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The Preliminary Design of Radial Inflow Turbines

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ABSTRACT

A procedure is described which develops the non-dimensional design of a radial inflow turbine rotor. The design is developed, for any specified non-dimensional power ratio, with the objective of minimising the inlet and discharge Mach numbers so that the passage losses are minimised. Initially state of the art efficiencies are assumed but are later modified through the specification of empirical losses. The resultant non-dimensional design can be transformed to absolute dimensions through the specification of the inlet stagnation conditions and the mass flow rate of the working fluid.

Notation

A	Flow area
a	Speed of sound
C	Absolute velocity
h	Enthalpy
M	Mach number
M'	Relative Mach number
$\dot{m}$	Mass flow rate
P	Pressure
R	Gas constant
r	radius
S_w	Power ratio
T	Temperature
U	Blade velocity
W	Relative velocity
$\dot{W}$	Power
Z_B	Number of blades
α	Absolute flow angle
β	Relative flow angle
γ	Ratio of specific heats
η	Efficiency
θ	Non-dimensional mass flow
ρ	Density

Subscripts

o	Stagnation condition
1	Turbine stage entry
2	Rotor inlet
3	Rotor discharge
s	Shroud position

1 INTRODUCTION

The design of radial inflow turbines is described by Rohlik(1968,1975), Glassman(1976),Rodgers and Geiser(1987), Rodgers(1987) and Whitfield and Baines(1989). These procedures are developed through the application of the fundamental turbomachinery and gas dynamic equations, similarity techniques (particularly specific speed), and empirical loss models. The procedure developed by Rohlik(1968) led to a series of correlations with specific speed which could be used as design charts. These results are, however, restricted by the limitations built into the original analysis procedure.

At the initial stage of a turbine design proposal the designer will have available a range of specified parameters. These may include the power output, mass flow rate, inlet stagnation conditions, rotational speed, and possible restraints on the overall size. Alternatively only the required power output may be specified, and it will be necessary for the designer to specify parameters so that the design can proceed. The objectives for each design may differ, however a common aim will generally be to maximise the efficiency and/or develop a compact power unit. It may be necessary to compromise the efficiency in order to achieve a compact design; in such cases the cost in terms of reduced efficiency needs to be known so that a well judged compromise can be made.

Precise prediction of efficiencies at the initial design stage, or at any other stage, is difficult, and the designer usually relies upon empirical loss models and correlations, see Rohlik(1968,1975), Glassman(1976) and Rodgers(1987). The predicted efficiencies are then as good as the under lying empirical loss models. Efficiency is maximised when the irreversibilities associated with the flow process are minimised. The energy loss is generally a function of the square of the velocity relative to the component under consideration, and if the losses are to be minimised it is essential that the flow velocities are no larger than necessary. The procedure developed here assesses the rotor inlet and discharge conditions with a view to minimising the velocities in terms of the absolute and relative Mach numbers. The resultant design should then give minimum losses irrespective of the loss model, and associated uncertainties, used.

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2 INITIAL DESIGN SPECIFICATION

In order to develop a general procedure it will be assumed initially that only the desired power output is specified.

The power developed by the turbine is given by

$$\dot{W}/\dot{m} = h_{o1} - h_{o3} = \frac{\gamma R}{\gamma - 1}(T_{o1} - T_{o3})$$

and can be developed to the non-dimensional power ratio

$$S_w = \frac{\dot{W}}{\dot{m} h_{o1}} = 1 - \frac{T_{o3}}{T_{o1}} \qquad [1]$$

If the required power output, mass flow rate and inlet stagnation temperature are known then S_w can be calculated directly. If, however, only the power output is known then S_w must be specified and the design analysis used to select an appropriate magnitude; this may be an iterative procedure. Once a magnitude of power ratio has been determined the required mass flow rate follows for any specified inlet stagnation temperature, which is usually limited by rotor material and stress capabilities.

The design procedure will therefore be developed from a specification of the power ratio S_w. As the mass flow rate is not necessarily known initially it will not be possible to calculate the absolute dimensions of the rotor, and the objective of the analysis is to develop a non-dimensional design, that is the dimensions will be non-dimensionalised by the rotor inlet radius, and the fluid and rotor velocities by the inlet stagnation speed of sound. As the design proceeds and it becomes possible to quantify the mass flow rate it will then be possible to convert the non-dimensional design to give the required size.

The power ratio is related to the stage expansion ratio through the efficiency defintion, which is given, on a total to static basis, by

$$\eta_{ts} = \frac{1 - T_{o3}/T_{o1}}{1 - (P_{o1}/P_3)^{(\gamma-1)/\gamma}}$$

The derived pressure ratio is shown in Fig.1. The pressure ratio can be interpreted as a total to total pressure ratio provided the efficiency is defined in a like manner. A good order of magnitude for the power ratio can be obtained from Fig.1 if the turbine expansion ratio is known.

