HOMER J. WOOD

Consulting Mechanical Engineer,
H. J. Wood and Associates,
West Los Angeles, Calif.
Mem. ASME

Current Technology of Radial-Inflow Turbines for Compressible Fluids

Radial turbines have been used for hydraulic, steam, and gas turbine service. In hydraulic turbine practice, a substantial regime of specific speed is reserved exclusively for Francis turbines. No similar situation is found with compressible fluids, where axial turbines are used for the entire specific speed range of practical interest. Francis turbines consistently achieve efficiencies of 90–94 percent, but their counterparts in compressible fluid machines generally are thought to be inferior to axial turbines in performance.

Review of available data indicates that radial inflow turbines with compressible fluids can approach or equal the performance of equivalent hydraulic turbines and that, in the specific speed range to which they are applicable, they can match axial turbine performance. Criteria are given which may be used for determination of the suitability of radial turbines for specific requirements.

Introduction

Designers of compressible fluid turbomachinery tend to select radial turbines only when considerations of cost outweigh the desire for high performance. There are valid reasons why large gas turbines utilize axial flow paths, but efficiency limitations are not among them.

Technologies have a way of becoming compartmentalized to such a degree that mutually exclusive conclusions can be reached in different industries by failing to place a particular line of evolution in its proper perspective. Water turbines and steam turbines both antedate gas turbine technology but there has been little cross-fertilization of information, and some myths have been established with respect to the usefulness of radial turbines which are overdue for exposure. Axial, mixed, and radial flow paths are found in all three of the major turbomachinery technologies, but radial turbines have remarkably different regimes of application in each.

Comparative Morphology of Power Turbines

Discussion of "radial turbines" raises difficulties in terminology. References [1 and 2][1] cover varieties of geometries which have been used, and note comparisons with water turbine practice. When mentioned herein, *radial turbines are those which have no appreciable axial component of fluid velocity entering the rotor.* This paper primarily covers the 90 deg Inward Flow Radial turbine.[2] Fig. 1 shows 90 deg geometry contrasted with a cantilever radial turbine. Selection of the 90 deg IFR turbine for emphasis is based upon practical considerations. This rotor form has the highest structural strength, and in many applications (specifically in gas turbines) stress considerations are dominant. For equal specific speeds, the cantilever radial turbine is much weaker. Also 90 deg IFR turbines are manufactured in larger numbers than other radial types.

Some writers refer to the 90 deg IFR turbine in Fig. 1(*b*) as a "mixed-flow," in that the discharge from the rotor is axial even though the entrance is radial; whereas the cantilever type has the blade flow path entirely radial. The author terms Fig. 2 a mixed-

[1] Numbers in brackets designate References at end of paper.

Contributed by the Gas Turbine Power Division and presented at the Gas Turbine Power Conference and Exhibit, Houston, Texas, March 4–8, 1962, of The American Society of Mechanical Engineers. Manuscript received at ASME Headquarters, January 9, 1962. Paper No. 62—GTP-9.

[2] Inward Flow Radial hence abbreviated to IFR.

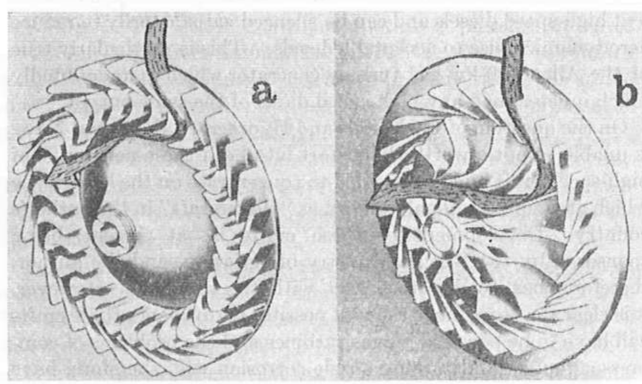

Fig. 1 Inward-flow radial turbines

Nomenclature

A_3 = turbine rotor exit area normal to meridional flow path, ft²

A_d = total disk area of rotor, $\frac{\pi}{4} D_2^2$, ft²

a = blade section area normal to radial element, ft²

a_r = blade section area at blade root, ft²

a_t = blade section area at blade tip, ft²

C_0 = isentropic spouting velocity, ft/sec

C_3 = absolute gas velocity at rotor exit, ft/sec

C_{m2} = meridional gas velocity at rotor inlet, ft/sec

C_{m3} = meridional gas velocity at rotor exit, ft/sec

D_2 = outside diameter of rotor, ft

D_R = hub diameter of blade root diameter of axial rotor, ft

D_s = specific diameter nondimensional

g = acceleration of gravity, ft/sec²

H_t = isentropic or hydraulic head across turbine stage, based on *total* pressures at both stations, ft

H_s = head across stage based on inlet *total* and exhaust *static* pressure, ft

M_3 = exhaust flow Mach number, nondimensional

N = rotative speed, rpm

N_s = specific speed, nondimensional

N_0 = rotative speed at best efficiency point, rpm

P_1 = stage inlet total pressure, psia

P_3 = stage outlet *total* pressure, psia

P_{3s} = stage outlet *static* pressure, psia

R = gas constant of working fluid, $\frac{\text{ft-lb}}{\text{lb deg F}}$

(Continued on next page)

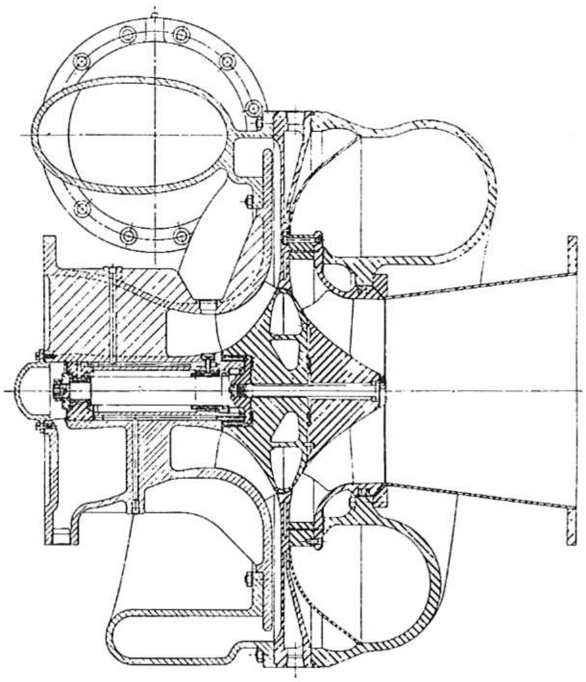

Fig. 2 Flow path of mixed-flow turbomachinery

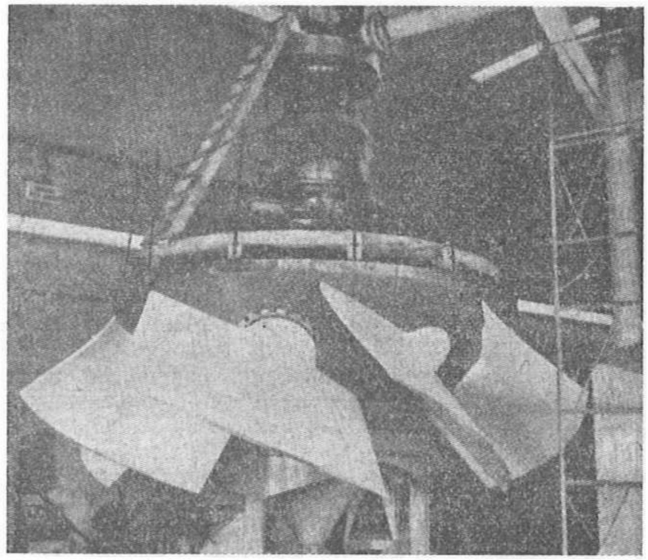

Fig. 3 Mixed-flow "Deriaz" hydraulic turbine rotor with adjustable blading ($N_s = 220$)

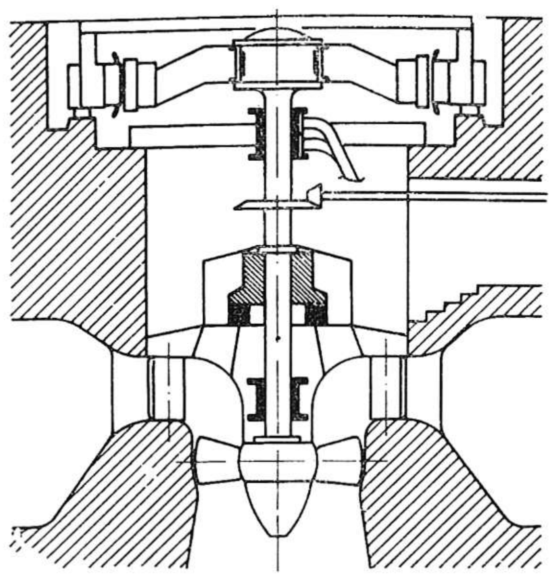

Fig. 4 Typical Kaplan turbine profile

flow type, since the meridional flow has both axial and radial components at the rotor *entrance*. Its counterpart in water turbines would be the Deriaz machine [3] shown in Fig. 3. Mixed-flow rotors can have as good structural integrity as 90 deg IFR rotors, since major blade elements may lie along radial lines. They may include geometries with specific speeds as high as axial types.

Three classes of hydraulic turbines are in general use, known as the "Pelton," "Francis," and "Kaplan" [4]. Pelton turbines have a special geometry which has no true counterpart in compressible fluid turbomachinery. Francis machines have radial inflow rotors with adjustable nozzles. In Kaplan turbines both rotor and nozzle bladings are adjustable, rotor entrance meridional flow is axial, but the variable-nozzle flow is radial, and the transition from radial to axial flow is made without guide vanes, Fig. 4. Each has found its proper application through competitive evolutions, which caused Peltons to be associated with very low, Francis turbines with medium, and Kaplans with very high specific speeds.

