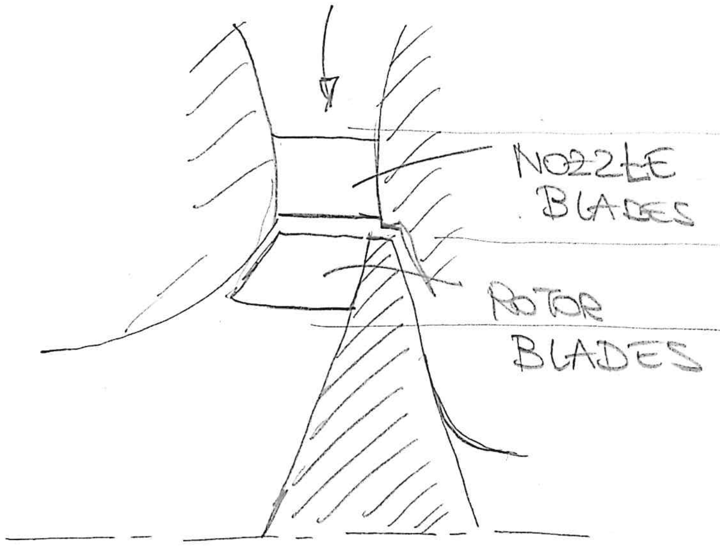
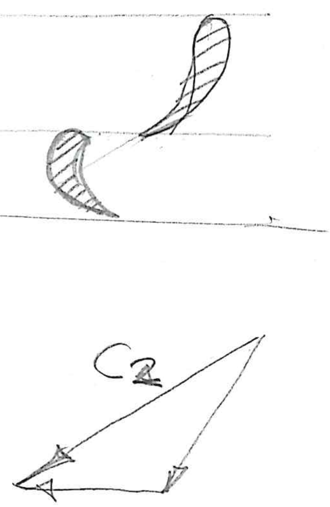
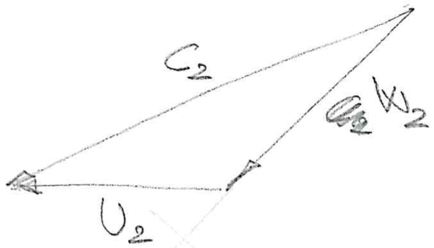
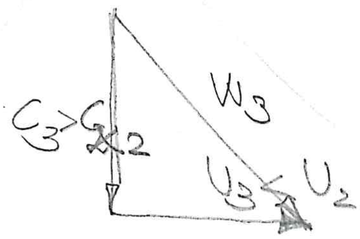
RADIAL TURBINES







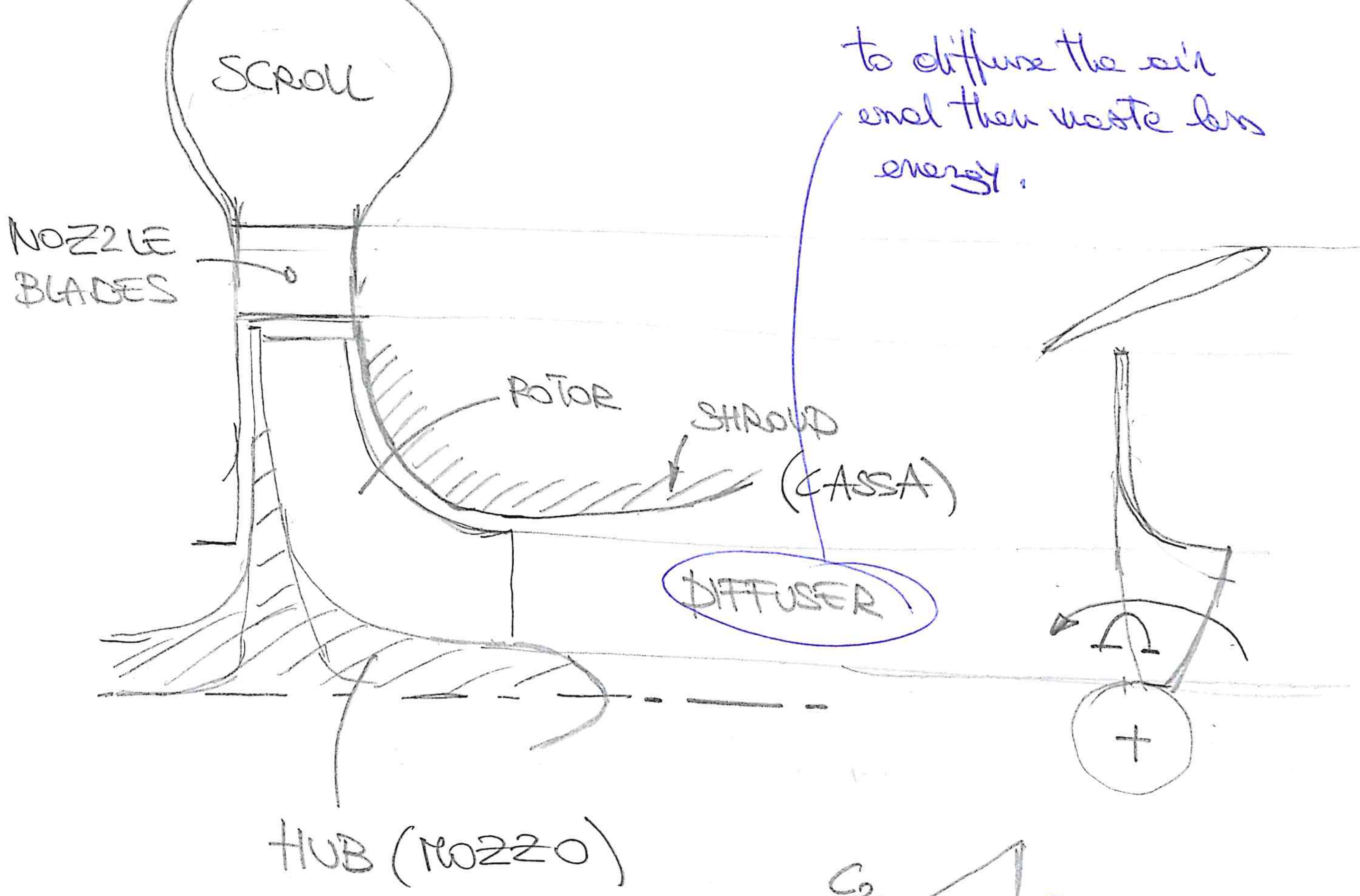


The vast majority of radial turbines are centripetal. In order to produce work, $U_3 c_{\theta 3} < U_2 c_{\theta 2}$

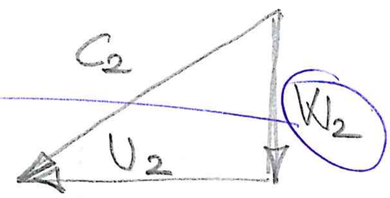
$$w = -\Delta h = U_2 c_{\theta 2} - U_3 c_{\theta 3}.$$

Usually, this is obtained by imparting a large tangential velocity by means of a row of stator blades, and then having no swirl at the exit (exit k. energy as low as possible).

