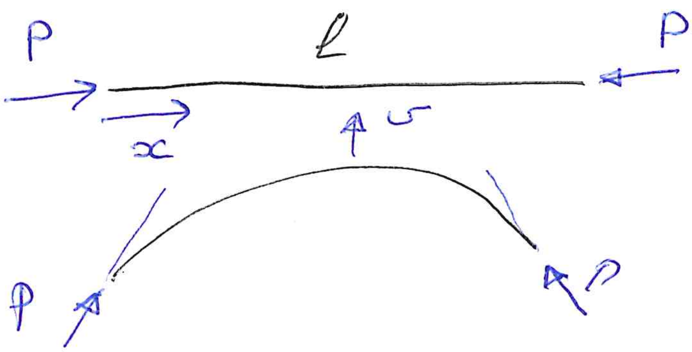
La trave con due forze follower



P_c - ?

$v = v(x,t)$ $\qquad$ $EIv^{(4)} + Pv'' + m\ddot{v} = 0$

le condizione al contorno

$$\begin{cases} v''(0) = 0 \\ v'''(0) = 0 \end{cases} \qquad v''(\ell) = 0 \qquad v'''(\ell) = 0$$

$$v(x,t) = V(x)e^{\lambda t}$$

$V:$ $\quad EIV^{(4)} + PV'' + \lambda^2 m V = 0$

$$V = C_1 \cos \alpha x + C_2 \sin \alpha x + C_3 \cosh \beta x + C_4 \sinh \beta x$$

$$\alpha^2 = \frac{\mu}{2} + \sqrt{\frac{\mu^2}{4} - \lambda^2 m/EJ}$$

$$\beta^2 = -\frac{\mu}{2} + \sqrt{\frac{\mu^2}{4} - \lambda^2 m/EJ}$$

$$\begin{cases} EI\,v'''' + P\,v'' + \lambda^2 m\, v = 0 \\ v''(0) = 0 \qquad v''(\ell) = 0 \\ v'''(0) = 0 \qquad v'''(\ell) = 0 \end{cases}$$

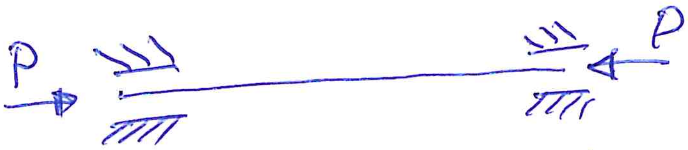
$$z = \underline{z(x)} = v''(x)$$

$$\frac{d^2}{dx^2}: \quad EI\,\underset{z^{(4)}}{v^{(6)}} + P\,\underset{z''}{v^{(4)}} + \lambda^2 m\,\underset{z}{v''} = 0$$

$$\rightarrow \quad \underline{EI\,z^{(4)} + P z'' + \lambda^2 m\, z = 0}$$

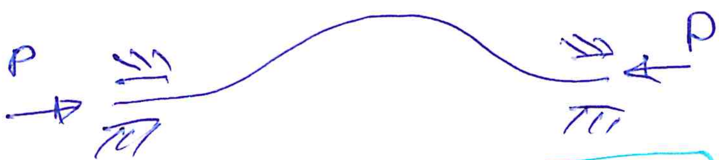
al contorno: $z(0) = 0 \qquad z(\ell) = 0$

$z'(0) = 0 \qquad z'(\ell) = 0$



$v(0) = 0 \qquad v(\ell) = 0$

$v'(0) = 0 \qquad v'(\ell) = 0$



$$EI\,z^{(4)} + P z'' + \boxed{\cancel{\lambda^2 m\, z}} = 0$$

$$EIz^{(4)} + Pz'' = 0$$

$$z = v(0) = 0 \qquad v(l) = 0$$

$$v'(0) = 0 \qquad v'(l) = 0$$

$$z(x) = C_1 \cos \gamma x + C_2 \sin \gamma x + C_3 x + C_4$$

$$z \sim e^{\gamma x} \Rightarrow EI\gamma^4 + P\gamma^2 = 0$$

$$\gamma = 0, \qquad \gamma^2 = -\frac{P}{EI} \Rightarrow \gamma = \pm i\sqrt{\frac{P}{EI}}$$

$$C_3 x + C_4 \qquad C_1 \cos\sqrt{\frac{P}{EI}}\,x + C_2 \sin\sqrt{\frac{P}{EI}}\,x$$

$$z(0) = 0 \Rightarrow C_1 + C_4 = 0$$

$$z'(0) = 0 \Rightarrow C_2\gamma + C_3 = 0$$

$$z(l) = 0 \Rightarrow C_1 \cos\gamma l + C_2 \sin\gamma l + C_3 l + C_4 = 0$$

$$z'(l) = 0 \Rightarrow -C_1\gamma \sin\gamma l + C_2\gamma\cos\gamma l + C_3 = 0$$

$$C_4 = -C_1, \quad C_3 = -C_2\gamma$$

$$\begin{cases} C_1[\cos\gamma l - 1] + C_2[\sin\gamma l - \gamma l] = 0 \\ C_1[-\gamma\sin\gamma l] + C_2[\gamma\cos\gamma l - \gamma] = 0 \end{cases}$$

