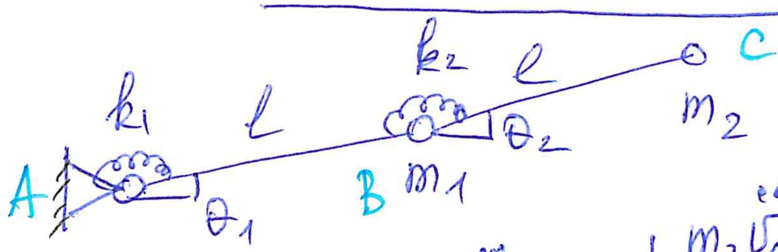
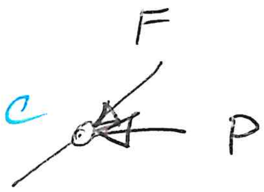
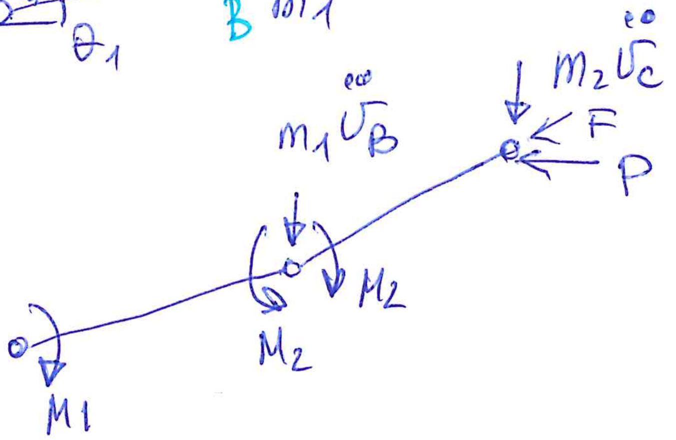
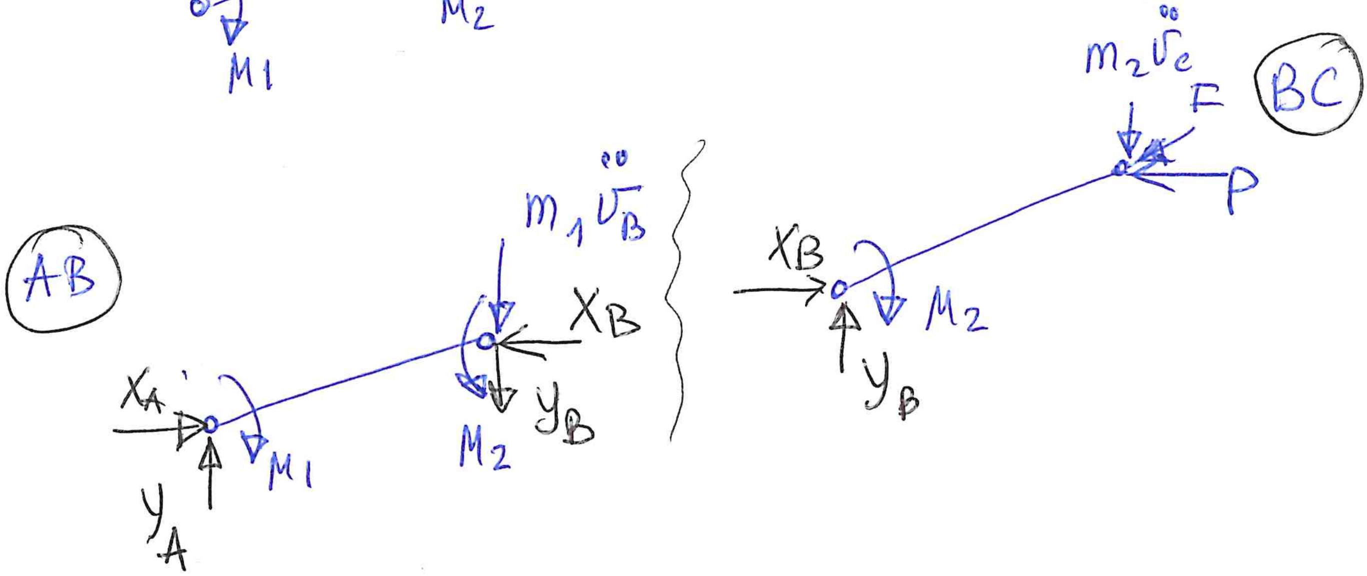
# La colonna di Ziegler




1) $\theta_1 \approx 0, \ \theta_2 \approx 0 \implies \sin\theta_{1,2} = \theta_{1,2}$