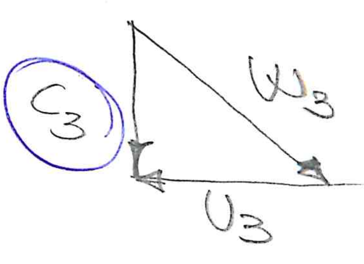
The turbine in the figure is of the CANTILEVER type (i.e. with radial flow both at entry and exit). Cantilever blades have usually low reaction (IMPULSE), as having a reaction blade would require an increase in area, which is difficult to accommodate in a small blade. Usually the ratio $\frac{r_3}{r_2}$ is close to unity, which means that the blading is similar to that of an axial machine.



The relative velocity at inlet is ~~axial~~ radial (mechanical reasons)



The absolute velocity at outlet (exit) is axial. (low exit kin. energy)



In the stator $\qquad h_0 = \text{const} \implies h_1 - h_2 = \frac{1}{2}(c_2^2 - c_1^2)$

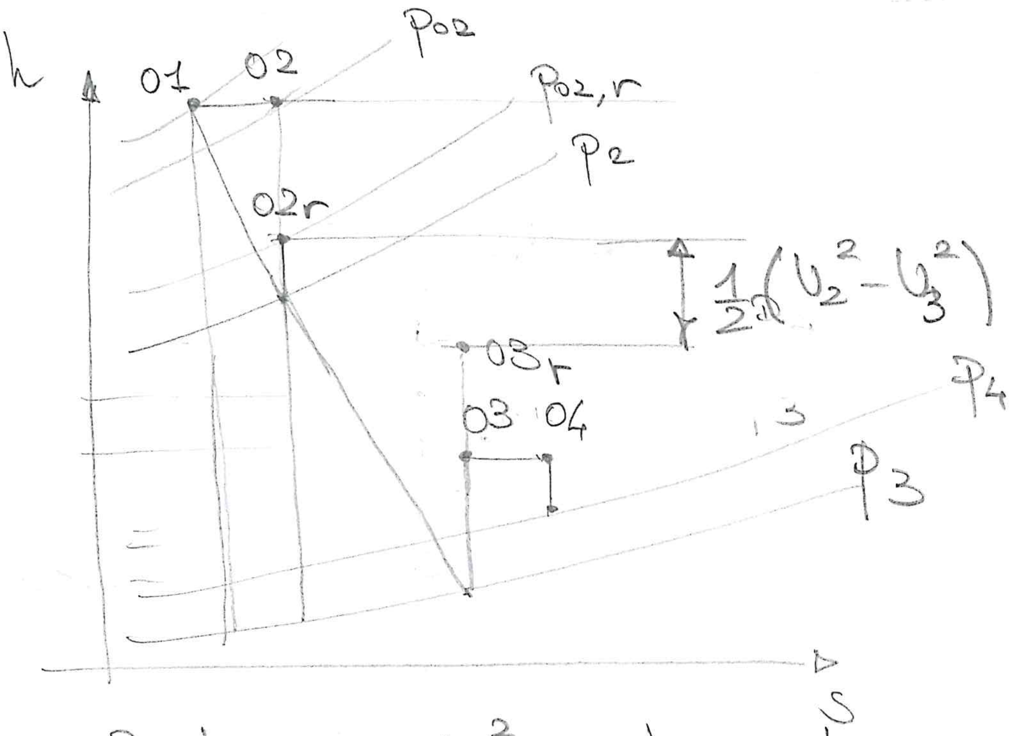
In the rotor $\qquad I = \text{const} \implies h_{02,rel} - \frac{1}{2}U_2^2 = h_{03,rel} - \frac{1}{2}U_3^2$

rothalpy

$$h_2 - h_3 = \frac{1}{2}(w_3^2 - w_2^2) + \frac{U_2^2 - U_3^2}{2} =$$
$$= \frac{1}{2}\left[(U_2^2 - U_3^2) - (w_2^2 - w_3^2)\right]$$

In the diffuser $\qquad h_0 = \text{const} \qquad h_3 - h_4 = \frac{1}{2}(c_4^2 - c_3^2)$

$$w = \frac{\dot{W}}{\dot{m}} = U_2 c_{\theta 2} - U_3 c_{\theta 3} = h_{02} - h_{03} =$$
$$= \frac{1}{2}(c_2^2 - c_3^2) + \frac{1}{2}(U_2^2 - U_3^2) + \frac{1}{2}(w_2^2 - w_3^2)$$

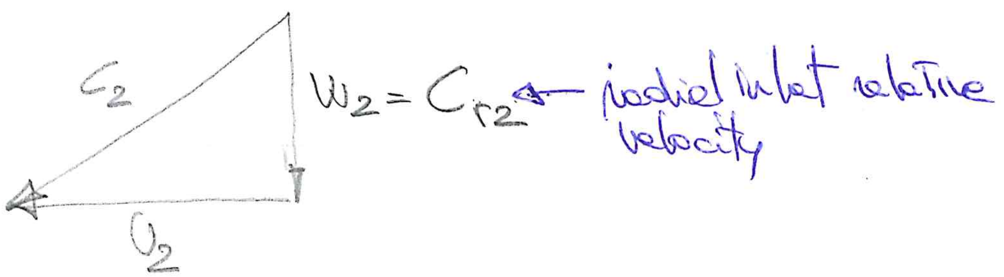


$$h_{02,r} - \frac{1}{2}U_2^2 = h_{03,r} - \frac{1}{3}U_3^2 \implies h_{02,r} = h_{03,r} + \frac{1}{2}(U_2^2 - U_1^2)$$

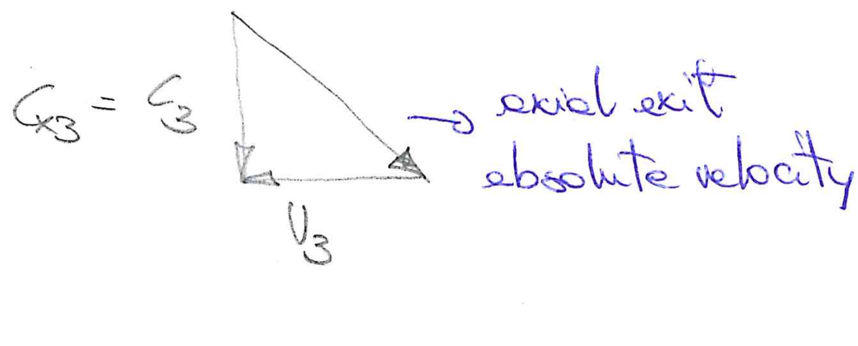
· A significant contribution to the specific work comes from the term $\frac{1}{2}(U_2^2 - U_3^2)$; this is the main reason for the better performance of radial turbines in terms of their specific work.

- The term $\frac{1}{2}(w_3^2 - w_2^2)$ gives a positive contribution when the exit (relative) velocity is higher than the inlet velocity, i.e. when the flow is accelerating.
- The third term $\frac{1}{2}(c_2^2 - c_3^2)$ indicates that the absolute inlet velocity should be larger than the exit velocity.