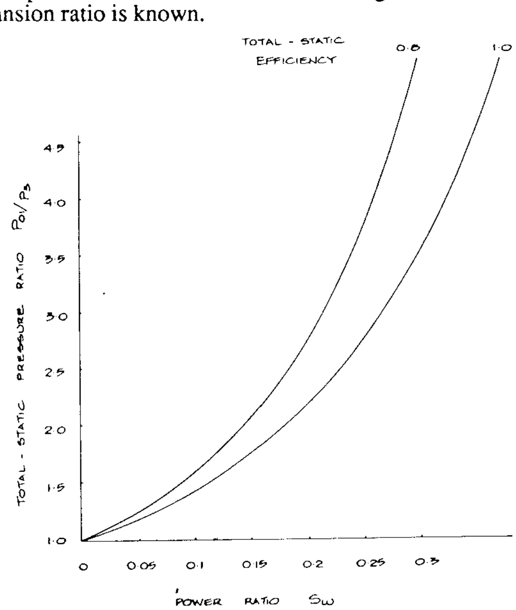


FIG. 1 TURBINE EXPANSION RATIO AS A FUNCTION OF POWER RATIO S_w

3 ROTOR INLET DESIGN

A typical turbine rotor is illustrated in Fig.2. The rotor blades are usually radial at inlet as this provides the best condition with respect to stressing; this will be assumed to be the case here. At the best efficiency point early design procedures assumed that the inlet relative velocity vector was aligned with the radial blade thereby giving a right angled velocity triangle. It has, however, been shown both experimentally and by incidence loss models that at the best efficiency point a significant angle of incidence occurs and the velocity triangle of Fig.3 is more appropriate. It is necessary, therefore, to specify the incidence angle at which the best efficiency point will occur. Rodgers(1987) quotes incidence angles of the order of -20 deg, and Rohlik gives magnitudes as high as -40 deg. Assessment of an appropriate magnitude for the optimum inlet flow angle is given in the next section after the inlet velocity triangle has been established.

At the best efficiency point it is usually assumed that the fluid discharges from the rotor in an axial direction, that is there is no discharge swirl. The Euler turbomachinery equation then becomes

$$\dot{W}/\dot{m} = U_2 C_{\theta 2} - U_3 C_{\theta 3} = U_2 C_{\theta 2} \qquad [2]$$

and it follows that

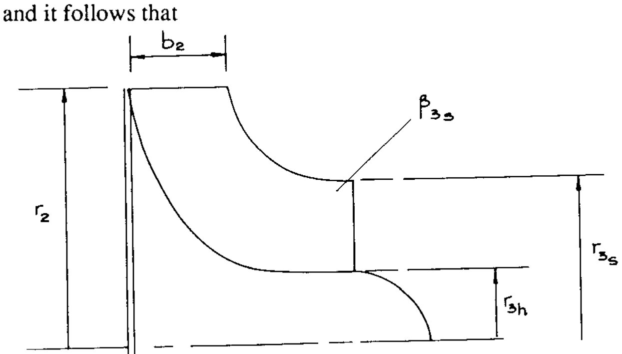


FIG 2 TURBINE ROTOR GEOMETRY

$$\frac{U_2 C_{\theta 2}}{a_{o1} a_{o1}} = \frac{S_w}{\gamma - 1} \qquad [3]$$

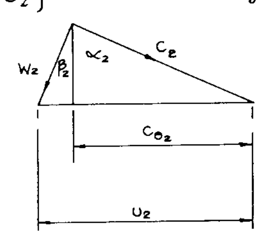
The objective now is calculate all parameters associated with the inlet velocity triangle as this information is necessary not only for the following rotor design, but also for the preceding stator.

From the velocity triangle, Fig.3

$$C_{\theta 2} = U_2 + C_{m2}\tan\beta_2 = U_2 + \frac{C_{\theta 2}}{\tan\alpha_2}\tan\beta_2 \qquad [4]$$

which can be developed to

$$\tan^2\alpha_2\left\{1 - \frac{U_2 C_{\theta 2}}{a_{o1}^2}\frac{a_{o1}^2}{C_2^2}\right\} - \tan\alpha_2\tan\beta_2 - \frac{U_2 C_{\theta 2}}{a_{o1}^2}\frac{a_{o1}^2}{C_2^2} = 0 \qquad [5]$$



(c) INLET VELOCITIES

FIG. 3 ROTOR VELOCITY COMPONENTS

Equation [5] is a quadratic equation in $\tan\alpha_2$ which can be solved if the stagnation Mach number C_2/a_{o1} is known or specified. This is equivalent to specifying the inlet Mach number as it can be derived through

$$M_2^2 = M_{o2}^2 \Big/ \left(1 - \frac{\gamma-1}{2} M_{o2}^2\right)$$

with the assumption of adiabatic flow through the stator, that is $T_{o2}=T_{o1}$.