Fig. 6 shows a high-head Francis turbine [5], and selection of the 90 deg IFR turbine as the geometry of interest invites comparison. Figs. 6 and 7 [6] show two extremes of geometries currently found in Francis rotors; the blade forms of the high-specific-speed rotor bear little resemblance to Fig. 1(*b*) and would have a poor structural integrity for high tip speeds. Also, the high-specific-speed water turbine has an outlet diameter greater than its inlet diameter, which is not implied by the 90 deg IFR terminology. This suggests that the 90 deg IFR turbine may not be capable of covering the specific speed range associated with Francis turbines, although, as Figs. 5 and 6 show, there are water turbines which are similar in flow path to the 90 deg IFR concept.

In hydraulic turbines, the "degree of reaction" (which is reflected in the value of U/C_0 at the best efficiency point) is a free variable, although there is a trend toward higher values of U/C_0 with higher specific speeds, and Fig. 7 shows a rotor of higher reaction than Fig. 6. 90 deg IFR turbines have very little varia-

Nomenclature

Re = Reynolds number, nondimensional
r_r = radius of rotor blade root, ft
r_t = radius of rotor blade tip, ft
S_r = blade root stress, psi
T_1 = stage inlet total temperature, deg R absolute
T_{s3} = exhaust static temperature, deg R absolute
U = rotor tip speed, ft/sec
U_{AR} = tangential velocity of blade roots of axial flow rotor, ft/sec
V_3 = exhaust volume flow from turbine rotor, ft³/sec
w_2 = rotor inlet relative gas velocity, ft/sec
w_3 = rotor outlet relative gas velocity, ft/sec
W = gas flow, lb/sec
γ = ratio of specific heats, nondimensional
δ = $P_1/14.7$, nondimensional
η_s = stage efficiency, based on inlet *total* and exhaust *static* pressures, nondimensional[a]
η_t = stage efficiency, based on inlet and exhaust *total* pressures, nondimensional[a]
θ = $T_1/519$, nondimensional
ρ = specific weight of blade material, lb/in.³
τ = torque, lb-in.

[a] See "Parameter Groups."

(Continued on next page)

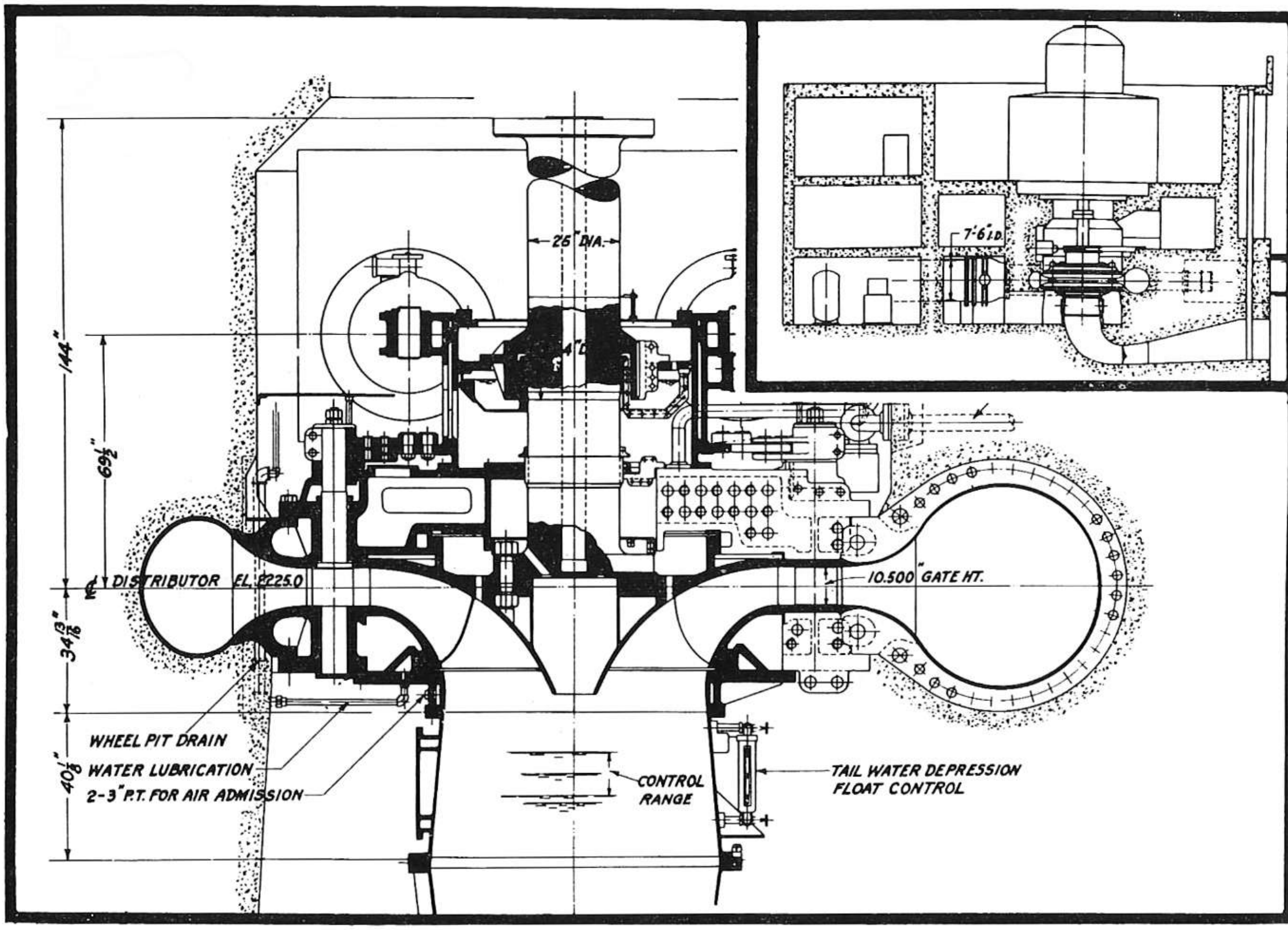

Fig. 5 Typical Francis turbine flow profile (N_s = 62)

tion of reaction with specific speed and cannot cover as broad a range of specific speed as Francis turbines for this reason.

Fig. 8(*a*) illustrates a 90 deg IFR rotor of modern design which would be suitable for gas turbine service at high tip speed and high temperature. For structural reasons, the rotor is unshrouded, and the back disk is deeply scalloped. The mating nozzle shown in Fig. 8(*b*) has variable vane angles.

Both radial and axial flow steam turbines have long histories, with radial-outflow being more common than radial-inflow. Nothing comparable to Francis or 90 deg IFR turbines has found wide acceptance in steam practice. In a typical modern central power plant, multistage axial turbines are used, with stage specific speeds in a single flow path ranging from $N_s = 10$ to $N_s = 1000$. Stage efficiencies above 90 percent have been reported.

Balje [7][3] has presented a comprehensive picture of the turbine performance spectrum, showing that axial turbines (with proper design parameters) can cover the entire specific speed range of practical interest, with efficiencies equal to those of any other turbine form. This would mean that Kaplan turbines could be designed for the same specific speed range as is normally covered

[3] Symbolism used herein is the same insofar as practicable.

Nomenclature

τ_0 = torque at best efficiency point, lb-in.
ω = rotative speed, radians/sec

Parameter Groups

$C_0 = (2gH_t)^{1/2}$
$(C_3/C_0)^2$ = exhaust energy factor
U/C_0 = rotor velocity ratio, radial
U_{AR}/C_0 = rotor velocity ratio, axial
A_3N^2 = centrifugal stress factor
A_3/A_d = rotor exhaust area ratio

$$\left.\begin{aligned} H_t &= \frac{\gamma}{\gamma - 1} RT_1 \left[1 - \left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right] \\ H_s &= \frac{\gamma}{\gamma - 1} RT_1 \left[1 - \left(\frac{P_{3s}}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right] \end{aligned}\right\} \text{for compressible fluids}$$

$$M_3 = \frac{C_3}{(\gamma g R T_{s3})^{1/2}} = \text{exhaust Mach number}$$

$\dfrac{WN}{\delta(\theta)^{1/2}} \cdot 10^{-9}$ = flow factor

$\dfrac{S_r/\delta}{A_3N^2}$ = blade stress parameter

$$\eta_t = \frac{550\ (\text{output horsepower})}{W \cdot H_t}$$

$$\eta_s = \frac{550\ (\text{output horsepower})}{W \cdot H_s}$$

$$\frac{\eta_t}{\eta_s} - 1 = \left(\frac{C_3}{C_0}\right)^2 \left\{\frac{\left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \eta_t\left[1 - \left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}\right\} \approx \left(\frac{C_3}{C_0}\right)^2$$

$$N_s = \frac{N(V_3)^{1/2}}{H_t^{3/4}}$$

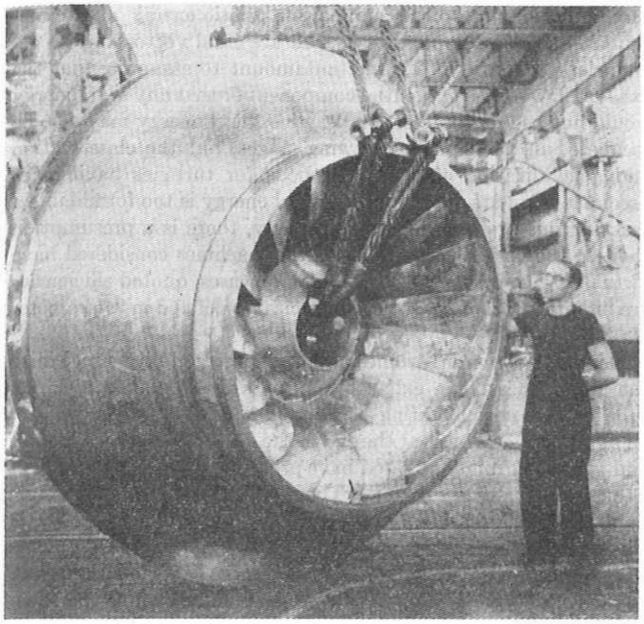

Fig. 6 High-head Francis rotor (N_s = 60)