NOMINAL DESIGN



$$\Longrightarrow w = -\Delta(U c_\theta) = U_2^2$$



SPOUTING VELOCITY

It is defined as the velocity that has an associated kinetic energy equal to the isentropic enthalpy drop from turbine inlet stagnation conditions to final exhaust conditions :

$$\frac{1}{2} C_0^2 = h_{0\cancel{1}} - h_{3ss}$$

Hence

$$\frac{1}{2} C_0^2 = U_2^2$$

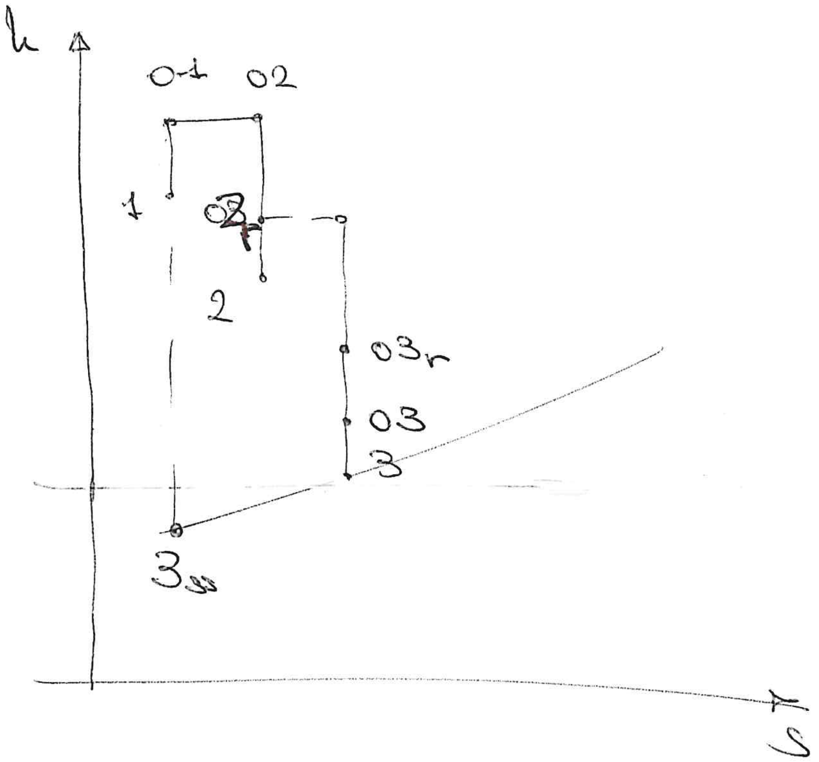
$$\Rightarrow U = \frac{\sqrt{2}}{2} C_0 = 0.707\, C_0$$

In practice, the best efficiency point of a radial turbine is often $0.68 < \frac{U_2}{C_0} < 0.71$

NOMINAL DESIGN POINT EFFICIENCY

We look at the total to static efficiency

$$\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}}$$

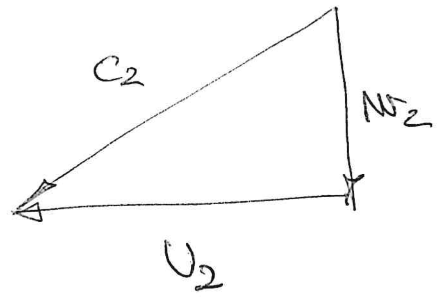


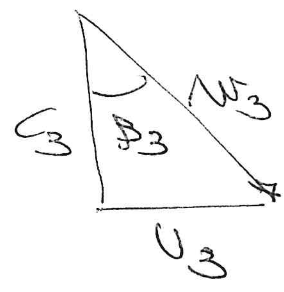
$$h_{01} - h_{3ss} = h_{01} - h_{03} + (h_{03} - h_3) + (h_3 - h_{3ss})$$

$$\eta_{ts} = \frac{h_{01} - h_{03}}{(h_{01} - h_{03}) + (h_{03} - h_3) + (h_3 - h_{3ss})} =$$

$$= \left[1 + \frac{(h_{03} - h_3) + (h_3 - h_{3ss})}{w_t} \right]^{-1}$$

Considero

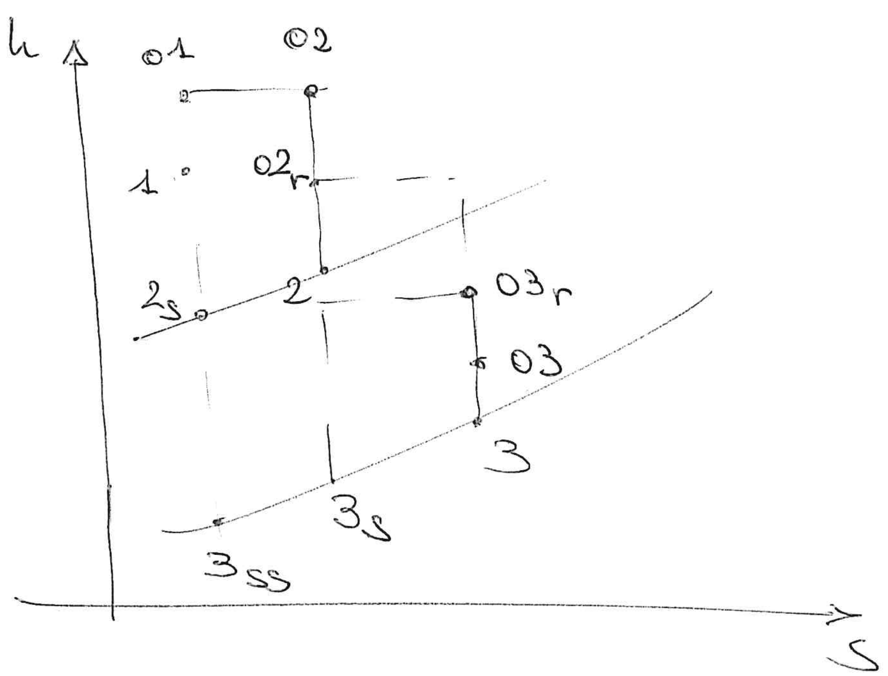




$$w_t = U_2^2$$

$$h_{03} - h_3 = \frac{c_3^2}{2} = \frac{U_3^2}{2\tan^2\beta_3}$$

$$h_3 - h_{3ss} = (h_3 - h_{3s}) + (h_{3s} - h_{3ss})$$



$$h_3 - h_{3s} = \zeta_R \cdot \frac{w_3^2}{2}$$

$$h_{3s} - h_{3ss} = T_3(s_2 - s_1) \quad \Big| \quad h_2 - h_{2s} = T_2(s_2 - s_1)$$

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Therefore:

$$h_{3s} - h_{3ss} = \frac{T_3}{T_2}\left(h_2 - h_{2s}\right)$$

and

$$h_2 - h_{2s} = \zeta_s \frac{c_2^2}{2}$$

$$h_3 - h_{3ss} = \zeta_R \cdot \frac{w_3^2}{2} + \frac{T_3}{T_2} \zeta_s \frac{c_2^2}{2}$$

And

$$\eta_{ts} = \left\{1 + \frac{1}{2}\left[\left(\frac{c_3}{U_2}\right)^2 + \zeta_3\left(\frac{w_3}{U_2}\right)^2 + \frac{T_3}{T_2}\zeta_s\left(\frac{c_2}{U_2}\right)^2\right]\right\}^{-1}$$

$$\frac{c_3}{U_2} = \frac{c_3}{U_3} \cdot \frac{U_3}{U_2} = \frac{r_3}{r_2} \cdot \frac{1}{\tan\beta_3}$$

$$\frac{w_3}{U_2} = \frac{w_3}{U_3} \cdot \frac{U_3}{U_2} = \frac{r_3}{r_2} \cdot \frac{1}{\sin\beta_3}$$