However, eqn [5] can only be solved if the stagnation Mach number is sufficiently large to yield a positive root. The limiting value of C_2/a_{o1} is such that

$$\tan^2\beta_2 + 4\left(1 - \frac{U_2 C_{\theta 2}}{a_{o1}^2}\frac{a_{o1}^2}{C_2^2}\right)\frac{U_2 C_{\theta 2}}{a_{o1}^2}\frac{a_{o1}^2}{C_2^2} = 0$$

Substituting from eqn [3] it follows that

$$4\left(\frac{S_w}{\gamma-1}\frac{a_{o1}^2}{C_2^2}\right)^2 - 4\frac{S_w}{\gamma-1}\frac{a_{o1}^2}{C_2^2} - \tan^2\beta_2 = 0 \qquad [6]$$

The solution of which yields

$$\frac{C_2^2}{a_{o1}^2} = \left(\frac{S_w}{\gamma-1}\right)\frac{2\cos\beta_2}{1+\cos\beta_2} \qquad [7]$$

The application of equation [7] enables the minimum absolute Mach number of the incidence gas to be derived through the specification of the power ratio, S_w, and the optimum inlet angle relative to the rotor, β_2. The minimum inlet Mach number is shown in Fig.4 as a function of power ratio, S_w, for an incidence angle of -30 deg. The effect of the assumed incidence angle on the minimum Mach number is small relative to that of the power ratio. The associated turbine expansion ratio can be observed through Fig.1. Mach numbers in excess of that given through eqn [7] can, of course, be specified by the designer. However, high Mach numbers will lead to increased stator losses, increased relative Mach numbers and consequently increased incidence and rotor losses, and increased rotational speeds of the rotor.

With the inlet Mach number specified or derived through eqn [7] the rotor inlet velocity triangle is fully defined. The absolute flow angle is given by eqn [5], and for the specific case where the minimum Mach number is used it can be shown that

$$\tan\alpha_2 = \frac{\sin\beta_2}{\cos\beta_2 - 1} \qquad [8]$$

Also for the minimum Mach number condition the non-dimensional speed of the rotor is given by

$$\left(\frac{U_2}{a_{o1}}\right)^2 = \left(\frac{1}{\gamma-1}\right)\frac{S_w}{\cos\beta_2} \qquad [9]$$

Equation [5] is illustrated in Fig.5 where the absolute flow angle is presented as a function of the inlet Mach number for different magnitudes of power ratio and inlet relative flow angle. From this it can be seen that as the inlet Mach number is increased beyond the minimum permissible the absolute flow angle decreases. However, as shown in Fig.6 the non-dimensional speed of the rotor and the relative Mach number both increase as the absolute Mach number is increased.

The solution to eqn [5] does, of course, contain two roots. The negative root yields a flow angle which increases rapidly to 90 deg, see Fig.5. As a consequence the relative Mach number approaches zero, and if the design is continued using these flow angles an unacceptably large rotor diameter is needed in order to provide sufficient flow area to carry the required mass flow.

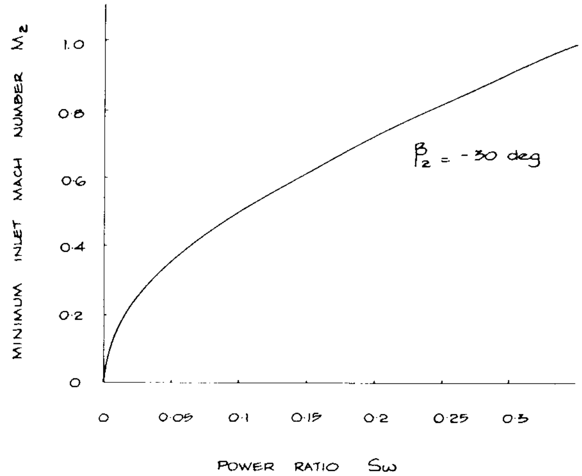


FIG. 4 ROTOR MINIMUM INLET MACH NUMBER

Clearly the minimum Mach number condition provides a satisfactory base from which the turbine design can be developed. If it is eventually found advantageous to increase the Mach number this can be readily done and the proposed design modified.

3.1 Calculation of the optimum inlet relative flow angle

The reason for the high optimum incidence angles of radial inflow rotors is attributed to the fact that the incoming flow has to board a rotor passage which is sustaining a relative eddy. This is analogous to the phenomenon which gives rise to slip in centrifugal compressor impellers. For the radial inflow turbine studies by Woolley and Hatton(1973) and Sugimoto et al(1975) confirm this pattern. An incidence factor is usually defined in a similar manner to the specification of the slip factor for centrifugal compressors. The analysis by Stanitz(1952) for the two-dimensional inviscid blade to blade flow in a centrifugal compressor impeller (which is equally valid for the turbine rotor if all velocity vectors are reversed) gave the tangential component of velocity as, (see also Rohlik(1968))