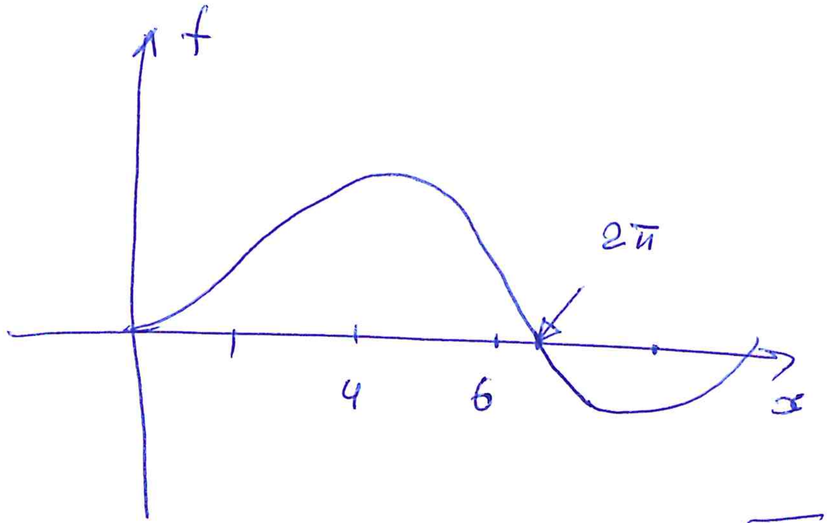
$$\det \begin{vmatrix} \cos\gamma l - 1 & \sin\gamma l - \gamma l \\ -\cancel{\gamma}\sin\gamma l & \cancel{\gamma}\cos\gamma l - \cancel{\gamma} \end{vmatrix} = 0$$

$$(\cos\gamma l - 1)^2 + \sin\gamma l\,(\sin\gamma l - \gamma l) = 0$$

$$\cos^2\gamma l - 2\cos\gamma l + 1 + \sin^2\gamma l - \gamma l \sin\gamma l = 0$$

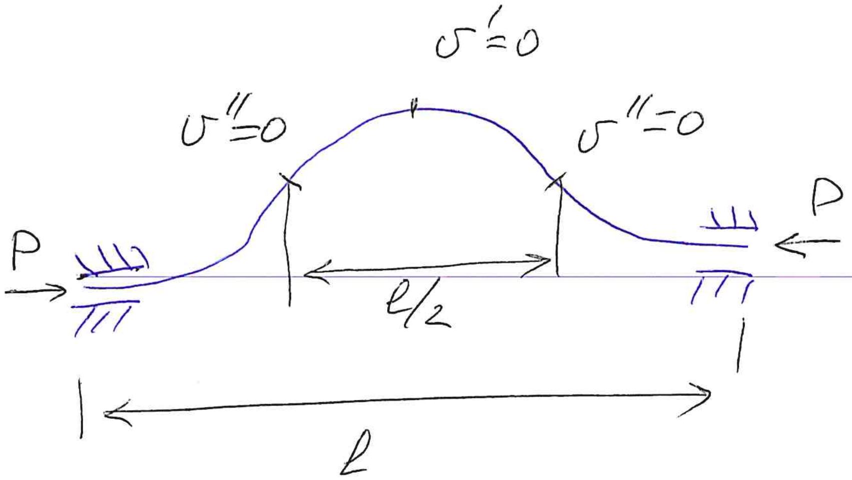
$$2 - 2\cos\gamma l - \gamma l \sin\gamma l = 0$$

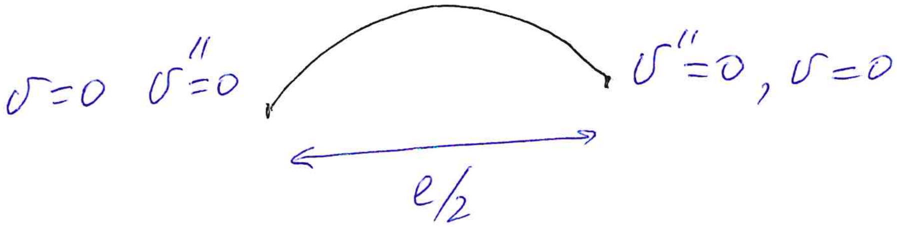
$$f(x) = 2 - 2\cos x - x\sin x$$



$$\gamma l = 2\pi \implies \gamma = \frac{2\pi}{l} \quad , \quad \gamma = \sqrt{\frac{P}{EI}}$$

$$\boxed{P = \frac{4\pi^2}{l^2} EI}$$





$$v(x) = C_1 \sin\beta x + C_2 \cos\beta x$$

$$v(0)=0 \Rightarrow C_2 = 0$$

$$v(l/2)=0 \Rightarrow \sin\beta\frac{l}{2} = 0 \Rightarrow$$

$$\Rightarrow \beta\frac{l}{2} = \pi, \quad \beta = \frac{2\pi}{l}$$

$$\beta = \sqrt{\frac{P}{EI}} \Rightarrow P_c = \frac{4\pi^2}{l^2} EI$$