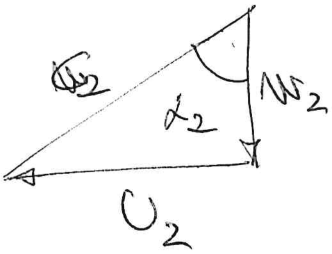
$$\frac{c_2}{U_2} = \frac{1}{\sin\beta_2}$$

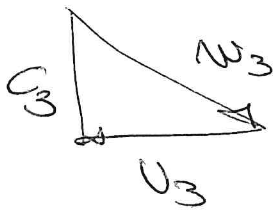
$$\eta_{ts} = \left\{ 1 + \frac{1}{2} \left[\left(\frac{r_3}{r_2}\right)^2 \cdot \left(\frac{1}{\tan^2\beta_3} + \zeta_3 \frac{1}{\sin^2\beta_2} \right) \right] + \frac{T_3}{T_2} \cdot \zeta_s \cdot \frac{1}{\sin^2\beta_2} \right\}^{-1}$$

$$h_{02_r} - h_{03_r} = \frac{1}{2}\left(U_2^2 - U_3^2\right)$$

$$h_2 - h_3 = \frac{1}{2}\left(U_2^2 - U_3^2\right) + \frac{1}{2}\left(w_3^2 - w_2^2\right) =$$

$$= \frac{1}{2}\left[\left(U_2^2 - w_2^2\right) - \underbrace{\left(U_3^2 - w_3^2\right)}_{-c_3^2}\right]$$





$$= \frac{1}{2}\left[\left(U_2^2 - w_2^2\right) + c_3^2\right] =$$

$$= \frac{U_2^2}{2}\left(1 - \left(\frac{w_2}{U_2}\right)^2 + \left(\frac{c_3}{U_2}\right)^2\right)$$

$$1 - \frac{T_3}{T_2} = \frac{U_2^2}{2 c_p T_2}\left(1 - \frac{1}{\tan^2\alpha_2} + \left(\frac{r_3}{r_2}\right)^2 \frac{1}{\tan^2\beta_3}\right)$$

$$\frac{T_3}{T_2} = 1 - \frac{\gamma-1}{2}\frac{U_2^2}{a_2^2}\left[1 - \frac{1}{\tan^2\alpha_2} + \left(\frac{r_3}{r_2}\right)^2\frac{1}{\tan^2\beta_3}\right]$$

$\frac{T_3}{T_2}$ is often close to unity, and it has a small effect on η_{ts}, therefore:

$$\eta_{ts} = \left\{1 + \frac{1}{2}\left[\zeta_s\frac{1}{\sin^2\alpha_2} + \left(\frac{r_3}{r_2}\right)^2\left(\frac{1}{\tan^2\beta_3} + \frac{\zeta_R}{\sin^2\beta_3}\right)\right]\right\}^{-1}$$

If (for any reasons) i want the total to total efficiency

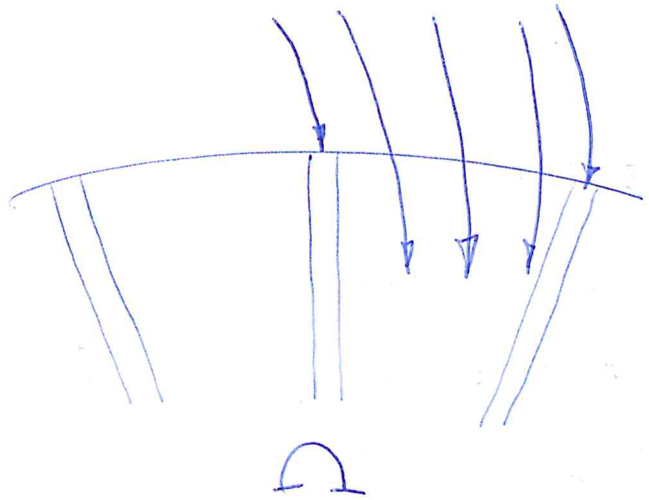
$$\eta_{tt} = \left\{1 + \frac{1}{2}\left[\zeta_s\frac{1}{\sin^2\alpha_2} + \left(\frac{r_3}{r_2}\right)^2\left(\frac{\zeta_R}{\sin^2\beta_3}\right)\right]\right\}^{-1}$$

$$\eta_{tt} = \frac{\Delta W_t}{\Delta W_{ts} - \frac{c_3^2}{2}} = \frac{1}{\frac{1}{\eta_{ts}} - \frac{c_3^2}{2\Delta W_t}} =$$

$$= \frac{1}{\frac{1}{\eta_{ts}} - \frac{c_3}{2U_2^2}} = \frac{1}{\frac{1}{\eta_{ts}} - \frac{c_3^2}{2U_3^2}\cdot\frac{U_3^2}{U_2^2}} =$$

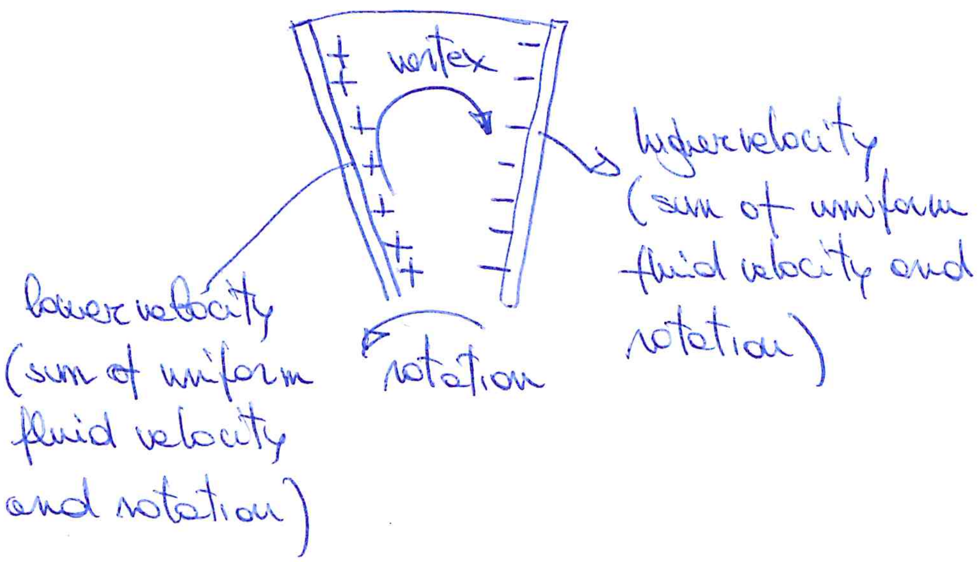
$$= \frac{1}{\frac{1}{\eta_{ts}} - \left(\frac{r_3}{r_2}\right)^2\cdot\frac{1}{2\tan^2\beta_3}}$$

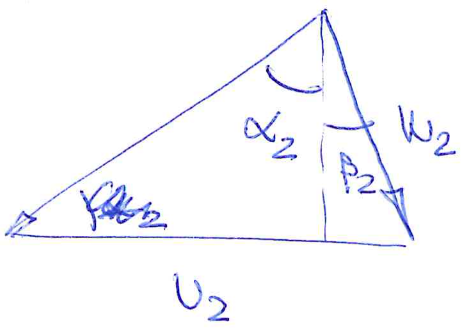
OPTIMUM EFFICIENCY



We have said that radial turbine blades have radial inlets ($\beta_2' = 90°$) to reduce stresses. But what is the real incidence of the flow on the blade?