$$C_{\theta 2} = U_2\left(1 - \frac{0.63\pi}{Z_B}\right) \qquad [10]$$

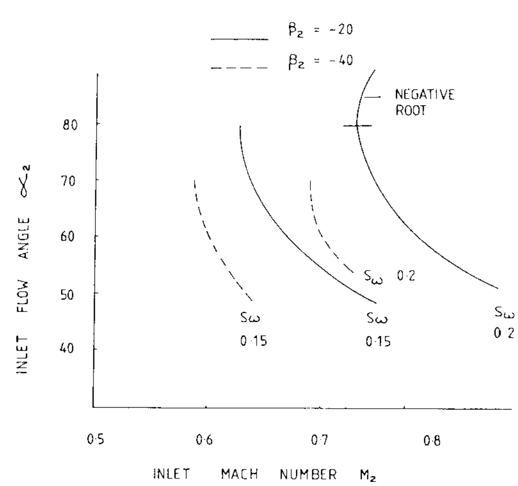


FIG. 5. VARIATION OF INLET FLOW ANGLE WITH MACH NUMBER

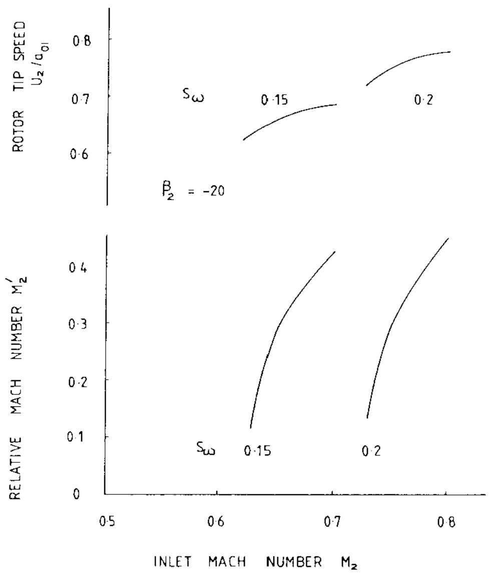


FIG. 6. EFFECT OF INLET MACH NUMBER ON RELATIVE MACH NUMBER AND ROTOR TIP SPEED.

From eqn [4] it follows that

$$C_{\theta 2} = U_2 \frac{\tan \alpha_2}{\tan \alpha_2 - \tan \beta_2} \qquad [11]$$

At the minimum Mach number condition eqn [8] can be combined with eqns [10] and [11] to give

$$\cos \beta_2 = \left(1 - \frac{0.63\pi}{Z_B}\right) \qquad [12]$$

This expression is shown in Fig.7. For radial turbine rotors with 12 to 20 blades optimum relative flow angles between -33 and -26 deg are predicted.

The optimum relative flow angle of eqn [12] can be combined with eqn [8] to give the corresponding absolute flow angle as

$$\cos^2 \alpha_2 = \frac{0.63\pi}{2Z_B} \qquad [13]$$

This expression is shown in Fig.8. For comparison the expressions given by Jamieson(1955) and Glassman(1976) are also included. Jamieson gave the relationship

$$Z_B = 2\pi \tan \alpha_2$$

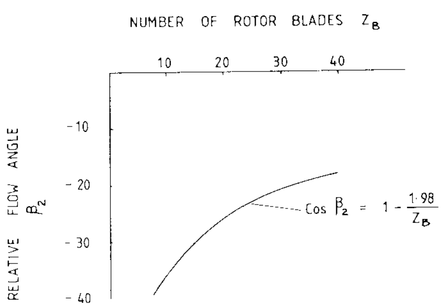


FIG. 7 EFFECT OF BLADE NUMBER ON OPTIMUM RELATIVE FLOW ANGLE

This relationship is strictly applicable only if induced incidence effects are neglected. Glassman(1976) considered that the Jamieson expression gave too many blades and modified it to

$$Z_B = \frac{\pi}{30}(110 - \alpha_2)\tan \alpha_2$$

Equation [13], based on the minimum inlet Mach number condition and eqn [10], compares well, over the range of number of blades most commonly adopted, with the expression given by Glassman(1976). For any specified number of rotor blades eqn [12] enables the optimum relative flow angle to be calculated. This is dependent on the empirical incidence factor relationship chosen, and following centrifugal compressor practice for slip factors a number of alternative expressions could be quoted. Care should, however, be exercised in the use of compressor slip factor relationships as they have often been developed empirically to provide good order of magnitude values for the separated flows at compressor impeller discharge. Such conditions do not apply at the turbine rotor inlet, and in general compressor slip factor correlations may not be applicable. The correlation used above was developed from a two-dimensional inviscid analysis and did not attempt to include separated flows. Equation [12] can only be used as a guide to the optimum incidence angle. Judging from the limited data given by Rodgers(1987) the values given by eqn [12] are a little high. Inclusion of blade and boundary layer blockage will reduce the effective passage size and hence the magnitude of the optimum incidence angle.

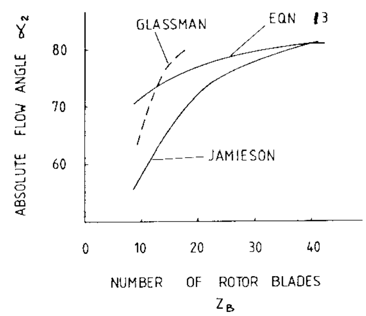


FIG. 8 ABSOLUTE FLOW ANGLE AS A FUNCTION OF THE NUMBER OF BLADES

4 ROTOR DISCHARGE DESIGN

For the design of a compressor impeller inducer a procedure was described by Stanitz(1953), and Coppage et al(1956) which led to the derivation of the inlet relative flow angle which ensured that the non-dimensional mass flow rate was achieved with the minimum inlet relative Mach number. A similar analysis can be applied to the rotor exducer in order to ensure that the desired mass flow rate is achieved with the minimum discharge relative Mach number. For the case of zero discharge swirl, the relative Mach number can be shown to be given by

$$M_3'^2 = M_3^2 + \frac{\theta M_u^2}{1 - \nu^2}\frac{1}{M_3}\left\{1 + \frac{\gamma - 1}{2}M_3^2\right\}^{0.5}\sqrt{\frac{T_{o1}}{T_{o3}}\frac{P_{o1}}{P_3}} \qquad [14]$$

where θ is the non-dimensional mass flow rate $\dfrac{\dot{m}}{\rho_{o1} a_{o1} \pi r_2^2}$