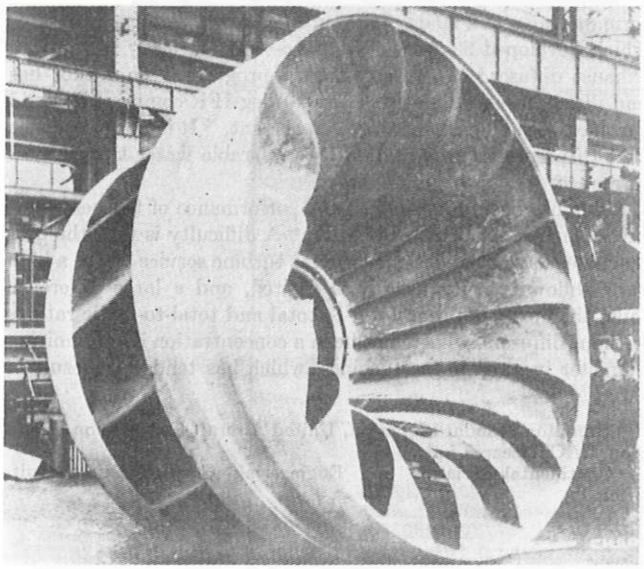

Fig. 7 Low-head Francis rotor (N_s = 160)

by Francis turbines. However, there is difficulty in attempting to use the design methods of [7] for water turbines having specific speeds normally covered by Pelton wheels. The analogy fails because the Pelton wheel solves the problem of high disk friction associated with low specific speeds by avoiding submergence of the disk in the working fluid. Thus, although it would be possible to build water turbines corresponding to the partial-admission axial stages of very low specific speed described in [7], their performance would be inferior to that of Pelton wheels. Turbines which must operate with rotors submerged in the working fluid tend to have efficiency losses dominated by disk friction and leakage in the low-specific-speed range, and by losses associated with high through-flow velocity in the high-specific-speed regions.

A low ratio of blade height to disk diameter is optimum for low specific-speed axial turbines [7]. When low blade height ratios are utilized, flow path efficiency is independent of whether the meridional flow at rotor entrance is axial, radial, or mixed; thus a cantilever radial turbine of low specific speed, such as appears in Fig. 1(a), would produce the same performance as its axial flow counterpart, since the blade geometries would, for all practical purposes, be identical and the disk friction similarly equal; fluid

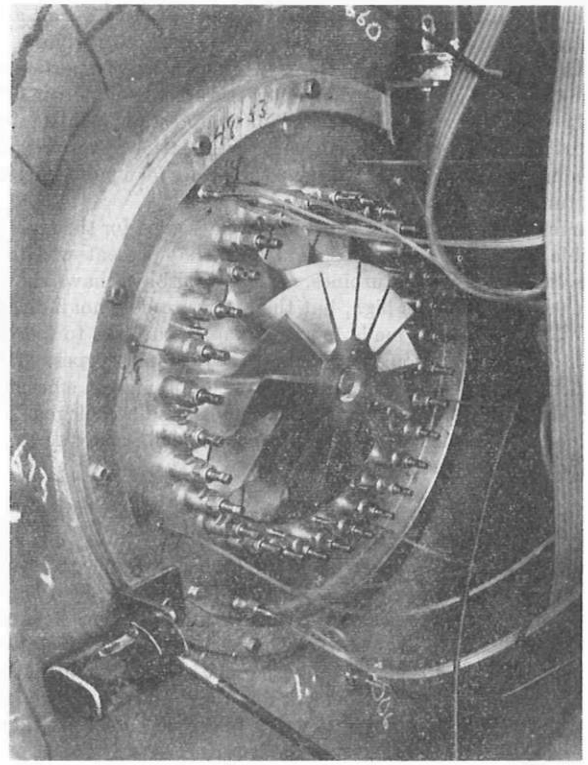

Fig. 8(a) Model test 90 deg IFR rotor in rig (N_s = 99)

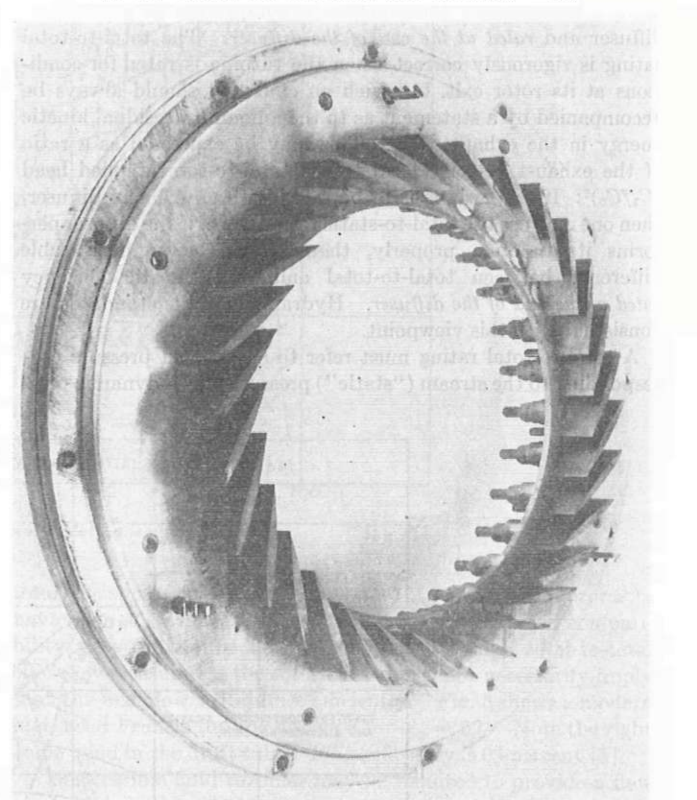

Fig. 8(b) Adjustable nozzle for test rig

dynamics considerations do not then favor one sort of turbine or another, and matters of stress or flow path convenience may be regarded as determinate.

In the last twenty years, 90 deg IFR turbines have been used in substantial production for small gas turbines, automotive turbochargers, pneumatic starters for jet engines, and various aircraft auxiliary services. Their specific speeds lie generally in the range reserved for Francis turbines in hydraulic practice, and morphological similarities are apparent. However, in both steam and gas turbines (particularly in larger sizes) axial turbines of equal specific speed are used. Thus the reservation of a particular specific speed regime for radial inflow turbines accepted in hydraulic technology is not reflected in compressible fluid design practice. There has been a tendency in the latter case to regard the radial turbine as merely a cheap substitute for a more efficient axial turbine.

Large hydroelectric power turbines are designed for the highest possible operating efficiencies, and above 90 percent was first achieved with Francis turbines. Kaplan turbines have shown efficiencies virtually as high, but the literature does not indicate that efforts have been made to apply Kaplan turbines to specific speed regimes traditionally dominated by Francis types. Hydraulic turbine designers are making no compromise in efficiency when they select Francis turbines. Why should homologous gas turbines be regarded as inferior to axial machines in the same specific speed regime?

How High Is High Efficiency?

Lack of technical liaison between steam, hydraulic, and gas turbine designers is compounded by confusion as to just what is an "honest" efficiency. There are important differences between total-to-total and total-to-static efficiency ratings [7]. Total-to-static efficiency has the flaw that a perfect isentropic machine so rated would not have 100 percent efficiency unless its throughflow were zero. It is not rational to presume that the kinetic energy of the flow leaving a turbine rotor necessarily is lost or is not otherwise useful; yet this is what the total-to-static rating really means, unless the machine is equipped with an exhaust diffuser and *rated at the end of the diffuser*. The total-to-total rating is rigorously correct when the turbine is rated for conditions at its rotor exit, but such an efficiency should always be accompanied by a statement as to the amount of residual kinetic energy in the exhaust flow. This may be expressed as a ratio of the exhaust velocity head to the total-to-total applied head $(C_3/C_0)^2$: If the turbine is to be rated with an exhaust diffuser, then one may use the total-to-static rating since, if the diffuser performs its function properly, there should be no measurable difference between total-to-total and total-to-static efficiency *rated at the end of the diffuser*. Hydraulic turbine standards are consistent with this viewpoint.

A total-to-total rating must refer to an exhaust pressure corresponding to the stream ("static") pressure plus a dynamic pressure corresponding to the summation kinetic energy of the meridional velocity components (which are axial vectors in the geometries of interest). This is tantamount to assuming that the exhaust flow has no tangential component or that any swirl energy would not be recoverable. In practice, swirl energy may also be recovered in diffusers or following stages, but the classification and generalization of design methods for turbines having appreciable amounts of recoverable swirl energy is too formidable a task for this presentation. Accordingly, there is a presumption that, at the best efficiency points, the machines considered have virtually swirl-free exhaust flow. (In all cases, quoted efficiencies are "isentropic shaft power" efficiencies, as defined in "Parameter Groups.")