Similarly to what happens in centrifugal compressors, the rotation of the blade creates a counter vortex in the passage, which increases the pressure on the pressure side of the blade, and decreases it on its suction side





A factor λ, analogous to the slip factor in centrifugal compressors is defined as follows:

$$\lambda = \frac{C_{\theta 2}}{U_2}$$

The model from Stanitz (1952) is often used in radial turbines

$$\lambda = 1 - \frac{0.63\pi}{Z} \approx 1 - \frac{2}{Z}$$

Thus

$$\tan\beta_2 = \frac{2\,U_2}{Z\,C_{m2}}$$

Design for optimum efficiency

$$w_t = \frac{\dot{W}}{\dot{m}} = h_{01} - h_{03} = c_p\left(T_{01} - T_{03}\right) =$$

$$= \frac{\gamma R}{\gamma - 1}\left(T_{01} - T_{03}\right)$$

$$= \frac{\gamma R T_{01}}{\gamma - 1}\left(1 - \frac{T_{03}}{T_{01}}\right)$$

$$S = \frac{w_t}{h_{01}} = 1 - \frac{T_{03}}{T_{01}}$$

where S is a non-dimensional factor called POWER-RATIO

WHITFIELD's METHOD

At the design point, it is assumed that the flow discharges axially:

$$w_t = U_2 c_{\theta 2} \qquad 2 \text{ (inlet to rotor)}$$

$$\frac{U_2 c_{\theta 2}}{h_{01}} = \frac{U_2 c_{\theta 2}}{\gamma R T_{01}}(\gamma - 1) = S$$

$$\Rightarrow \frac{U_2 c_{\theta 2}}{a_{01}^2} = \frac{S}{(\gamma - 1)}$$

From the velocity triangle at rotor inlet:

$$U_2 - c_{\theta 2} = c_{m2} \tan\beta_2 = c_{\theta 2}\frac{\tan\beta_2}{\tan\alpha_2}$$

Multiplying both sides by $\frac{c_{\theta 2}}{c_m^2}$

$$\frac{U_2 c_{\theta 2}}{c_{m2}^2} - \frac{c_{\theta 2}^2}{c_{m2}^2} = \frac{c_{\theta 2}^2}{c_{m2}^2} \frac{\tan \beta_2}{\tan \alpha_2}$$

$$c_{\theta 2} = c_{m2} \tan \alpha_2$$

$$\frac{U_2}{c_{m2}} \tan \alpha_2 - \tan^2 \alpha_2 - \tan \alpha_2 \tan \beta_2 = 0$$

$$\frac{U_2 c_{\theta 2}}{c_{m2}^2} = \frac{U_2 c_{\theta 2}}{c_2^2 \cos^2 \alpha_2} = \frac{U_2 c_{\theta 2}}{c_2^2} (1 + \tan^2 \alpha_2)$$

$$\frac{U_2 c_{\theta 2}}{c_2^2} (1 + \tan^2 \alpha_2) - \tan^2 \alpha_2 - \tan \beta_2 \tan \alpha_2 = 0$$

Hence, if we define: $c = \frac{U_2 c_{\theta 2}}{c_2^2}$ $b = \tan \beta_2$

$$(c-1) \tan^2 \alpha_2 - b \tan \alpha_2 + c = 0$$

$$\tan \alpha_2 = \frac{b \pm \sqrt{b^2 - 4(c-1)c}}{2(c-1)}$$

In order to have real solutions $b^2 - 4(c-1)c \geq 0$

We take the situation $b^2 - 4(c-1)c = 0$, which means

$$c^2 - c - \frac{b^2}{4} = 0$$

Solving for $c \rightarrow$ $c = \frac{1 \pm \sqrt{1 + b^2}}{2}$

$$c = \frac{1}{2} \pm \frac{\sqrt{1+b^2}}{2} = \frac{1}{2} \pm \frac{\sqrt{1+\tan^2\beta_2}}{2} =$$

$$= \frac{1}{2}\left(1 \pm \frac{1}{\cos\beta_2}\right) = \frac{U_2 c_{\theta 2}}{c_2^2}$$

The corresponding solution for ~~&~~ $\tan\alpha_2$ is $(\Delta = 0)$

$$\tan\alpha_2 = \frac{b}{2(c-1)} = \frac{\tan\beta_2}{-1 \pm \frac{1}{\cos\beta_2}} =$$

$$= \frac{\sin\beta_2}{-\cos\beta_2 \pm 1}$$

We want $\alpha_2 > 0$ an therefore

$$\tan\alpha_2 = \frac{\sin\beta_2}{1 - \cos\beta_2}$$

It can be shown that $\alpha_2 = 90° - \frac{\beta_2}{2}$

The stagnation Mach number at inlet

$$M_{02}^2 = \frac{c_2^2}{a_{01}^2} = \frac{U_2 c_{\theta 2}}{a_{01}^2} \cdot \frac{c_2^2}{U_2 c_{\theta 2}} =$$

$$= \frac{S}{\gamma - 1} \cdot \frac{2\cos\beta_2}{1+\cos\beta_2}$$

~~and~~ [illegible] =

$$\frac{c_{\theta 2}}{U_2} = \frac{\cancel{c_{\theta 2}}\, c_{\theta 2}}{c_{\theta 2} + w_{\theta 2}} =$$

$$= \frac{1}{1 + \frac{w_{\theta 2}}{c_{\theta 2}}} = \frac{1}{1 + \frac{\tan \beta_2}{\tan \alpha_2}}$$

$$\frac{\tan \alpha_2}{\tan \beta_2} = \frac{1}{\frac{1}{\cos \beta_2} - 1}$$

$$\frac{\tan \beta_2}{\tan \alpha_2} = \frac{1}{\cos \beta_2} - 1$$

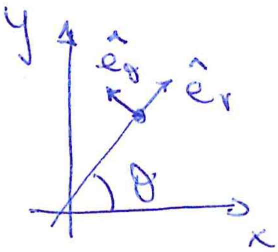
$$\frac{c_{\theta 2}}{U_2} = \cos \beta_2 = 1 - \frac{2}{Z}$$

$$\cos 2\alpha_2 = \cos(180 - \beta_2) = -\cos \beta_2$$

$$\cos^2 \alpha_2 = \frac{1 - \cos \beta_2}{2} = \frac{\cancel{1} - \cancel{1} + 2/Z}{2} = \frac{1}{Z}$$

