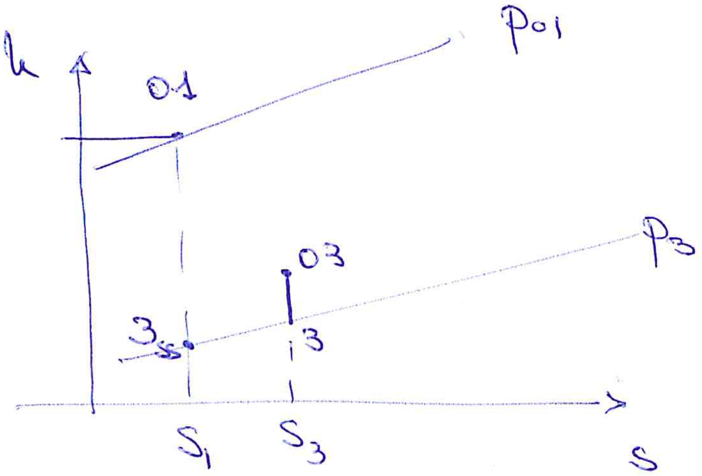
Esempio: progetto preliminare di una turbina a flusso misto

Specifiche: $T_{01} = 1050\ K \qquad c_p = 1150 \dfrac{J}{kg\,K} \qquad \gamma = 1.33$

$$\begin{cases} p_{01} = 3.1\ bar \\ p_3 = 1\ bar \end{cases} \qquad \dot m = 10\ kg/s$$



Calcoliamo:
$$h_{01} - h_{3ss} = c_p\left(T_{01} - T_{3ss}\right) =$$
$$\cancel{= c_p\left(T_{01} - T_{3ss}\right)}$$
$$= c_p T_{01}\left(1 - \frac{T_{3ss}}{T_{01}}\right) =$$
$$= c_p T_{01}\left(1 - \beta^{-\frac{\gamma-1}{\gamma}}\right) =$$
$$= 1150 \cdot 1050\left(1 - 3.1^{(-0.33/1.33)}\right)$$
$$= 295.5\ kJ/kg$$

LAVORO SPECIFICO

$$w_t = \eta_{ts}\left(h_{01} - h_{3ss}\right) = 254.2\ kJ/kg$$

scelgo (ipotizzo) $\eta_{ts} = 0.86$ da Rholik,
quindi sto fissando $\nu_s = 0.6$

A questo punto determino

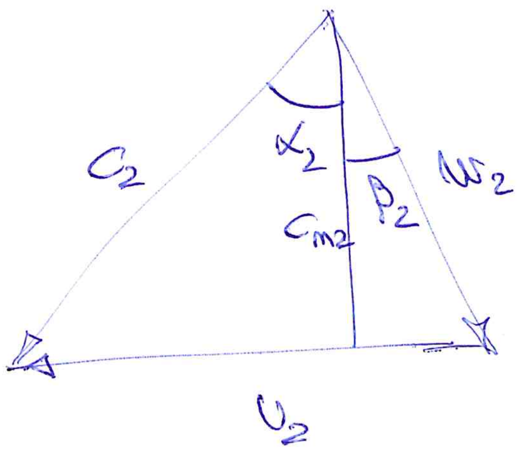
$$C_0 = \sqrt{2(h_{01} - h'_{3ss})} = 768.3 \ m/s$$

$$U_2 = C_0 \cdot 0.7 = 538.2 \ m/s$$

Posso calcolare

$$c_{u2} = \frac{w_t}{U_2} = 472.3 \ m/s$$

Pertanto il triangolo di velocità sarà:



α_2 sarà circa 74°, e quindi il numero di pale Z:

$$Z = \frac{1}{\cos^2 \alpha_2} = 13.16$$

(Whitfield method)

Fisso $Z = 13$, e quindi avrò: $\alpha_2 = \cos^{-1}\sqrt{\frac{1}{Z}} = 73.9°$

A questo punto:

$$c_{m2} = c_{u2} / \cancel{t}\, tg\ 73.9° = 136.3 \ m/s$$

$$c_2 = c_{u2} / \sin 73.9° = 491.6 \ m/s$$

$$w_{u2} = U_2 - c_{u2} = 65.9$$

$$\beta_2 = \arctan\left(\frac{w_{u2}}{c_{m2}}\right) = 25.8°$$

$$w_2 = \sqrt{w_{u2}^2 + c_{m2}^2} = 151.4 \ m/s$$

Numeri di Mach :