Designers of water turbines having submerged rotors recognize the importance of an exhaust diffuser which is called a "draft tube." The normal rating of a water turbine quotes efficiency on a total-to-total basis at the end of the draft tube. Both Francis and Kaplan turbines so rated have produced overall efficiencies of 94 percent or slightly better. The best comparable compressible fluid data directly available to the author is a total-to-static efficiency, without an exhaust diffuser, of a 90 deg IFR turbine which made 89 percent at a pressure ratio of 6:1. This turbine had an exhaust energy factor $(C_3/C_0)^2$ of 0.05, which would then give it a total-to-total efficiency of 93 percent.[4] It is likely that, with an exhaust diffuser, this same turbine would have produced a total-to-static efficiency of 92 percent. Fig. 9 shows a performance map of a 90 deg IFR turbine, shown in Figs. 8(*a*), 8(*b*), which developed 93 percent total-to-total efficiency without an exhaust diffuser.[5] In three separate programs, the author has had direct access to test data on 90 deg IFR compressible fluid turbines with efficiencies above 90 percent. It is no coincidence that this figure checks well with comparable water turbine performance.

Little has been published on the performance of the best compressible fluid axial turbines [8, 9]. A difficulty is that the best machines have been designed for gas turbine service where a high throughflow velocity usually is desired, and a large difference must then exist between total-to-total and total-to-static ratings without diffusers. There has been a concentration on minimizing diameter in aircraft gas turbines, which has tended to result in

[4] Hamilton Standard Division, United Aircraft Corporation. Data from UAC Research Labs.

[5] Continental Aviation and Engineering Corporation, Detroit, Mich.

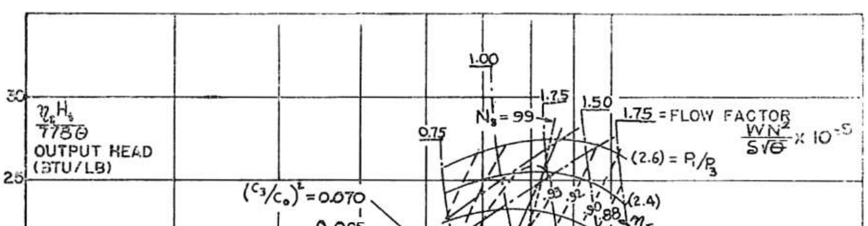

Fig. 9 90 deg IFR turbine performance (N_s = 99)

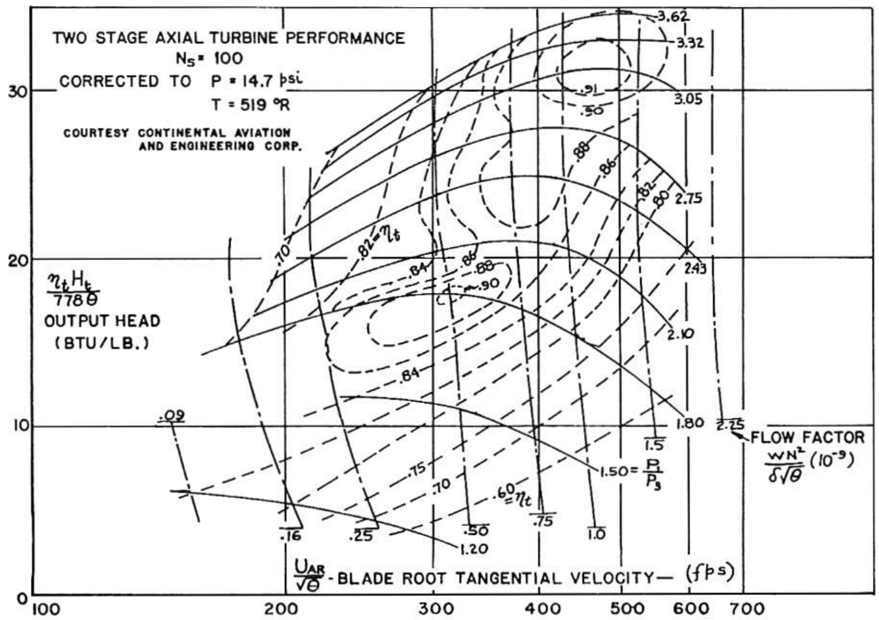

Fig. 10 Two-stage axial-turbine performance (N_s = 100)

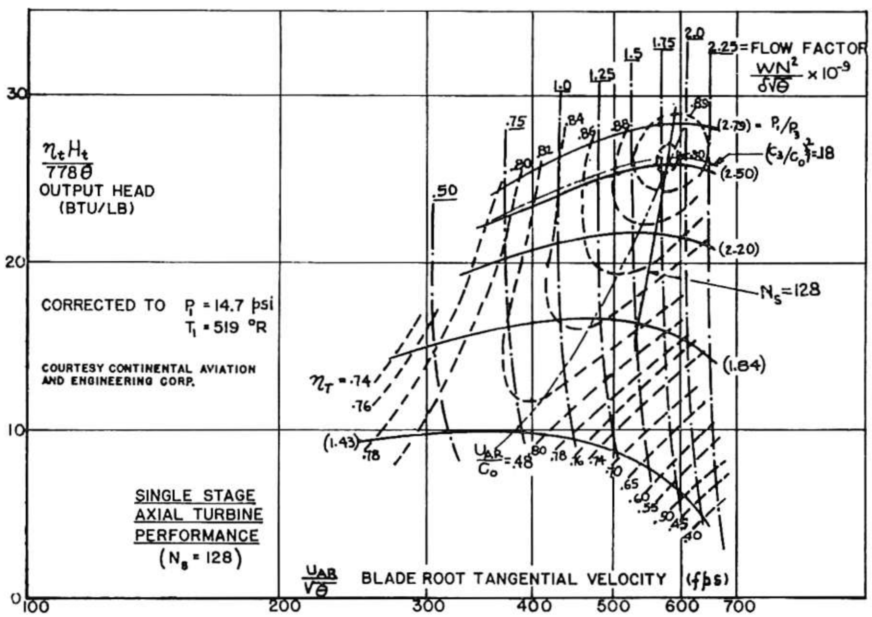

Fig. 11 Single-stage axial-turbine performance (N_s = 128)

efficiencies somewhat less than might be expected at the corresponding specific speed. (See [7] re the "Specific Diameter" influence.) These circumstances, combined with security restrictions and reluctance to release information, have provided little published high-performance axial turbine data. The writer has seen instances of axial turbines showing total-to-total efficiencies above 90 percent. Fig. 10[5] shows a performance map of a two-stage axial turbine exceeding 90 percent, and Fig. 11[5] shows a single-stage machine which made 90 percent. In Fig. 11, a total-to-static efficiency of only 77 percent would be implied if there were no exhaust diffuser, and the specific speed of this turbine lies within a range which would be reserved for a Francis configuration in the water turbine tradition.

Controversy regarding total-to-total versus total-to-static efficiency ratings can be resolved only in terms of functional analyses of the exhaust flow. In water turbines, where only single stages are used and the need for most efficient use of available hydraulic power is dominant, it is natural to regard turbines without diffusers as essentially incomplete. Accordingly, although total-to-total efficiences (rated at the *rotor outlet*) of 95–96 percent probably have been achieved, this is of little significance, since compatibility with draft tube flow is essential. A high total-to-total efficiency measured at the rotor outlet does not necessarily imply that the exit flow will diffuse efficiently. Fig. 5 shows a modern high-head Francis turbine designed for N_s = 62. Note the right angle bend in the draft tube. Peak efficiency is 93 percent [5].

Compressible fluid turbines may be required to provide a flow compatible with many arrangements (such as following stages, a jet nozzle festooned with structure, or a piping system catering more to space limitations than efficiency). In consequence, the motivation for exhaust diffusion as a standard rating procedure is not strong, and the tendency has been to pay insufficient attention to this matter. In many instances, diffusers have been left off systems where the performance benefits would be substantial, with too much emphasis on reduction of rotor exit velocities. High specific speeds may require exhaust velocities which represent serious kinetic energy losses if not diffused efficiently.

Medium-to-high-specific-speed turbines should be developed to have uniform flow velocities at the rotor exit with little or no swirl, and design methods which ignore exhaust flow can be quite wasteful. When diverging conical exhaust diffusers are used, some swirl in the fluid next to the outer wall can be helpful in improving performance. Such normally occurs in a nominally zero-swirl exhaust flow from a turbine rotor because the peripheral leakage always has a substantial tangential flow component.

In comparing "best available" performance data of compressible and incompressible fluid turbines, specific cases have specific inhibitions on design configuration which may reduce performance. Reynolds number effects may be prominent in either case, and Mach number influences have first-order significance in compressible fluid flow paths. Detail analyses of the physics of boundary-layer flow show subtleties related to fluid properties to a degree that it might be concluded that water and gas turbines would *not* show similar performance; but the recorded facts support the conclusion that, for equivalent specific speeds, optimum specific diameters, and high Reynolds numbers, the best water and gas turbines have equivalent efficiencies. Furthermore, the "best available" data show total-to-total efficiencies of 90–94 percent for specific speeds ranging from 60 to 600.

Significance of Specific Speed

Balje [7] has given an extensive mathematical treatment to the specific speed (N_s) and specific diameter (D_s) concepts. The basic mathematics are not repeated here.