In order to use eqn [14] the pressure ratio P_{o1}/P_3 must be derived through the specification of a total to static efficiency for the turbine. Then

$$\left(\frac{P_3}{P_{o1}}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{1 - T_{o3}/T_{o1}}{\eta_{ts}} = 1 - \frac{S_w}{\eta_{ts}} \qquad [15]$$

Equation [14] is presented in Fig.9. It can be seen that the relative Mach number is a minimum when the relative flow angle is of the order of -55 deg. The use of larger flow angles (e.g. -70 deg) cannot, however, be dismissed as this leads to a reduction in the absolute Mach number. There is, therefore, a compromise to be made between low relative Mach number to minimise the rotor losses, and a reduced absolute Mach number to reduce the exit loss which is directly proportional to the square of the discharge Mach number. This choice cannot be made at this stage, and the above analysis can only be used to provide a good order of magnitude for the relative flow angle; magnitudes in the range 0 to -50 deg can be ruled out as unsatisfactory. The analysis is, therefore, continued with the relative flow angle simply specified.

To proceed further and derive the discharge velocity triangle it is necessary to specify a further parameter. Rohlik(1968) assumed a relative velocity ratio, W_3/W_2, and maintained this constant at 2.0. In axial turbine design the degree of reaction is often specified. Rodgers and Geiser(1987) correlated the efficiency against the discharge velocity ratio C_{m3}/U_2 and showed that optimum values lay between 0.2 and 0.3. Any of these parameters can be selected and the others derived. For convenience the relative velocity ratio, $W_R=W_3/W_2$, is adopted here, and will be systematically increased from unity to any desired upper limit.

The discharge relative Mach number can be derived through

$$M'^2_3 = W_R^2 M'^2_2 \frac{T_2}{T_{o2}} \frac{T_{o2}}{T_{o3}} \frac{T_{o3}}{T_3} \qquad [16]$$

where T_{o3}/T_3 is given by

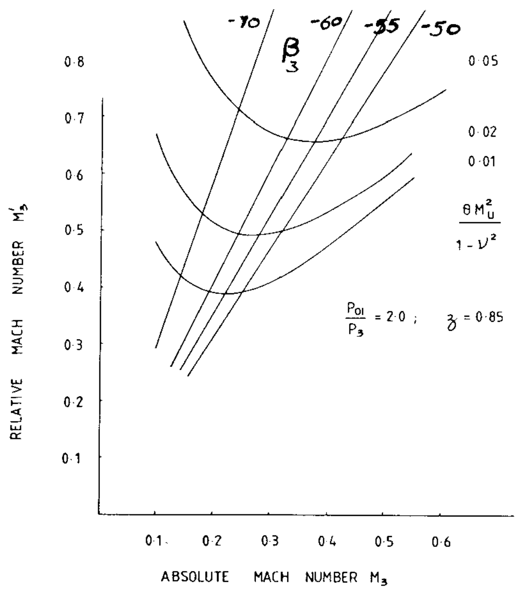


FIG. 9 RELATIVE FLOW ANGLE FOR MINIMUM RELATIVE MACH NUMBER

$$\frac{T_3}{T_{o3}} = 1 - \frac{\gamma-1}{2} \frac{W_3^2 \cos \beta_3^2}{a_{o3}^2}$$

and

$$\frac{W_3}{a_{o3}} = W_R \frac{W_2}{a_{o1}} \sqrt{\frac{T_{o1}}{T_{o3}}}$$

The absolute Mach number is then

$$M_3 = M'_3 \cos \beta_3 \qquad [17]$$

and the discharge velocity triangle is established. The other parameters, degree of reaction and discharge velocity ratio, C_{m3}/U_2, can be readily found.

5 CALCULATION OF THE ROTOR NON-DIMENSIONAL GEOMETRY

Through the specification of the relative velocity ratio the discharge velocity triangle is fully defined with all velocity vectors non-dimensionalised by the stagnation speed of sound, a_{o3}, or the speed of sound, a_3. As the stagnation temperature ratio across the rotor is known through the initial specification of the power ratio, eqn [1], the velocity vectors of the discharge velocity triangle can be modified so that they are non-dimensionalised by the inlet stagnation speed of sound. The radius ratio of the rotor can then be found through

$$\frac{r_{3s}}{r_2} = \frac{U_3}{a_{o1}} \frac{a_{o1}}{U_2} \qquad [18]$$

For the specific case where the minimum inlet Mach number is used it can be shown that the rotor radius ratio is given by

$$\left(\frac{r_{3s}}{r_2}\right)^2 = W_R^2 \frac{1-\cos\beta_2}{1+\cos\beta_2} \sin^2\beta_3 \qquad [19]$$

