\sigma_s l}{E} + \frac{3}{4} \frac{\sigma_s l}{E} = \frac{15}{4} \frac{\sigma_s l}{E}}$$

$$P_{2c} = ?$$

$$N_1 + N_2 + N_3 = P$$

$$\sigma_s A + \sigma_s A + \left(\frac{1}{2} \sigma_s A\right) = P$$

$$P_{2c} = \frac{3}{2} \sigma_s A$$



$$P_{1c} = \frac{17}{15} \sigma_s A$$

Stato 3.

$$\underline{\underline{P > P_{2c}}}$$

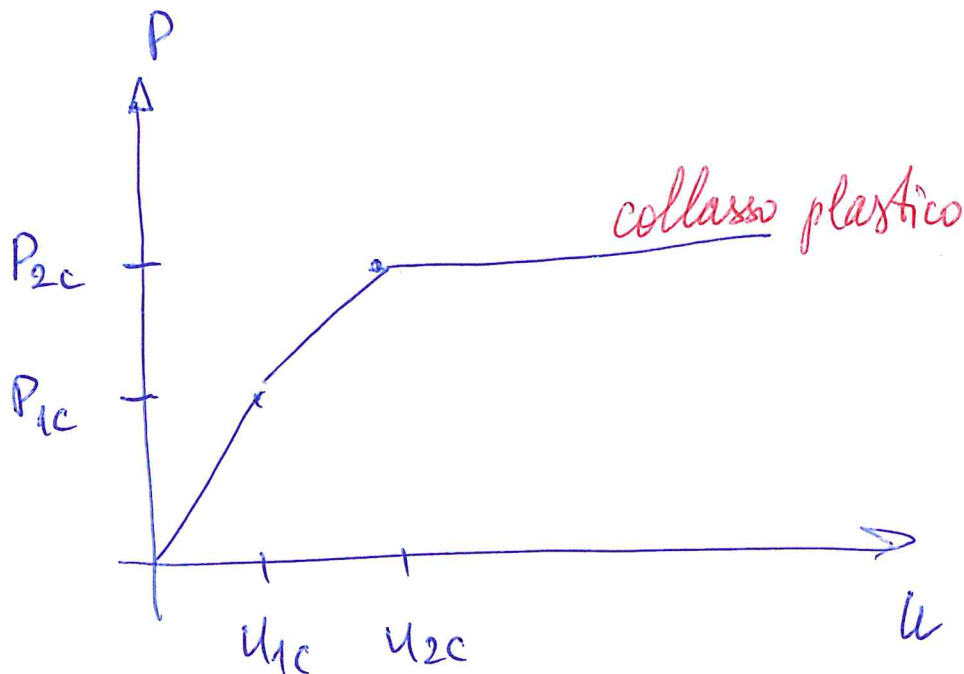
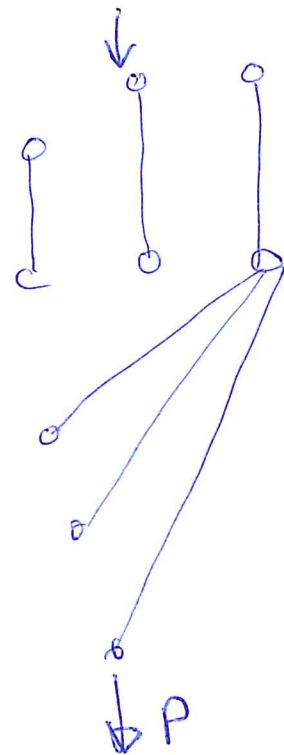
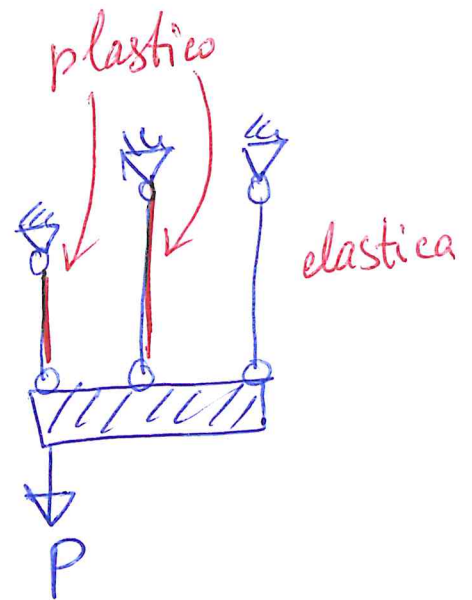
$$N_1 = \sigma_s A, \quad N_2 = \sigma_s A$$