$$a_{01} = \sqrt{\gamma R T_{01}} = \sqrt{(\gamma-1) c_p T_{01}} =$$

$$= \sqrt{0.33 \cdot 1150 \cdot 1050} = 631.2 \; m/s$$

$$a_2 = \sqrt{\gamma R T_2} = \sqrt{(\gamma-1) c_p T_2}$$

$$T_2 = \underset{\underset{T_{01}}{\parallel}}{T_{02}} - \frac{c_2^2}{2c_p} = 1050 - \frac{(491.6)^2}{2 \cdot 1150} = 945 \; K$$

$$a_2 = \sqrt{0.33 \cdot 1150 \cdot 945} = 598.8 \; m/s$$

$$M_2 = \frac{c_2}{a_2} = \frac{491.6}{598.8} = 0.82$$

$$M_{2\,rel} = \frac{w_2}{a_2} = \frac{151.6}{598.8} \simeq 0.25$$

Dimensioni della girante :

$$\Omega_s = \Omega \frac{Q_3^{1/2}}{(\Delta h_0)^{3/4}}$$

Voglio $\Omega_s = 0.6$ (ottimo) $\quad \Delta h_0 = 254.2 \; kJ/kg$

$$Q_3 = \dot{m}/\rho_3$$

$$\rho_3 = \frac{P_3}{R T_3} \simeq \frac{P_3}{R T_{03}} = \frac{10^5}{\frac{0.33}{1.33} \cdot 1150 \left(1050 - \frac{254200}{1150}\right)} =$$

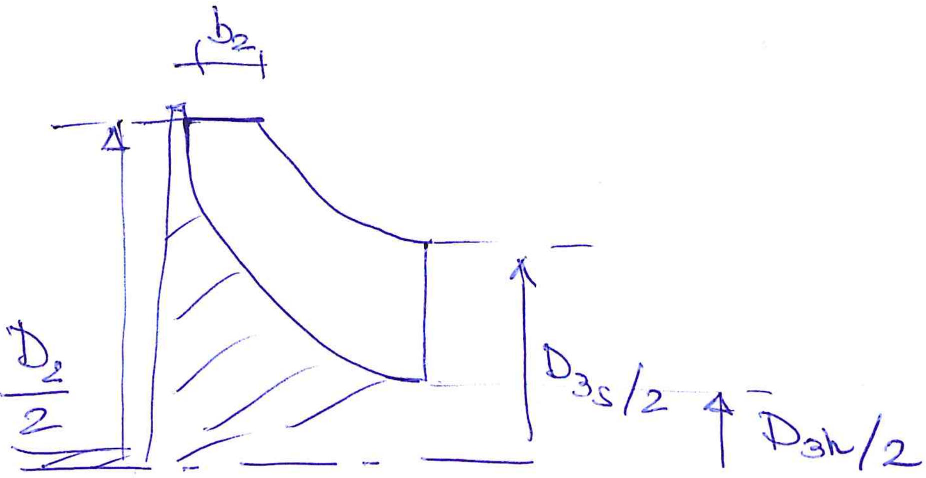
$$= 0.42 \; kg/m^3$$

e quindi $Q_3 = 23.65 \; m^3/s$

$$\Omega = \Omega_s \cdot \frac{\Delta h_0^{3/4}}{Q_3^{1/2}} = 0.6 \cdot \frac{254200^{3/4}}{23.65^{1/2}} = 1396 \ \frac{rad}{s}$$

$$N = \Omega \cdot \frac{60}{2\pi} = 13{,}338 \text{ rpm}$$

$$D_2 = \frac{2U_2}{\Omega} = \frac{2 \cdot 538.2}{1396} = 0.77 \text{ m}$$



$$b_2 = \frac{\dot{m}}{\rho_2 \, c_{m2} \, \pi \, D_2}$$

$$\rho_2 = \rho_{02} \Big/ \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{1}{\gamma - 1}}$$

$$= \frac{p_{02}}{R T_{02}} \Big/ \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{1}{\gamma-1}}$$

$$= \frac{310000}{\frac{0.33}{1.33} \cdot 1150 \cdot 1050} \Big/ \left(1 + \frac{0.33}{2} \cdot 0.82^2\right)^{\frac{1}{0.33}}$$

assumo $p_{02} \simeq p_{01}$

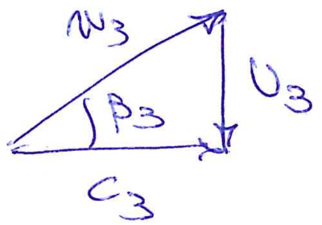
$$= 0.75 \ kg/m^3$$

Quindi $$b_2 = \frac{10}{0.75 \cdot 136.3 \cdot \pi \cdot 0.77} = 0.04 \text{ m}$$

E in uscita?