The radius ratio is a linear function of the relative velocity ratio for any specified inlet and discharge relative flow angle, provided the minimum inlet Mach number condition is applied (it is not a function of any assumed stage or rotor efficiency). In terms of the discharge velocity ratio, C_{m3}/U_2, the discharge velocity triangle with zero swirl gives

$$\frac{r_{3s}}{r_2} = \frac{C_{m3}}{U_2} \tan\beta_3 \qquad [20]$$

The geometric parameter remaining to be calculated relates to the rotor inlet blade height, b_2, which can be determined in the non-dimensional form b_2/r_2. The area ratio across the rotor can be determined through the application of the continuity condition between rotor inlet and discharge . As the inlet and discharge Mach numbers are known the non-dimensional mass flow at each station, defined as

$$\theta = \frac{m}{\rho_o a_o A}$$

can be derived through

$$\theta_2 = \cos\alpha_2 M_2 \left[1 + \frac{\gamma-1}{2} M_2^2\right]^{-\frac{\gamma+1}{2(\gamma-1)}}$$

at the inlet, and at the discharge

$$\theta_3 = M_3 \left[1 + \frac{\gamma-1}{2} M_3^2\right]^{-\frac{\gamma+1}{2(\gamma-1)}} \qquad [21]$$

The rotor area ratio is, therefore, given by

$$\frac{A_3}{A_2} = \frac{\theta_2 \rho_{o2} a_{o2}}{\theta_3 \rho_{o3} a_{o3}} = \frac{\theta_2 \rho_{o2}}{\theta_3 \rho_{o3}} \sqrt{\frac{T_{o1}}{T_{o3}}} \qquad [22]$$

Before eqn [22] can be solved for the area ratio it is necessary to determine the stagnation density ratio across the rotor. This can only be done if the irreversibilities associated with the complete stage and the stator are quantified. This is most easily done by specifying appropriate efficiencies. Through the state equation the density ratio can be rewritten in terms of pressure and temperature ratios to give

$$\frac{\rho_{o2}}{\rho_{o3}} = \frac{P_{o2} T_{o3}}{P_{o3} T_{o2}} = \frac{P_{o2} P_{o1} T_{o3}}{P_{o1} P_{o3} T_{o1}} \qquad [23]$$

The pressure ratio P_{o1}/P_{o3} can be derived through the combination of P_{o1}/P_3, given by eqn [15], and P_{o3}/P_3, which can be derived from the known discharge Mach number. The stagnation pressure ratio across the stator, P_{o2}/P_{o1}, must be derived through a specification of the stator efficiency. With the efficiency defined as

The exducer hub to shroud radius ratio is specified through consideration of the number of blades which must be accommodated around the hub, and through stress considerations to ensure an adequate diameter and may have to be refined as the design progresses. The results shown in Figs.10 and 11 were obtained with a hub shroud radius ratio of 0.4.

6 DESIGN OPTIMISATION

An optimum incidence angle of -30 deg was generally adopted to develop alternative designs. For comparative purposes results with an optimum incidence angle of -20 deg are also included. Design selection reduces to the specification of the relative velocity ratio which will maximise the rotor efficiency. Rohlik(1968) used a relative velocity ratio of 2, based on the mean radius of the exducer. Rodgers and Geiser(1987) showed that peak efficiencies occurred at exit velocity ratios, C_{m3}/U_2, between 0.2 and 0.3. These criteria can be used to select the desired design. If this is not considered suitable the losses associated with the flow process must be calculated in order to identify the peak efficiency point.

The internal loss processes usually identified for the rotor are those due to passage friction and curvature, blade-shroud clearance, blade loading, and exit kinetic energy. Empirical equations for the evaluation of these losses are given by Rohlik(1968), Glassman(1976), and Rodgers(1987). The empirical loss equations adopted followed those given by Rodgers(1987) with the exception that the loss due to passage curvature was arbitrarily modified from

$$\Delta q_k = \left(\frac{\frac{b_2}{r_2}+\frac{b_3}{r_2}}{1-\frac{r_3}{r_2}}\right)\left(\frac{W_2^2+W_3^2}{2a_{o1}^2}\right)\left(\frac{a_{o1}^2}{U_2^2}\right)$$

to

$$\Delta q_k = 0.5\left(\frac{\frac{b_2}{r_2}+\frac{b_3}{r_2}}{1-\left(\frac{r_3}{r_2}\right)^2}\right)\left(\frac{W_2^2+W_3^2}{2a_{o1}^2}\right)\left(\frac{a_{o1}^2}{U_2^2}\right)$$