$$\begin{cases} N_1 + N_2 + N_3 = P \\ N_2 + 2N_3 = 0 \end{cases}$$

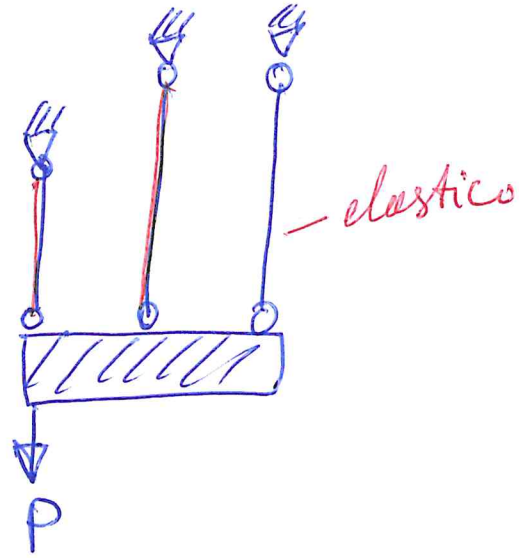
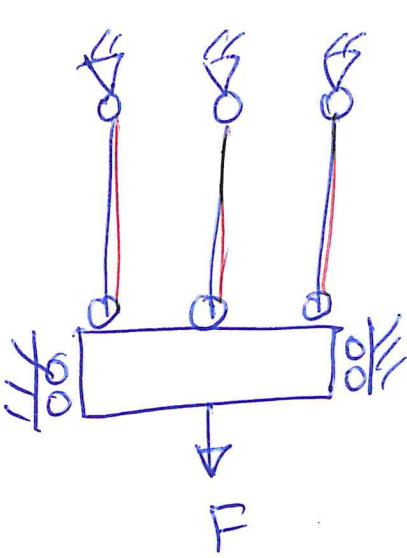
$$\begin{cases} \sigma_s A + \sigma_s A + N_3 = P \\ \sigma_s A + 2N_3 = 0 \end{cases}$$

\Rightarrow no soluzione !!!
non c'è l'equilibrio

il collasso plastico



Differenza:

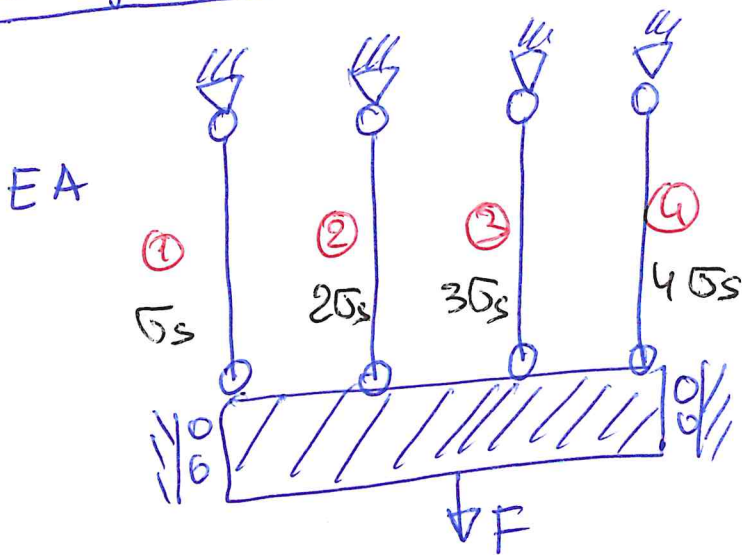


il collasso plastico

tutti
plastici

due plastici

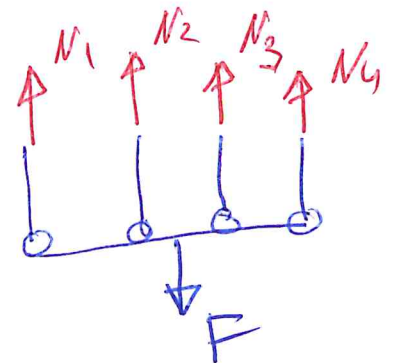
Esempio 3



$$\Delta l_1, \Delta l_2, \Delta l_3, \Delta l_4$$

$$N_1 + N_2 + N_3 + N_4 = F$$

$$\Delta l_1 = \Delta l_2 = \Delta l_3 = \Delta l_4 = \Delta l$$



Stato 1: limite elastica

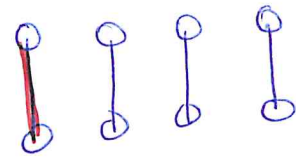
$$N_1 = \sigma_s A, \quad F_{1c} = 4\sigma_s A, \quad \Delta l = \frac{\sigma_s l}{E}$$

Stato 2: $F > F_{1c}$

stato elasto-plastico

$$N_2 = 2\sigma_s A$$

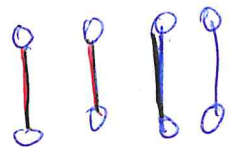
$$F_{2c} = \sigma_s A + 2\sigma_s A + 2\sigma_s A + 2\sigma_s A = 7\sigma_s A$$



Stato 3: elasto-plastico: $F > F_{2c} = 7\frac{\sigma_s A}{E}$

$$N_3 = 3\sigma_s A$$

$$F_{3c} = \underbrace{\sigma_s A}_{N_1} + \underbrace{2\sigma_s A}_{N_2} + \underbrace{3\sigma_s A}_{N_3} + \underbrace{3\sigma_s A}_{N_4}$$



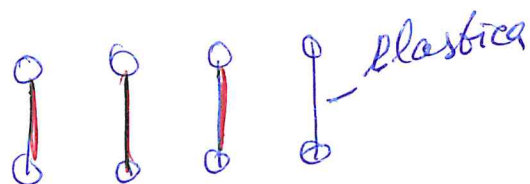
$$F_{3c} = 9\sigma_s A$$

Stato 4 : $F > F_{3c}$

$$F_{4c} = \overset{N_1}{\sigma_s A} + \overset{N_2}{2\sigma_s A} + \overset{N_3}{3\sigma_s A} + \overset{N_4}{4\sigma_s A},$$

$$N_4 = 4\sigma_s A$$

$$\underline{F_{4c} = 10\sigma_s A}$$



Stato 5: il collasso plastico

$$F > F_{4c}$$

$$F_{1c} = 4\sigma_s A$$

$$F_{2c} = 7\sigma_s A$$

$$F_{3c} = 9\sigma_s A$$

$$F_{4c} = 10\sigma_s A$$