For 90 deg IFR turbines it is helpful to factor N_s as follows:

$$N_s = \frac{N(V_3)^{1/2}}{H_t^{3/4}} \alpha \left(\frac{V_3}{ND_2^3}\right)^{1/2} \cdot \left(\frac{U}{C_0}\right)^{3/2} \qquad (1)^6$$

The term

$$\frac{U}{C_0} = \frac{U}{(2gH_t)^{1/2}} \qquad (2)$$

is found to vary only over a small range for the complete spectrum of practical 90 deg IFR turbines at the best efficiency point *if* the best efficiency point occurs with zero exhaust swirl. Thus, in practice, at the best efficiency point:

$$0.69 < \frac{U}{C_0} < 0.725$$

or

[6] In hydraulic turbine practice, specific speed $(N_s)^1$ is commonly quoted as $(N_s)^1 = N(HP)^{1/2}/(H_t)^{5/4}$. To convert $(N_s)^1$ for *water* turbines to N_s, the following relationship may be used: $N_s/(N_s)^1 = 3/(\eta_t)^{1/2}$.

$$(U/C_0)_{\text{bep}} = \left(\frac{1}{2}\right)^{1/2} \approx 0.71 \qquad (3)$$

(H_t is the total-to-total head, but V_3 is the actual volume flow leaving the turbine.)

To a close approximation, for 90 deg IFR turbines

$$N_s \alpha \left[\frac{(V_3)}{ND_2^3}\right]^{1/2} \qquad (4)$$

(No such simplification would be possible if C_0 were based on a total-to-static head.)

Data, from $N_s = 30$ to 100, show no important trend in U/C_0 at best efficiency, although efficiency does vary, as indicated in Fig. 12. Disk friction and leakage are the major loss factors in 90 deg IFR turbines at $N_s < 60$.

To see the practical significance of equation (1) we write:

$$V_3 = C_3A_3 \text{ (presumes } C_3 \text{ is uniform)} \qquad (5)$$

$$C_0^2 = 2gH_t \qquad (6)$$

$$N_s = (C_3A_3)^{1/2} \cdot \frac{N}{(C_0)^{3/2}} \cdot (2g)^{3/4} \qquad (7)$$

$$= \left(\frac{C_3}{C_0}\right)^{1/2} \cdot \left(\frac{A_3N^2}{C_0^2}\right)^{1/2} (2g)^{3/4} \qquad (8)$$

$$= 22.7 \left(\frac{C_3}{C_0}\right)^{1/2} \cdot \left(\frac{A_3N^2}{U^2}\right)^{1/2} \cdot \left(\frac{U}{C_0}\right) \qquad (9)$$

Since $(U/C_0)_{\text{bep}} = 0.71$ for 90 deg IFR turbines

$$N_s = 16.04 \left(\frac{C_3}{C_0}\right)^{1/2} \left(\frac{A_3N^2}{U^2}\right)^{1/2} \qquad (10)$$

Since $U^2 = \dfrac{\pi^2N^2D_2^2}{3600}$

$$N_s = 271.5 \left(\frac{C_3}{C_0}\right)^{1/2} \left(\frac{A_3}{A_d}\right)^{1/2} \qquad (11)$$

Since $(C_3/C_0)^2$ represents the ratio of exhaust kinetic energy to total head, we can write:

$$\frac{\eta_t}{\eta_s} \approx 1 + \left(\frac{C_3}{C_0}\right)^2 \qquad (12)$$

In gas turbine practice,

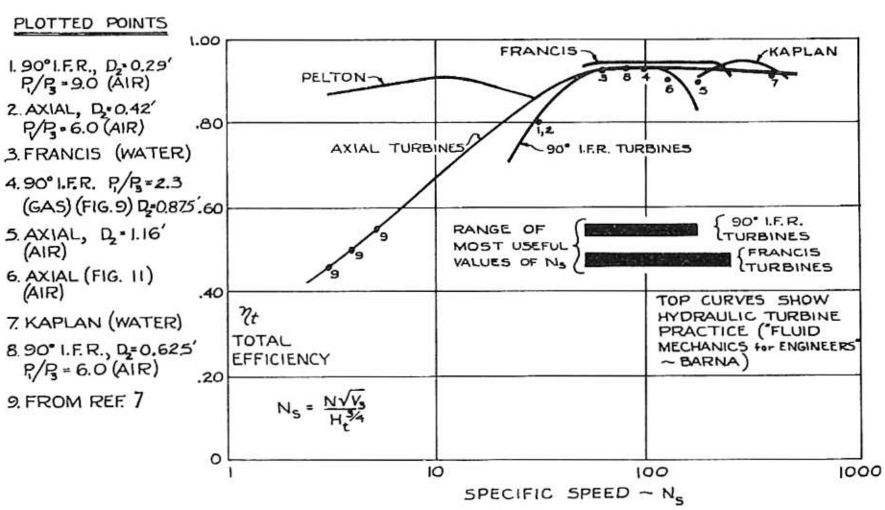

Fig. 12 Specific-speed characteristics—turbines (best current data for "submerged rotors")

$$0.04 < \left(\frac{C_3}{C_0}\right)^2 < 0.30 \qquad (13)$$

$$\text{Flow tends to be unstable with } \left(\frac{C_3}{C_0}\right) < 0.04 \qquad (14)$$

Also, practical geometries generally yield

$$0.1 < \frac{A_3}{A_d} < 0.5$$

Using $$\frac{A_3}{A_d} = 0.1, \quad \left(\frac{C_3}{C_0}\right)^2 = 0.04 \qquad (15)$$

$$N_s = 38.7$$

Using $$\frac{A_3}{A_d} = 0.5, \quad \left(\frac{C_3}{C_0}\right)^2 = 0.30$$

$$N_s = 142$$

Fig. 13 plots $N_s = \mathcal{f}\left[\left(\frac{A_3}{A_d}\right), \left(\frac{C_3}{C_0}\right)^2\right]$ for $U/C_0 = 0.71$ (applicable to 90 deg IFR turbines)

Noting that [7]

$$D_s = \frac{D_2 H_t^{1/4}}{V_3^{1/2}} = \frac{1}{(U/C_0)^{1/2}\cdot\left(\frac{V_3}{ND_2^3}\right)^{1/2}} \qquad (16)$$

Since $U/C_0 = 0.71$ (a constant)

$$D_s \alpha \frac{1}{N_s} \qquad (17)$$

Fig. 12 presents current state-of-the-art for hydraulic and compressible fluid turbines. The limiting efficiencies shown as functions of specific speed are the best which might be expected if Reynolds number and/or Mach number are favorable and no compromises are made to degrade efficiency in favor of weight, bulk, diameter, cost, or other compromising factors. Statistically, compressible fluid turbines and hydraulic turbines with submerged rotors do not have different performance limits, but Pelton turbines are much better than submerged rotor turbines of the same specific speed.

The best radial flow turbines match best axial flow turbine performance in a limited specific speed range, but from $N_s = 3.0$ to 10,000, no other form of turbine flow path exceeds the peak performance capability of the axial flow turbine. (This applies to incompressible fluid turbines as long as the specific speed is higher than the applicable range of Pelton turbines.) Axial turbines do not always represent the best design solution, but the specific speed criterion alone cannot be used to rule explicitly in favor of a radial flow path.

Stress Considerations

Stress patterns in 90 deg IFR turbines may be analyzed by treating the main disk and exducer as separate, even if a one-piece construction is used. The exducer is an axial flow member and may be treated as such. Centrifugal stresses in axial blading may be related to the parameter A_3N^2 as illustrated in Fig. 14.

Main disk stresses are more complex, particularly due to thermal gradients. The deeply scalloped construction shown in Fig. 8(a) permits drastic tapering of the radial blades to hold down centrifugal stress. It also serves to minimize thermal stresses (which can be very high in a full disk wheel). Use of these techniques permits tip speeds (U) of over 2000 ft/sec with high gas temperatures, and many gas turbines have been built in the last decade with tip speeds of 1700 fps and 1650 deg F inlet gas temperature. However, it is necessary to cool the hub, since this zone is highly stressed. Failure to use hub cooling seriously limits the 90 deg IFR turbine for gas turbine service, and this point has been overlooked in several engines using 90 deg IFR stages.

Specific speed can be written in stress-limiting terms as follows [rewriting equation (10)]:

$$N_s = 16.04\frac{(A_3N^2)^{1/2}}{U}\cdot\left(\frac{C_3}{C_0}\right)^{1/2} \qquad (18)$$

In 90 deg IFR turbines, *main disk stresses* primarily are determined by the tip speed (U), and *exducer blade root stresses* by the factor A_3N^2.

In gas turbine practice, an upper limit value of A_3N^2 would be about $3.8\cdot10^8$, and a value of (U) of about 1800 fps is well justified. Using $\left(\frac{C_3}{C_0}\right)^2 = 0.30$, a value of specific speed is obtained of:

$$N_s = \frac{16.04(0.30)^{1/4}}{1800}(3.8\cdot10^8)^{1/2}$$

$$= \underline{128.5}$$

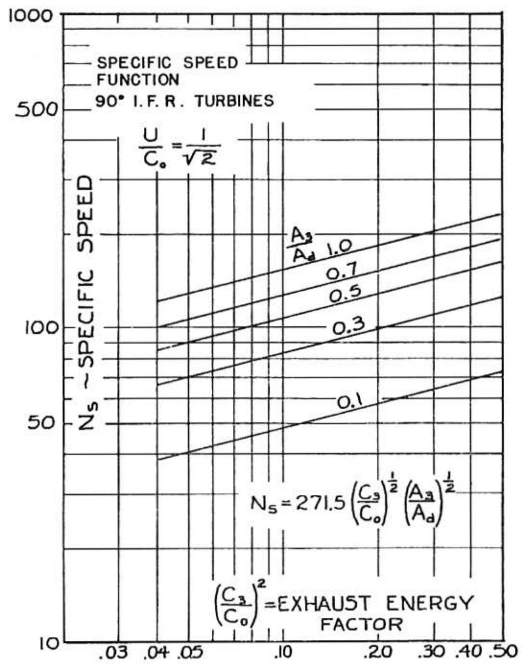

Fig. 13 Specific-speed function—90 deg IFR turbines

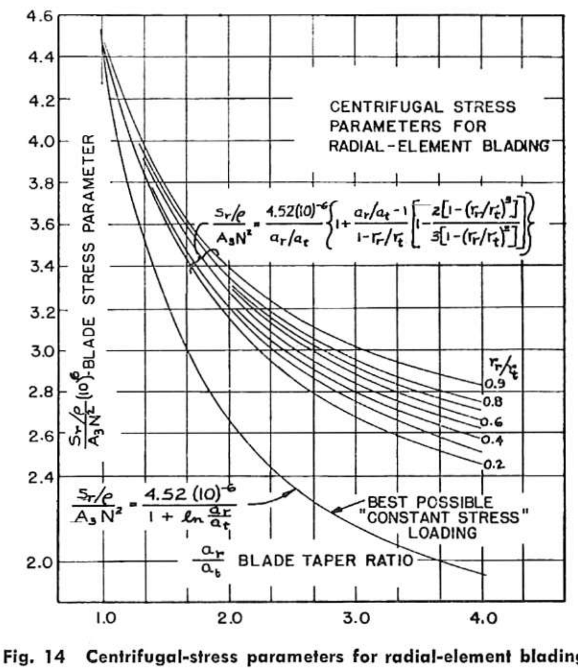

Fig. 14 Centrifugal-stress parameters for radial-element blading

Note that $N_s \alpha \frac{1}{U}$

Since exducer and main disk represent separate structural problems, it often can be established that the use of high tip speeds to match high heads per stage *reduces* the value of specific speed which can be utilized. The numerical example is to illustrate an analytical technique rather than to set a finite limit.

$$\text{Noting that } (A_3N^2/U^2) = 286(A_3/A_d). \tag{19}$$

Fig. 13 may be used to solve equation (18).

Axial Versus Radial Stage Loadings

Although the many variables of detail design make direct comparisons of axial and radial turbines difficult, some insight may be gained, noting that axial turbine blade root stresses in gas turbines also are directly related to the parameter A_3N^2 and disk stresses are related to U_{AR}, where U_{AR} is the tangential velocity of the blade roots. For *efficient* axial turbines, $U_{AR}/C_0 \geq 0.47$, which means that the roots run at "impulse" conditions, or with some "reaction."

Equation (8) is valid for axial turbines and equation (9) may be rewritten as:

$$N_s = \left(\frac{C_3}{C_0}\right)^{1/2} \cdot \left(\frac{A_3N^2}{U_{AR}^2}\right)^{1/2} \cdot \left(\frac{U_{AR}}{C_0}\right) \cdot (2g)^{3/4} \tag{20}$$

Using $\frac{U_{AR}}{C_0} = 0.47$, and substituting $(2g)^{3/4} = 22.7$,

$$N_s = 10.7 \left(\frac{C_3}{C_0}\right)^{1/2} \cdot \left(\frac{A_3N^2}{U_{AR}^2}\right)^{1/2} \tag{21}$$

Since

$$U_{AR}^2 = \frac{\pi^2N^2D_R^2}{3600} \tag{22}$$

and

$$A_d = A_3 + \frac{\pi}{4} D_R^2 \tag{23}$$ [7]

$$N_s = 181 \left(\frac{C_3}{C_0}\right)^{1/2} \left(\frac{1}{A_d/A_3 - 1}\right)^{1/2} \tag{24}$$

Equation (24) may be compared with equation (11).
Using

$$\frac{A_3}{A_d} = 0.5, \quad \left(\frac{C_3}{C_0}\right)^2 = 0.30$$

$$N_s = 134$$

Fig. 15 plots equation (24). Comparing equations (21) and (10) for a fixed design requirement, $(C_3/C_0)^2$, N_s, and A_3N^2 would be the *same* for either an axial or a 90 deg IFR turbine. In such case $U/U_{AR} = 16.04/10.7 = 1.5$. ($U$ is the *tip* speed of the radial turbine and U_{AR} is the *blade root speed* of the axial turbine!) Thus the tip speed of a 90 deg IFR turbine must be 50 percent higher than the blade root speed of an axial turbine. In practice, this can be achieved with *lower stresses* in the *radial turbine!* Accordingly, for equal specific speeds, the radial turbine can handle a higher head efficiently in a single stage.

As long as $(C_3/C_0)^2$ is the same for all turbine geometries considered, the power output per stage can be higher for the radial turbine, since it can match a higher head. However, if disk stresses to match a specified head are not critical, the turbine which can tolerate the higher value of $(C_3/C_0)^2$ must have higher power. For these and many other reasons, generalities are dangerous. Convenient comparison equations have been presented for consideration of specific problems.

[7] This ignores the difference between geometric and effective exhaust flow areas. The approximation is close enough for purposes of this argument.

The $(C_3/C_0)^2$ parameter is important, since it can be larger for an axial turbine than for a 90 deg IFR turbine because of the meridional turning losses in the latter. In practice $(C_3/C_0)^2 = 0.20$ produces no measurable performance loss in the total-to-total efficiency of a 90 deg IFR machine, and the upper limit has not been explored in data available to the author. Probably $(C_3/C_0)^2 = 0.30$ is somewhat high for the 90 deg IFR flow path, whereas many axial gas turbine stages run at or near this value. If exhaust diffusion is necessary to achieve a high *net* total-to-static efficiency, values of $(C_3/C_0)^2$ above 0.20 are questionable in either case.[8]

"Stage loading" has no generally accepted rigorous definition. It refers to the head per stage, and sometimes to the horsepower. Thus, for equal available heads within single stage capacity, the higher maximum N_s of the axial turbine implies a higher horsepower per stage; but the 90 deg IFR turbine can handle a higher head per stage, so "stage loading" comparisons can be ambiguous.

Another stage loading criterion is pressure ratio. For equal inlet temperatures, higher pressure ratios are related to higher heads, and this favors the radial turbine. However, for any class of turbine, increasing pressure ratios must mean higher relative Mach numbers. For equal specific speed, $(C_3/C_0)^2$, and pressure ratio, a radial turbine will have relative Mach numbers equal to or lower than an axial turbine. Furthermore, relative Mach numbers tend to increase with increasing specific speed. It can be argued that a 90 deg IFR turbine is thus better suited to operation at high pressure ratios, and data are available showing efficiencies above 80 percent at pressure ratios up to 20:1 at $N_s = 30$. Under these conditions, absolute tangential velocities of the gas entering the rotor are supersonic, but relative velocities throughout the rotor are subsonic. Also, converging nozzles are used, and gas acceleration to supersonic speeds occurs by free-vortex action or oblique expansion shock patterns in the nozzle-to-rotor clearance space. Thus a 90 deg IFR turbine avoids supersonic turns at pressure ratios requiring them in axial flow paths. A few years ago this was important, and it could be claimed that 90 deg IFR turbines were superior at high pressure

[8] High values of $(C_3/C_0)^2$ are not found in Francis turbines because of limits imposed by draft tube cavitation.

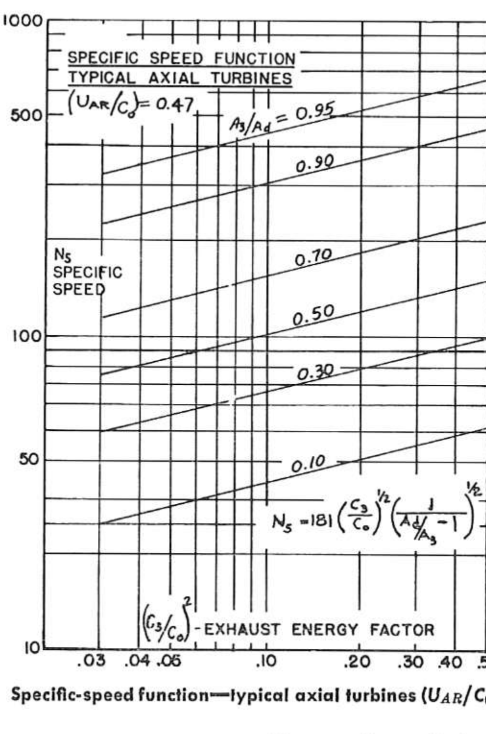

Fig. 15 Specific-speed function—typical axial turbines $(U_{AR}/C_0) = 0.47$

ratios, but it is now established that high-angle supersonic turning passages are at least as efficient as subsonic turns (provided an adequate supersonic design technique is used). Accordingly, for equal specific speeds, properly designed axial and radial turbines are equally capable of utilizing high pressure ratios per stage.