Inlet absolute angle as a function ~~vs n°~~ of blades

ACCELERATION IN CYLINDRICAL COORDINATES



POSITION $\vec{\rho} = r\hat{e}_r$

$$\frac{d\hat{e}_r}{dt} = \frac{d}{dt}\left(\cos\vartheta\,\bar{i} + \sin\vartheta\,\bar{j}\right) =$$

$$= -\sin\vartheta\,\dot{\vartheta}\,\bar{i} + \cos\vartheta\,\dot{\vartheta}\,\bar{j} = \dot{\vartheta}\,\hat{e}_\vartheta$$

$$\frac{d\hat{e}_\vartheta}{dt} = \frac{d}{dt}\left(-\sin\vartheta\,\bar{i} + \cos\vartheta\,\bar{j}\right) = -\sin\vartheta\,\dot{\vartheta}\,\bar{i} - \cos\vartheta\,\dot{\vartheta}\,\bar{j} =$$

$$= -\dot{\vartheta}\,\hat{e}_r$$

VELOCITY $\bar{v} = \frac{d\rho}{dt} = \dot{r}\hat{e}_r + r\frac{d\hat{e}_r}{dt} = \dot{r}\hat{e}_r + r\dot{\vartheta}\hat{e}_\vartheta$

ACCELERATION $\bar{a} = \frac{d^2\rho}{dt^2} = \ddot{r}\hat{e}_r + \dot{r}\frac{d\hat{e}_r}{dt} + \dot{r}\dot{\vartheta}\hat{e}_\vartheta + r\ddot{\vartheta}\hat{e}_\vartheta$

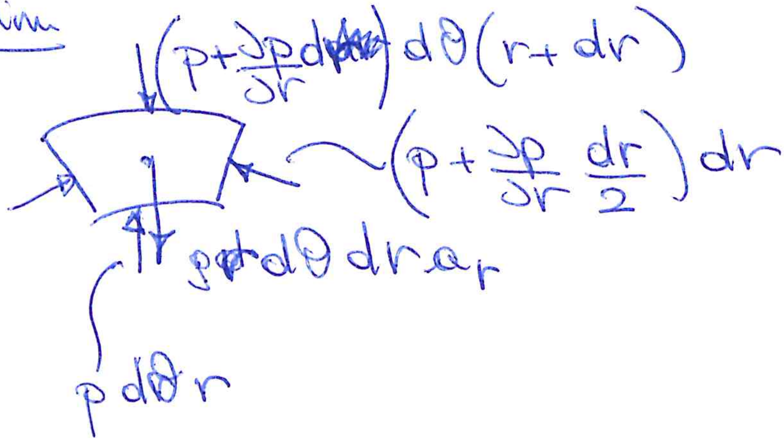
$$+ r\dot{\vartheta}\frac{d\hat{e}_\vartheta}{dt} = \ddot{r}\hat{e}_r + \dot{r}\dot{\vartheta}\hat{e}_\vartheta + \dot{r}\dot{\vartheta}\hat{e}_\vartheta + r\ddot{\vartheta}\hat{e}_\vartheta$$

$$- r\dot{\vartheta}^2\hat{e}_r =$$

$$= \left(\ddot{r} - r\dot{\vartheta}^2\right)\hat{e}_r + \left(r\ddot{\vartheta} + 2\dot{r}\dot{\vartheta}\right)\hat{e}_\vartheta =$$

$$= \left(\dot{V}_r - r\Omega^2\right)\hat{e}_r + \left(r\dot{\Omega} + 2V_r\Omega\right)\hat{e}_\vartheta$$

Equilibrium



RADIAL EQUILIBRIUM :

$$p\, d\theta\, r - \left(p+\frac{\partial p}{\partial r}dr\right)(r+dr)\,d\theta + \left(p+\frac{\partial p}{\partial r}\frac{dr}{2}\right)dr\, d\theta - (\dot{V}_r - r\Omega^2)\,dr\, d\theta\, \rho r = 0$$

$$p\cancel{r\, d\theta} - p\cancel{r\, d\theta} - p\cancel{dr\, d\theta} - \frac{\partial p}{\partial r} r\, dr\, d\theta + \text{higher order}$$

$$+ p\cancel{dr}\, d\theta - (\dot{V}_r - r\Omega^2)\,dr\, d\theta\, \rho r = 0$$

$$+ \frac{\partial p}{\partial r}\cancel{r}\cancel{d\theta}\cancel{dr} + (\dot{V}_r - r\Omega^2)\cancel{dr}\cancel{d\theta}\rho\cancel{r} = 0$$

$$\frac{1}{\rho}\frac{\partial p}{\partial r} + (\dot{V}_r - r\Omega^2) = 0$$

$$\dot{V}_r = \frac{dV_r}{dt} = \frac{\partial V_r}{\partial t} + \bar{V}\cdot\nabla V_r =$$

$$= \frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}$$

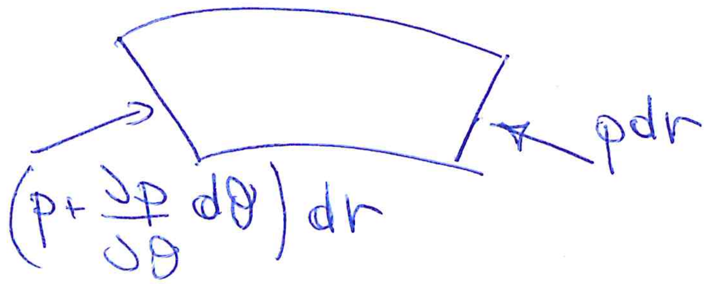
$$V_\theta = 0 \qquad \frac{\partial V_r}{\partial t} = 0 \quad \Rightarrow \quad \dot{V}_r = V_r\frac{\partial V_r}{\partial r}$$

R- EQUILIBRIUM

$$\frac{1}{\rho}\frac{\partial p}{\partial r} + V_r \frac{\partial V_r}{\partial r} = \Omega^2 r$$

Integrating ($\rho = \text{const}$) $\Rightarrow$ $\frac{p}{\rho} + \frac{1}{2} V_r^2 - \frac{1}{2} U^2 = \text{const}$

TANGENTIAL EQUILIBRUIM

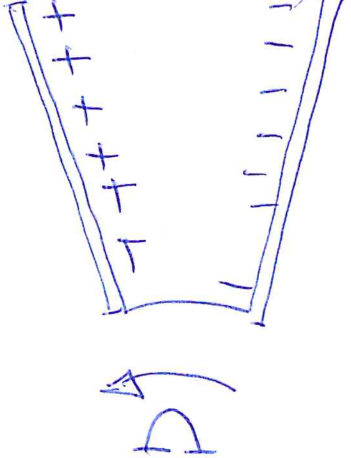


$$\left(\cancel{p} + \frac{\partial p}{\partial \theta} d\theta\right) dr - \cancel{pdr} + \left(r\cancel{\dot{\Omega}} + 2V_r\Omega\right)\rho r\, dr\, d\theta$$

$\Rightarrow \Omega = \text{const}$

$$\frac{\partial p}{\partial \theta} \cancel{dr}\,\cancel{d\theta} + 2 V_r \Omega \rho r \cancel{dr}\,\cancel{d\theta} = 0$$

$$\frac{1}{\rho}\frac{\partial p}{\partial \theta} = -2\Omega r V_r \qquad (V_r < 0)$$



There is a positive gradient in the θ direction.