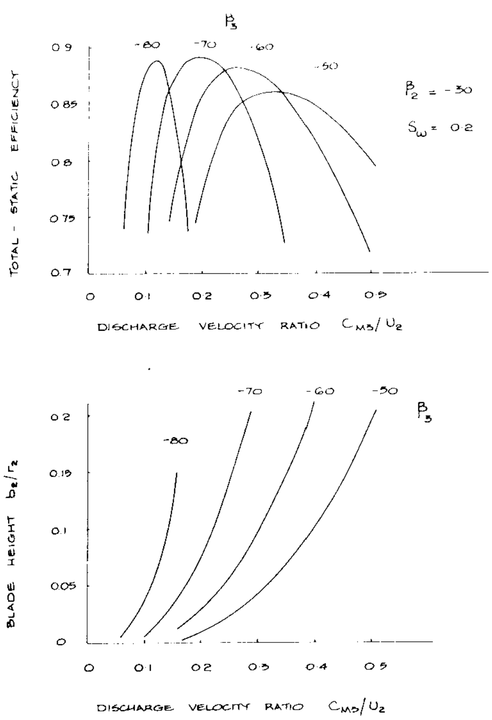


FIG 10. EFFICIENCY AND BLADE HEIGHTS

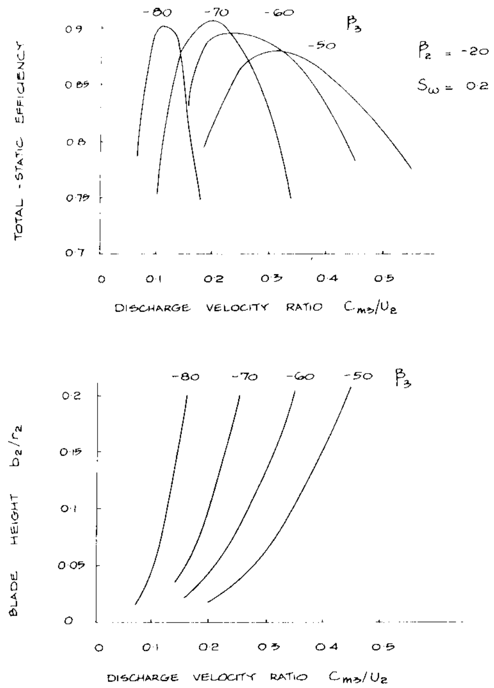


FIG 11 EFFICIENCY AND BLADE HEIGHTS

This was done as it was found that the original formulation led to low efficiencies at high radius ratios (in excess of 0.8), whilst turbocharger turbines operate with acceptable efficiencies with radius ratios in excess of 0.8.

The friction loss was given by

$$\Delta q_F = \frac{0.03((W_2/a_{o1})^2+(W_3/a_{o1})^2)}{4(D_h/L_h)(U_2/a_{o1})^2}$$

The blade loading loss by

$$\Delta q_{BL} = \frac{2(C_{\theta 2}/U_2)^2}{Z_B Z/r_2}$$

As this blade loading loss is a function of the inlet velocity triangle only it does not change as the relative velocity ratio is varied. For the clearance loss

$$\Delta q_{CL} = 0.4(\varepsilon/b_2)(C_{\theta 2}/U_2)^2$$

and the exit loss was given by

$$\Delta q_{EX} = 0.5(C_3/U_2)^2$$

To apply the clearance loss equation the clearance to radius ratio was assumed to be 0.008. For the computation of the hydraulic length to diameter ratio, required in the friction loss equation, an axial length to radius ratio of 0.7 and 15 blades were assumed. The objective here is not to assess or develop suitable empirical loss equations, but to illustrate the design procedure.

As the non-dimensional radius and blade height increase with increasing relative velocity ratio both the passage friction loss and the clearance loss will decrease. With the assumption that the exhaust kinetic energy is not recovered, or used, the losses which increase with increasing relative velocity ratio are the exit and passage curvature losses. The best efficiency point occurs where the falling friction and clearance losses combine with the exit and curvature losses to yield the minimum loss condition. Application of the empirical loss equations showed that the clearance loss, which falls more rapidly than the passage friction loss, plays the dominant role in locating the best efficiency point.

7 PRESENTATION OF RESULTS

All results presented were obtained at the minimum inlet Mach number condition, and whilst the analysis procedure used the specification of the relative velocity ratio, it was found that the results correlated better with the discharge velocity ratio. Predicted efficiencies are shown in Fig.10 along with the calculated inlet blade height for an inlet flow angle of -30 deg and a series of discharge flow angles. Comparative results are shown in Fig.11 for an assumed inlet flow angle of -20 deg. The location of the peak efficiency point was not changed by the assumed inlet flow angle (compare Figs.10 and 11), but it is clearly a function of the assumed discharge flow angle. The rapid increase in efficiency as C_{m3}/U_2 increases is due to a reduction in the clearance loss as the non-dimensional blade height increases; the rate at which the efficiency then falls again is a function of the increasing passage curvature and exit kinetic energy losses relative to the friction and clearance losses which continue to decrease. The results shown are all for a power ratio, S_w, of 0.2. The use of magnitudes of 0.1 and 0.3 did not significantly modify the location of the peak efficiency point, whilst the absolute magnitudes were decreased as S_w increased due to the increasing inlet Mach number. If the turbine installation is such that the exhaust kinetic energy can by recovered in an exhaust diffuser the exducer blade angle could be of the order of -55 to -50 deg, see Fig.8. The high exducer blade angles (e.g. -80°) could lead to excessive blade blockage at the hub. This would have to be assessed further as the design progressed.