All forms of turbine have pressure ratio limits imposed by the exhaust "choking" limit, which occurs when C_3 approaches sonic velocity (for simplicity, only the no-swirl case is considered). This condition is expressed by the equation:

$$\left(\frac{C_3}{C_0}\right)^2 = \left[\frac{1}{1 - \left(\frac{P_3}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} - \eta_t\right]\left[\frac{\frac{\gamma-1}{2}M_3^2}{1 + \frac{\gamma-1}{2}M_3^2}\right] \quad (25)$$

which reduces to:

$$\frac{P_1}{P_3} = \left[\frac{\left(\frac{C_3}{C_0}\right)^2 + \frac{\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_3^2}\eta_t}{\left(\frac{C_3}{C_0}\right)^2 - \frac{\frac{\gamma-1}{2}M_3^2}{1+\frac{\gamma-1}{2}M_3^2}(1-\eta_t)}\right]^{\frac{\gamma}{\gamma-1}} \quad (26)$$

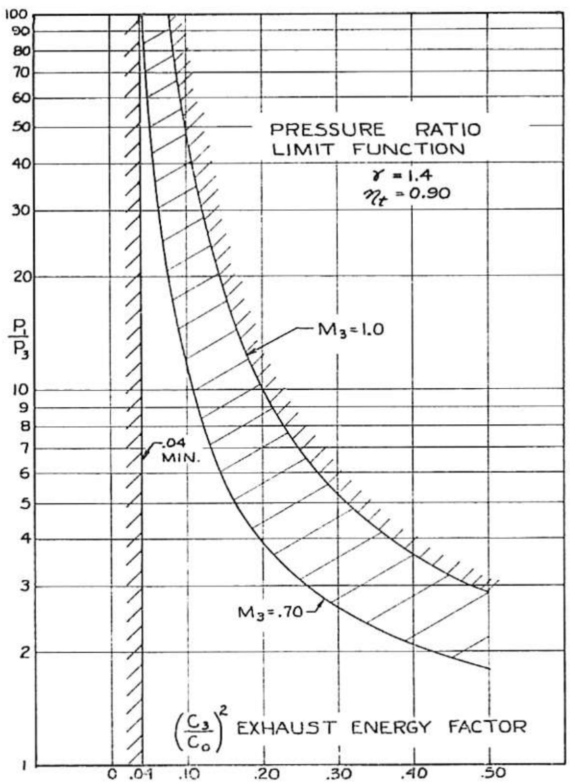

Fig. 16 Pressure-ratio limit function (γ = 1.4, η_t = 0.90)

In Fig. 16 this function is plotted for $\eta_t = 0.90$, $\gamma = 1.4$, and $M_3 = 1.0$ and 0.7. In practice, effective exhaust choking occurs at nominal values of M_3 equal to 0.7 or slightly higher instead of the ideal case of $M_3 = 1.0$. $(C_3/C_0)^2$ has a first-order effect on pressure ratio limits of single-stage, full-admission turbines. Exhaust swirl, if present, *lowers* the pressure ratios at choke. Fig. 16 also demonstrates that high pressure ratios tend to force the use of lower specific speeds, since, as illustrated by Fig. 13, N_s follows the trend of $(C_3/C_0)^2$. This is an important case where the water turbine analogy breaks down.

As used herein, "choking" refers to a condition where further increase in exhaust pressure produces no increase in output torque with exhaust conditions at or near zero swirl. In a plot like Fig. 9, choking sets an upper limit on the ordinate parameter. The phenomenon *may* occur when rotor exit relative velocities (w_3) are sonic, which means that C_3 must be subsonic. However, when the turbine rotor is suitably designed, w_3 may be supersonic, with efficient performance. In *all* cases, $M_3 \rightarrow 1.0$ sets a design limit, since the rotor cannot then sense further decreases in exhaust pressure. In practice, boundary-layer, wake, and distribution effects result in sonic velocity when $M_3 = 0.7$ for the ideal (no blockage) case.

Operating Characteristics

Confusion exists regarding the off-design operating characteristics of all turbines, and the 90 deg IFR class is no exception. Occasionally one or another particular design is claimed to have a "broader operating range," but the terminology is misleading.

Turbines using incompressible fluids can have their basic characteristics illustrated on single maps, provided there is only one element of variable geometry. As soon as compressible fluids are used, only fixed geometry can be represented on a single map, and a three-dimensional plot is required for even a single element of variable geometry. The picture is complicated by whether or not the exhaust swirl which may exist at off-design operation is recoverable, and it is necessary to define the exhaust flow path to present rational data.

If it is presumed that the exhaust flow path is not capable of either recovering swirl energy or developing abnormal losses with swirl (as occurs with a converging flow path), some generalizations may be made. An axial turbine of fixed geometry operating at constant head has less variation of efficiency with speed than its 90 deg IFR counterpart, and this is true for either compressible or incompressible fluids. Fig. 17 shows torque and power curves for both types to illustrate this point. The *linearity* of the axial turbine torque curve is confirmed by data. The extent to which the 90 deg IFR torque curve lies below the axial torque curve is largely determined by the ratio of the *mean* effective diameter

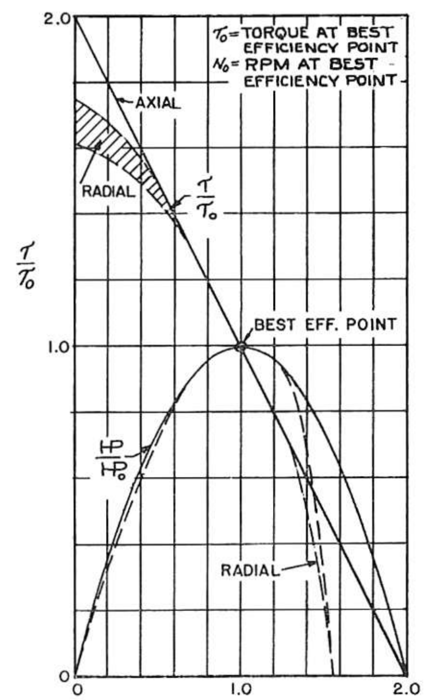

Fig. 17 Comparison of axial and radial-turbine torque characteristics

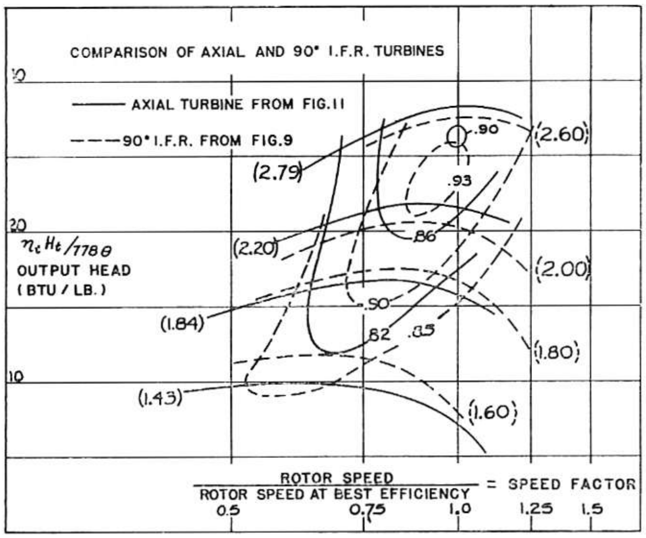

Fig. 18 Comparison of axial and 90 deg IFR turbines

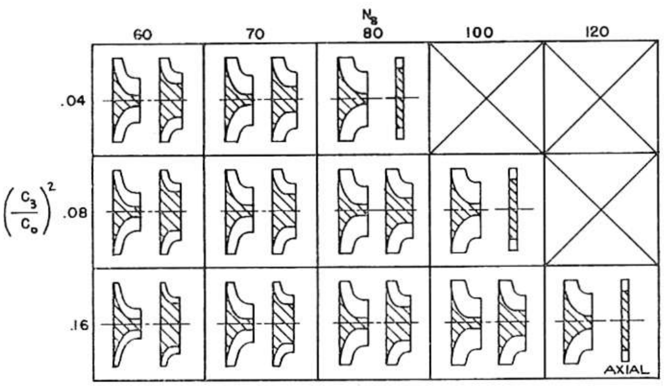

Fig. 19 Influence of N_s and $(C_3/C_0)^2$ on 90 deg IFR rotor profile geometry

of the exducer blading to the outside diameter of the radial turbine rotor; the lower off-design torque is really due to the lower angular momentum of the fluid leaving the 90 deg IFR rotor when swirl is present.

Fixed geometry axial and 90 deg IFR turbines using compressible fluid show different effects of pressure ratio when both are designed for the same best efficiency point. The best obtainable efficiencies at off-design pressure ratios are *higher* for the 90 deg IFR turbine, and this is shown in Figs. 9 and 11. Again the reason relates to exhaust swirl, with internal flow compatibility also being involved. In effect, at *off-design* pressure ratios, both 90 deg IFR and axial turbines develop peak performance with some exhaust swirl, but the swirl energy is less in the 90 deg IFR flow path. In this case, a small exducer mean diameter minimizes swirl energy losses. A "broader operating range" may be claimed for either type machine, depending upon the off-design variable. Fig. 18 illustrates the basic trends in a single plot by over-plotting Figs. 9 and 11.

With either radial or axial turbines, exhaust diffusers capable of recovering swirl energy improve off-design performance. Similarly, following stages with blading insensitive to inlet angle-of-attack can recover exhaust swirl energy.

A 90 deg IFR turbine of a given specific speed may have various meridional flow configurations of equal "best" efficiency, as illustrated in Fig. 19. The large-hub-exducer machine would behave like its axial flow counterpart, and 90 deg IFR turbines can have surprisingly different performance maps. Single-stage axial turbines show an amazing similarity when their maps are reduced to common parameters. Blunt versus sharp leading edges and design point relative Mach numbers have little influence on performance deterioration at off-design conditions, if exhaust axial Mach numbers are below 0.7.

Figs. 9 and 11 bring out clearly that the optimum values of U/C_0 and U_{AR}/C_0 shift with pressure ratio in fixed-geometry turbines. Whether lines of constant U/C_0 or lines of optimum efficiency are followed, the 90 deg IFR turbine shows less variation of efficiency with pressure ratio than does the axial machine. This trend is more marked at lower specific speeds.

Internal Fluid Dynamics

Water turbines have been developed by long evolution of scale models, but the end products show strong evolutionary convergence between independent firms. 90 deg IFR turbines for compressible fluids were first evaluated by running compressors backward, and the results,[9] showing about 90 percent efficiencies, were generally disbelieved. It was not until later that the importance of exducer design was appreciated, since designers were misled by the predictably small swirl losses. However, the exducer flow strongly influences conditions at the rotor tip, and these losses are significant. Also, the relative importance of internal flow losses with respect to disk friction and leakage at lower specific speeds often have been ignored or misunderstood [7].