Potrei supporre $\frac{D_{3S}}{D_2} = 0.7 \Rightarrow D_{3S} = 0.7 \cdot 0.77 = 0.539\ m$

Al raggio medio il triangolo di velocità "sarà":



Voglio $w_3 > w_2 \cdot 2$ Scelgo $w_3 = 2\, w_2 = 2 \cdot 151.4 = 302.8\ m/s$

$$U_3 = \frac{D_3 \cdot \Omega}{2} = \frac{\Omega\,(D_{3h} + D_{3S})}{4} =$$

$$= \Omega D_{3S} \frac{(1+\nu)}{4}$$

$$c_3 = \frac{4\, \dot m/\rho_3}{\pi\,(D_{3S}^2 - D_{3h}^2)} = \frac{4 \cdot \dot m/\rho_3}{\pi D_{3S}^2 (1-\nu^2)}$$

$$w_3^2 = U_3^2 + c_3^2 = \left[\Omega D_{3S} \frac{(1+\nu)}{4}\right]^2 + \frac{4\, \dot m/\rho_3}{\pi D_{3S}^2 (1-\nu^2)}$$

Nel sito c'è un programmino (sizing.m) per risolvere questo problema. Ottengo

$\nu = 0.45$ $\quad c_3 = 130.99\ m/s \quad U_3 = 272.9\ m/s$

$\beta_3 = \mathrm{atan}\,(U_3/c_3) = 64.3°$

E i Mach?

$$T_3 = T_{03} - \frac{c_3^2}{2c_p} = T_{01} - \frac{\overset{w_t}{\Delta H_0}}{c_p} - \frac{c_3^2}{2c_p}$$

$$= T_{01} - \left(\frac{\Delta H_0 + c_3^2/2}{c_p}\right) = 821.5\ K$$

$$a_3 = \sqrt{\gamma R T_3} = \sqrt{(\gamma-1) c_p T_3}$$

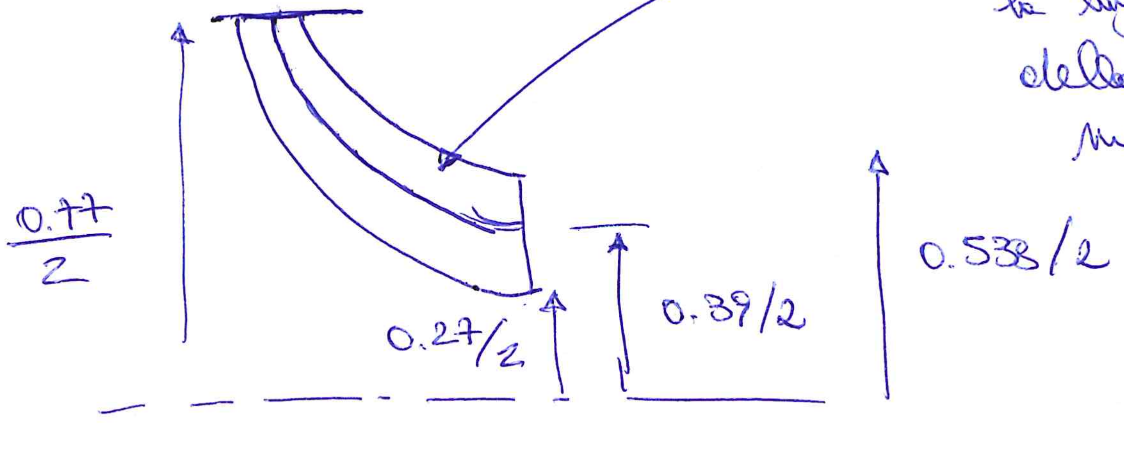
$$= \sqrt{0.33 \cdot 1150 \cdot 821.5} = 558.3\ m/s$$

$$M_3 = \frac{c_3}{a_3} = \frac{131}{558.3} = 0.23$$

$$M_{3,rel} = \frac{w_3}{a_3} = \frac{302.8}{558.3} = 0.54$$

Infine, per la scelta della lunghezza della pala posso utilizzare il grafico di Rodger e Geiser (nelle slides)

$$\frac{Z L}{D_2} = 6 \Rightarrow L = \frac{6 \cdot D_2}{Z} = 0.35$$



questo rappresenta la lunghezza della linea media.