Deriving the radial equilibrium equation:

$$\frac{1}{\rho}\frac{\partial p}{\partial \theta} + V_r \frac{\partial V_r}{\partial \theta} = 0$$

Hence:

$$-\cancel{V_r}\frac{\partial V_r}{\partial \theta} = -2\Omega r \cancel{V_r}$$

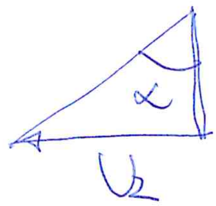
$$\frac{\partial V_r}{\partial \theta} = 2\Omega r$$

Hence, the radial velocity is not uniform across the passage, but it is ~~higher~~ larger for larger θ (i.e towards the pressure side). <u>Be careful</u>: larger but V_r is negative, hence V_r is lower in module towards the pressure side.

$$\cancel{V_{MAX} = \overline{W}}$$

Let's call $w = -V_r$

$$w_{MIN} = \overline{w} - \Omega r \Delta\theta$$



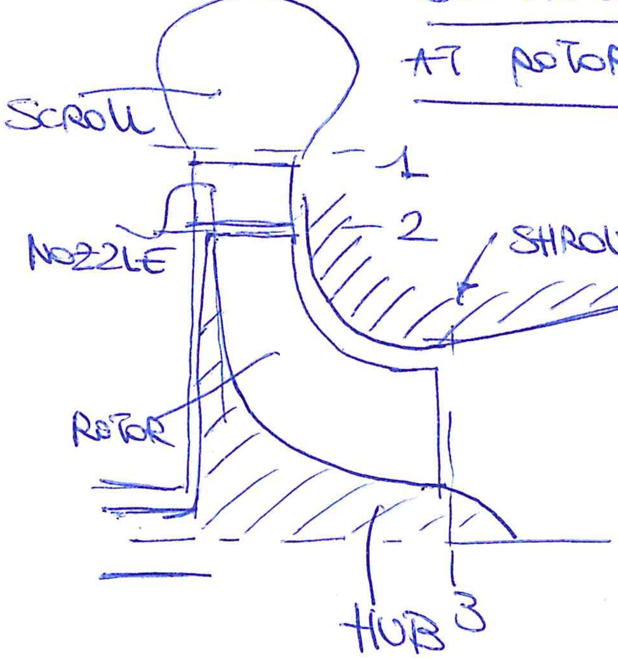
U_2

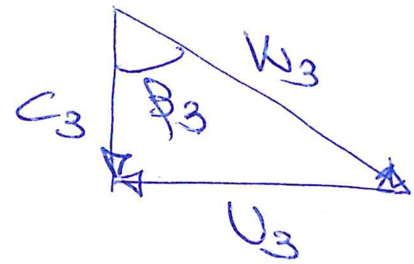
$$w_{MAX} = \overline{w} + \Omega r \Delta\theta$$

If I want $w_{MIN} > 0 \Rightarrow \Delta\theta = \frac{2\pi}{Z}$

$$\overline{w} - U_2 \cdot \frac{2\pi}{Z} > 0 \Rightarrow Z \geq \frac{2\pi U_2}{\overline{w}} = 2\pi \tan\alpha$$

DESIGN CONSIDERATION AT ROTOR EXIT





At the rotor exit (3), the flow should be axial to minimise the exit kinetic energy.

$$w_3^2 = c_3^2 + U_3^2$$

$$\beta_3 = \operatorname{atan}\left(\frac{U_3}{c_3}\right) = \operatorname{atan}\left(\frac{\Omega}{c_3 r}\right)$$

The meridional velocity ($c_3 = c_{m3}$) should be kept small in order to minimize the exit energy loss, unless an exhaust is fitted to the turbine.

Rodgers and Geiser (1987) correlated attainable efficiency levels with the ratio of blade tip speed - spouting velocity, and with the the exit flow coefficient, and noticed how the maximum efficiency was obtained for

$$\frac{U_2}{c_0} \simeq 0.7 \quad \text{and} \quad 0.2 \leq \phi_3 \leq 0.3$$

Another important guideline comes from Rohlik (1968) who suggested that ~~the~~:

- $\frac{r_3}{r_2} < 0.7$ → to avoid excessive curvature of the shroud
- $\frac{r_{3h}}{r_{3t}} > 0.4$ → to avoid blockage effects caused by closely spaced vanes at the hub.

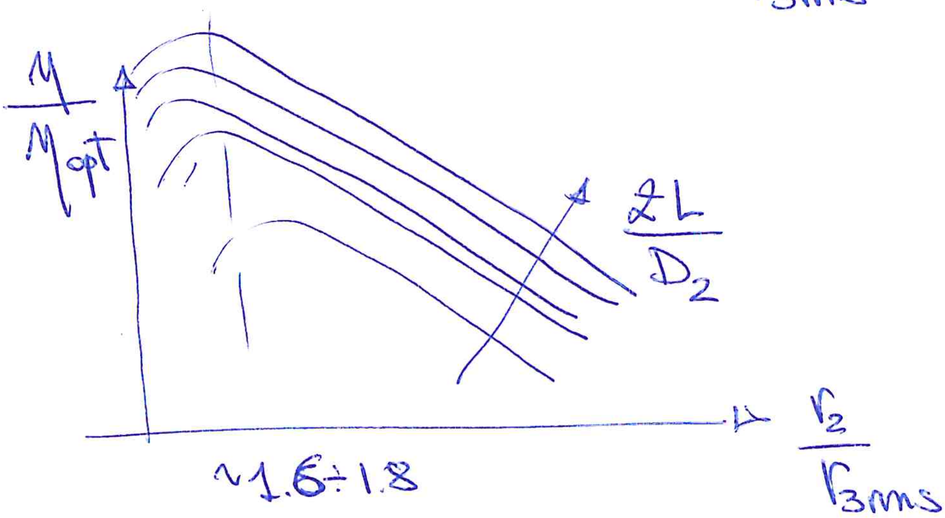
$$\frac{2\pi r_{3h}}{Z}\cos\beta > t_{3h}$$

~~AR~~ 2D-AREA FLOW PER VANE

thickness at the hub

(without considering boundary layers)

Rodgers and Geiser (1987) correlated efficiency with $\frac{r_2}{r_{3rms}}$ and $\frac{\ell L}{D_2}$



$$\frac{r_{3rms}}{r_2} = \frac{r_{3s}}{r_2}\left(\frac{1+\nu^2}{2}\right)^{1/2}$$

$$\nu = \frac{r_{3h}}{r_{3t}}$$

Rohlikh (1968) also suggested that the ratio of relative velocities $\left(\frac{w_3}{w_2}\right)$ should be large enough to ensure a low pressure loss. He gave a value of 2.0 as a guideline.

SPECIFIC SPEED

The specific speed for a (mixed-flow) turbine can be written as:

$$\Omega_s = \frac{\Omega\, Q_3^{1/2}}{(gH)_s^{3/4}}$$

where

$$gH_s = \frac{c_0^2}{2g}$$

$$Q_3 = c_3 A_3$$

And therefore:

$$\Omega_s = \frac{\Omega\,(c_3 A_3)^{1/2}}{\left(\frac{1}{2} c_0^2\right)^{3/4}} =$$

$$\Omega_s = 2^{3/4} \cdot \left(\frac{c_3}{c_0}\right)^{1/2} \cdot \left(\frac{A_3\, \Omega^2}{c_0^2}\right)^{1/2}$$

Given that $\Omega = \frac{2U_2}{D_2}$

$$\Omega_s = 2^{3/4} \left(\frac{c_3}{c_0}\right)^{1/2} \cdot \left(\frac{A_3 \cdot 4 U_2^2}{D_2^2\, c_0^2}\right)^{1/2} =$$

$$= 2^{3/4} \left(\frac{c_3}{c_0}\right)^{1/2} \cdot \left(\frac{U_2}{c_0}\right) \cdot \left(\frac{4 A_3}{D_2^2}\right)^{1/2}$$

We define $A_d = \frac{\pi D_2^2}{4}$

$$\Omega_s = \pi^{1/2} \cdot 2^{3/4} \left(\frac{c_3}{c_0}\right)^{1/2} \cdot \left(\frac{U_2}{c_0}\right) \cdot \left(\frac{A_3}{A_d}\right)^{1/2}$$

We can consider:

$$\left(\frac{U_2}{C_0}\right) \simeq \frac{1}{\sqrt{2}}$$

$$\left(\frac{A_3}{A_d}\right) = \frac{D_{3t}^2}{D_2^2} \cdot \left(1 - \left(\frac{D_{3h}}{D_{3t}}\right)^2\right)$$

D_{3h} ← exit hub diameter

D_{3t} ← exit tip diameter.