A one-dimensional analysis of an isentropic 90 deg IFR turbine with an infinite number of blades on a total-to-total basis shows that, for zero exit swirl and shockless rotor entry, $U/C_0 = 0.707$. All data available to the author, which cover a wide range of specific speeds and best efficiencies, show $U/C_0 \approx 0.71$ as long as the rotor is capable of providing near zero swirl and uniform exhaust flow at the best efficiency point. This suggests that the hydraulic efficiency of the main flow is high. The rotor internal flow path is not significant as long as the ratio C_3/C_{m2} is greater than unity at the best efficiency point and the exducer has converging helicoidal passages of high solidity. Also, deeply scalloped wheels are at least equal to full disk wheels. This is a definite departure from water turbine practice.

Consistent with the observation that $U/C_0 = 0.71$ at the best efficiency point, rotor entrance relative velocities must be nearly equal to C_{m2}. Exit relative velocities then represent the vector sum of C_3 and the rotor tangential velocity at the exit point. This means that the ratio $(w_3/w_2) = (w_3/C_{m2})$ must *increase* as the exducer radial station increases, and that the *acceleration* of the relative velocity must be more pronounced as the exducer tip is approached. Since a loss coefficient in a cascade *decreases* with increasing acceleration ratio, the assumption of a *constant loss coefficient* in [10] is not justified. Test information available to the author indicates that rotor internal flow losses are nearly constant from exducer hub to tip, except for deviations due to leakage and spillover at the blade tips.

Although Fig. 9 represents tests of a built-as-designed 90 deg IFR turbine with a design objective of swirl-free uniform exhaust flow, it is probably true that uniform exhaust velocity is not an absolute optimum. However, deviations from uniform flow are not justified in basic design theory, and the needs of a diffuser or following stage must influence any empirical corrections. The importance of such corrections becomes greater at higher specific speeds.

In general, radial inflow turbine nozzle blading needs no profile camber, and most water turbines follow this practice. Some curved nozzle vanes are good, but are no better than straight vanes [10].

Radial clearance between nozzle and rotor is not critical if the centripetal vortex angle is above 10 deg. Kaplan turbines, with radial nozzles and axial rotors, represent an extreme case, but similar results are found in 90 deg IFR air turbines. With vortex angles below 10 deg, side-wall friction losses become prominent with large clearances. Similarly, there is no "optimum" nozzle angle between 10 and 45 deg as long as a total-to-total analysis is used. The nozzle angle is determined by the ratio C_{m2}/U, and C_{m2} is related to $(C_3/C_0)^2$, which is basic parameter.

[9] Reference [1], Fig. 18, $\eta_s = 0.85$, $(C_3/C_0)^2 \approx 0.08 - 0.10$.

An adequate description of 90 deg IFR rotor internal flow conditions which would account for coriolis effects and three-dimensional gradients is not known to the author. The key factors lie in designing for reasonable values of N_s and $(C_3/C_0)^2$ and recognizing the following points:

A Straight nozzle blades of proper solidity have nozzle efficiencies of 97–99 percent.

B Nozzle clearance is not critical.

C Rotor internal hydraulic efficiency is 94–97 percent.

D C_7/C_{m2} should be between 1.0 and 1.5.

E Best efficiency can be obtained with zero exhaust swirl, uniform exhaust velocity, and $0.69 < U/C_0 < 0.725$.

F Exducer blading should be of high solidity and match desired exhaust conditions.

With these simple rules and reasonable attention to avoiding unduly abrupt changes in flow path contours, high efficiencies will be obtained as predicted from Fig. 12.

Why Use a Radial Turbine?

Since, at *any* value of N_s, an axial turbine can equal or better the performance of a 90 deg IFR machine, the question heading this section arises. There are special cases where design point performance of the 90 deg IFR turbine would be superior, and they relate to operation at low Reynolds number with $30 < N_s < 120$. The high degree of reaction and reduced blade numbers of the 90 deg IFR flow path render it less sensitive to Reynolds number influences, but low Reynolds number conditions are seldom found in compressible fluid design requirements for turbines.

The most common motivation for using 90 deg IFR turbines relates to production cost factors. Particularly in small rotor sizes, it is expensive to maintain the profile accuracies of axial rotor blading compatible with high efficiency. 90 deg IFR turbine performance is insensitive to flow surface deviations as long as basic flow area relationships are maintained. This, combined with the small number of blades required, permits relatively inexpensive fabrication for radial machines. Evolution of automotive turbochargers over the past decade demonstrates the superior production cost feature of 90 deg IFR turbines, since virtually all such units in service now are of that type. In larger rotors, the cost advantage of radial turbines is less significant unless a single-stage 90 deg IFR unit can substitute for two axial stages. Forging or casting of large radial turbine rotors presents formidable production problems, but Fig. 6 indicates that this has not inhibited the hydraulic industry.

Where pressure ratio and/or flow rates representing off-design conditions are important, the 90 deg IFR turbine has a broader range characteristic. When variable-nozzle stages are considered, a 90 deg IFR design can have significant performance advantages. A less widely appreciated factor relates to geometric difficulties in pivoting axial-flow nozzle blading. Efficient axial turbines of $N_s > 60$ normally use nozzle blade having high camber and some twist, seeking to generate free-vortex flow entering the rotor. When large changes in blade angle are required, geometric incompatibilities arise which distort the vortex flow. Radial nozzle vanes like those of Fig. 8(*b*) introduce no such problem when they are pivoted. This is consistent with the use of radial inflow nozzles and axial rotors in Kaplan water turbines. (A similar arrangement should work with compressible fluids.)

The flow path of a 90 deg IFR stage can have mechanical design advantages in a turbomachine. This cannot be generalized, but often it has been presumed that the use of the radial flow path entailed a performance penalty; such certainly need not be the case. On the other hand, multiple staging of 90 deg IFR turbines has never been proved attractive within the author's experience. A more natural arrangement is to use the 90 deg IFR stage as a *first* stage followed by axial stages. This reflects the trend toward high specific speeds in the latter stages and permits a smooth meridional flow path.

In general, weight, bulk, and diameter are greater for radial than axial turbines, but the differences are not as large as reputed, and mechanical design compatibility can reverse the difference for the complete turbomachine. In gas turbines, preferable combustor arrangements and/or reduction of stage numbers can favor a 90 deg IFR flow path.

The insensitivity of the 90 deg IFR turbine to flow profile deviations also can be advantageous where erosion or deposit formation must be tolerated. This is particularly evident in automotive applications, where "dirty" exhausts are common. Turbochargers in diesel trucking service have an outstanding record in spite of these conditions.

Conclusions

Use of radial turbines in compressible fluid turbomachines has been inhibited by lack of knowledge of structural and gas dynamic characteristics. In the specific speed range from 60 to 100, peak efficiencies have been demonstrated in the 90–94 percent regime. Furthermore, 90 deg IFR stages can handle higher adiabatic heads and pressure ratios efficiently than are possible with axial turbines. The real criteria for selecting the basic flow path for a given turbomachine must depend upon cost, design compatibility, off-design performance, and other secondary factors. Although the radial turbine is not the best for all turbomachines, it often has been overlooked for applications where its use would be advantageous.

Acknowledgments

The writer wishes to acknowledge the helpful cooperation of the following persons and organizations:

1 Continental Aviation and Engineering Corporation, Detroit, Michigan, for the majority of the compressible fluid turbine information utilized. Mr. E. H. Benstein of CAE has been especially helpful.

2 Hamilton Standard Division, United Aircraft Corporation, Windsor Locks, Connecticut, and Mr. S. G. Best of that organization, for cooperation and permission to publish certain performance information.

3 Dr. O. E. Balje, Consulting Mechanical Engineer, Hollywood, California, for his continuing cooperation and commentary.

References

1 W. T. von der Nuell, "Single-Stage Radial Turbines for Gaseous Substances With High Rotative and Low Specific Speed," Trans. ASME, vol. 74, no. 4, 1952, p. 499.

2 R. Birmann, "The Elastic-Fluid Centripetal Turbine for High Specific Outputs," Trans. ASME, vol. 76, no. 2, 1954, p. 173.

3 A. E. Aeberli, "Deriaz Type Reversible Pump-Turbine Installation at Sir Adam Beck-Niagara Forebay Pumped Storage Project," *Journal of Basic Engineering*, Trans. ASME, Series D, vol. 81, 1959, p. 505.

4 "Hydraulic Turbines," Marks' Handbook, sixth edition, pp. 9–207.

5 L. Brown, "High-Head Francis Turbines for Mammoth Pool," ASME Paper No. 61—Hyd-21.

6 G. A. Bovet, "Modern Trends in Hydraulic-Turbine Design in Europe," Trans. ASME, vol. 75, 1953, p. 975.

7 O. E. Balje, "A Study on Design Criteria and Matching of Turbomachines: Part A," Journal of Engineering for Power, Trans. ASME, vol. 84, Series A, 1962, pp. 83–102.

8 F. Baumgartner and R. Amsler, "Presentation of a Blade-Design Method for Axial-Flow Turbines, Including Design and Test Results of a Typical Axial-Flow Stage," Journal of Engineering for Power, Trans. ASME, Series A, vol. 82, 1960, p.19.

9 L. B. Mann, A. H. Bell, and G. W. Thebert, "Determination of Turbine Stage Performance for an Automotive Power Plant," ASME Paper No. 57—GTP-10.

10 N. Mizumachi, "A Study of Radial Gas Turbines," University of Michigan Report UM IP476 (translation).