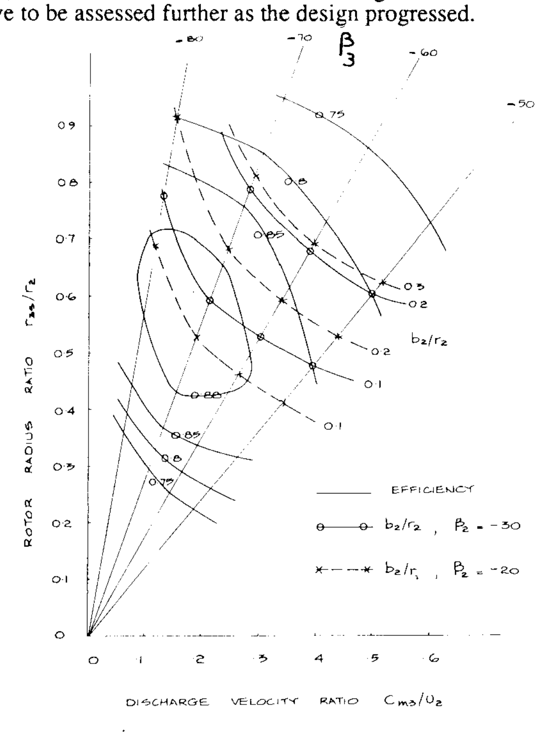


FIG 12 ROTOR EFFICIENCY CONTOURS

The rotor radius ratio is a linear function of the discharge velocity ratio for any specified discharge flow angle, see eqn [20], and is shown in Fig.12. The efficiency contours were added from Fig.10. Also included are contours of non-dimensional blade height from Figs.10 and 11. The main effect of modifying the assumed optimum incidence angle, either by design or due to uncertainties as to the correct magnitude, is to change the calculated non-dimensional blade height. The blade heights shown are based on the calculated flow areas required and the geometric area and blade height will be larger due to boundary layer blockage. Clearly Fig.12 could be replotted with any of the contours as the x-axis and C_{m3}/U_2 as a contour. As the inlet blade height is a function of the specified exducer hub to shroud radius ratio (here a value of 0.4 has been assumed), the rotor area ratio is used for the x-axis in Fig.13. Contours of discharge flow angle and efficiency are

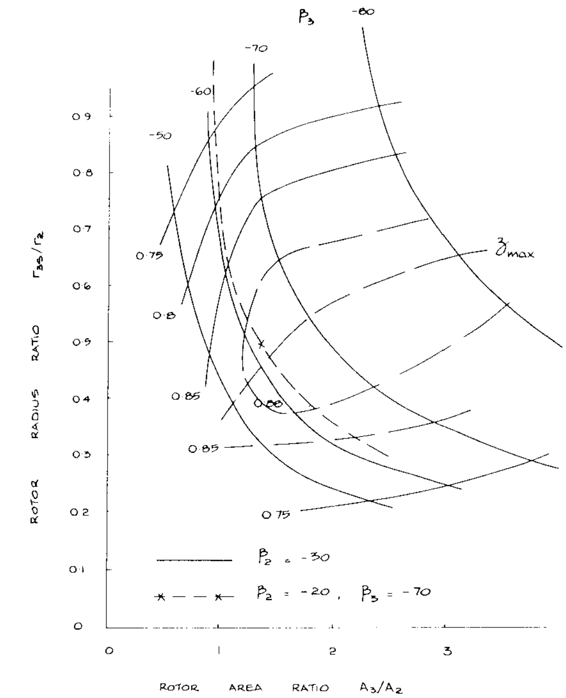


FIG 13 EFFICIENCY CONTOURS AS A FUNCTION OF ROTOR GEOMETRY

superimposed. The rotor inlet blade height can be calculated from the rotor area ratio through eqn [25] with the inclusion of a suitable boundary layer blockage factor. The rotor geometry can be obtained from the non-dimensional design with the specification of the gas mass flow rate and inlet stagnation conditions.

8 CONCLUSIONS

Through the specification of the power ratio, S_w, and the optimum angle of incidence on to a radial blade, the minimum absolute Mach number at rotor inlet can be determined. This leads to the rotor inlet velocity triangle and the minimum non-dimensional speed for the rotor. Consequently the inlet velocity triangle is no longer a variable during the design optimisation process. This not only simplifies the initial design procedure but also enables the results to be concisely presented in the form of contour plots.

The design optimisation can be carried out non-dimensionally without the need to specify the gas mass flow rate.

The empirical loss equations used showed that the magnitude of the discharge velocity ratio at which the peak efficiency occurred was a strong function of the discharge flow angle, but only a weak function of the assumed optimum angle of incidence.

When inlet Mach numbers in excess of the minimum were specified reduced efficiencies and non-dimensional inlet blade heights followed, and no justification for modifying the inlet Mach number could be found.

With the computation of the non-dimensional design of the turbine rotor all non-dimensional performance parameters can be determined and used in the design assessment. For example, the specific speed, specific diameter, non-dimensional mass flow, non-dimensional power coefficient, non-dimensional torque coefficient, and isentropic expansion ratio can be determined.

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