Typical values of $\frac{D_{3h}}{D_{3t}}$ range between 0.4 and 0.8 while $\frac{D_{3t}}{D_2}$ is usually between 0.4 and 0.7 (see Rohlik 1968)

$$0.4^2 \cdot (1 - 0.8^2) < \frac{A_3}{A_d} < 0.7^2 \left(1 - 0.4^2\right)$$

$$0.058 < \frac{A_3}{A_d} < 0.41$$

$\frac{C_3}{C_0}$ is typically between 0.2 and 0.55.

Therefore

$$\pi^{1/2} \cdot 2^{3/4} \cdot 0.2^{1/2} \frac{1}{\sqrt{2}} \cdot 0.058^{1/2} < A_s < \pi^{1/2} \cdot 2^{3/4} \cdot 0.55^{1/2} \frac{1}{\sqrt{2}} \; 0.41^{1/2}$$

$$\boxed{0.2 < A_s < 1}$$

These are the typical values of A_s for a mixed-flow turbine.

Often, the specific speed is replaced by a ~~po~~ quasi-non-dimensional specific speed, and the quantities can be expressed in ~~non~~ units that do not belong to the International system.

$$N_s = N \cdot \frac{Q^{1/2}}{H^{3/4}}$$

where $N = \Omega \cdot \frac{60}{2\pi}$ and H is measured in ft.

In this case

$$N_s = \Omega_s \cdot \frac{60}{2\pi} \cdot \underset{32.2\ ft/s^2}{g^{3/4}}$$

$$N_s \simeq 129\, \Omega_s .$$

In this case the typical values for N_s will be:

$$26 < N_s < 129$$

These values (and this definition) are used in Wood (1963) "Current technology of radial-inflow turbines for compressible fluids"

Efficiency.

Using the following assumptions :

$$w_3 / w_2 = 2$$

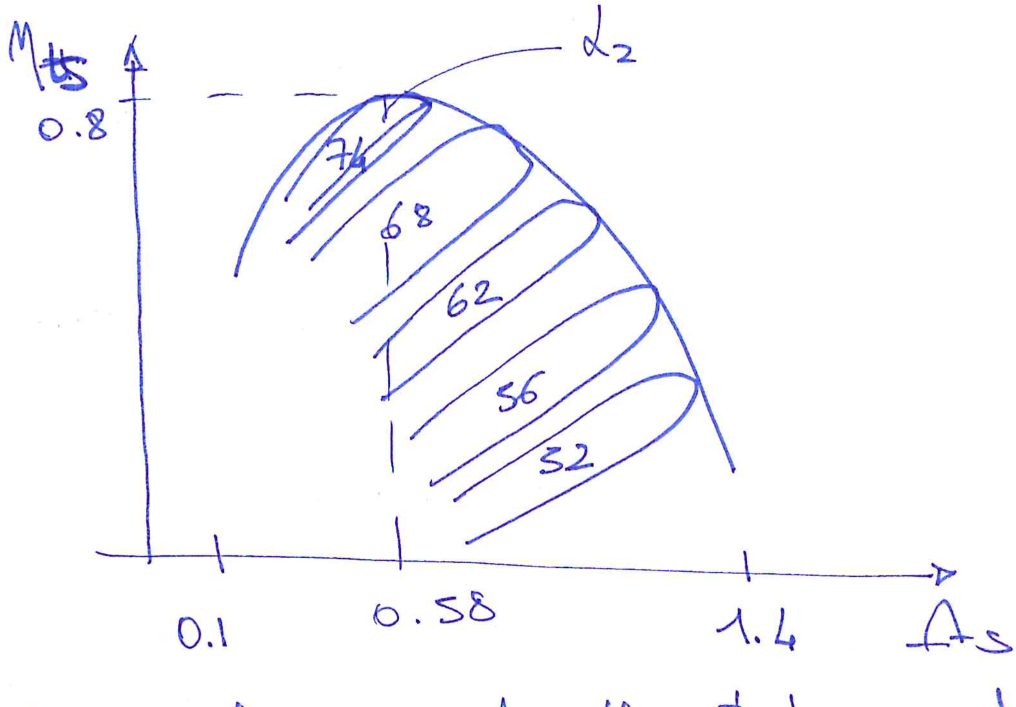
$$C_{u3} = 0 \text{ (axial flow at exit)}$$

$$\beta_2 = \beta_{2,opt}$$

$$r_{s3}/r_2 = 0.7$$

$$r_{3h}/r_{3t} = 0.4$$

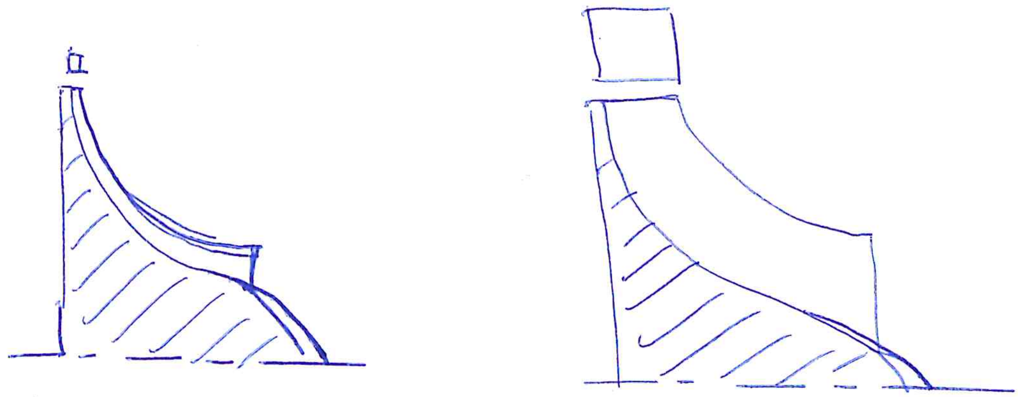
Rohlik (1968) derived a graph :



The optimal Ω_s for a mixed-flow turbine is about 0.58, with an α_2 of 74°.

DISTRIBUTION OF LOSSES

- At low $\Delta_s = \frac{\Omega Q^{1/2}}{\Delta h_{os}^{3/4}}$, friction losses dominate, as the ~~f~~ wetted area is large relatively to the flow area. At large Δ_s, the kin. energy losses dominate because of the large exit velocity.



low Δ_s large Δ_s