$\cos\theta_{1,2} = 1$

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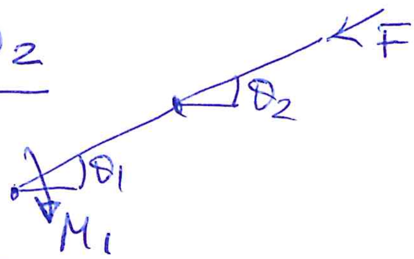
## Il criterio statico. $\boxed{P=0}$ $F \neq 0$

$\ddot{v}_C = 0, \ \ddot{v}_B = 0$

$M_2 = k_2(\theta_2 - \theta_1)$

(BC) $M_B$ : $M_2 = 0 \implies \underline{\theta_1 = \theta_2}$

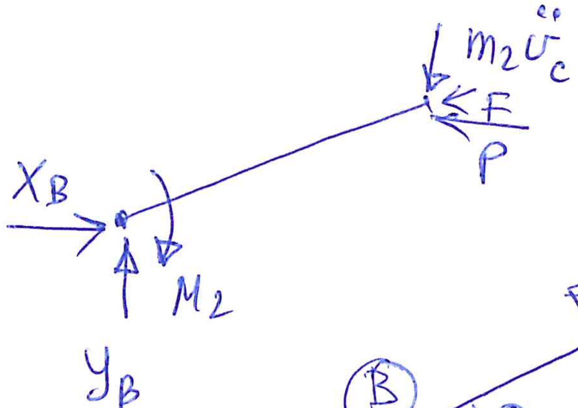


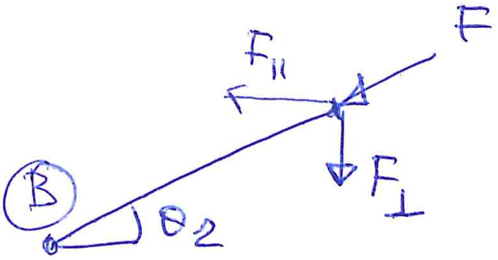

(AC) $M_A$ : $M_1 = 0$

$M_1 = k_1\theta_1 \implies \theta_1 = 0 \implies \theta_2 = 0$

$\implies$ <u>banale</u> !

BC



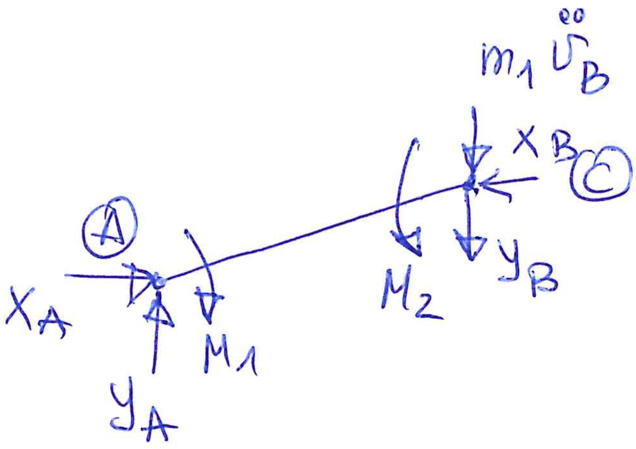

$$F_{\parallel} = F\cos\theta_2 \quad , \quad F_{\perp} = F\sin\theta_2$$
$$= F \qquad\qquad = F\theta_2$$

$$\begin{cases} X_B = F_{\parallel} + P = F + P \\ y_B = F_{\perp} + m_2\ddot{v}_c = F\theta_2 + m_2\ddot{v}_c \end{cases}$$

$M_B$: $\underline{M_2 + m_2\ddot{v}_c l - P l \theta_2 = 0}$ $\Big|$ $\underline{\cos\theta_2 \approx 1}$

$\sin\theta_2 = \theta_2$

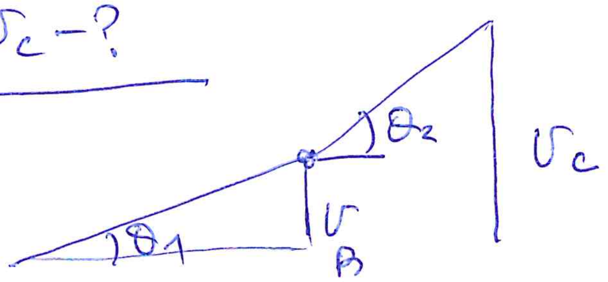
AC




$$X_A = X_B$$
$$y_A = m_1\ddot{v}_B + y_B$$

$M_A$: $\underline{M_1 - M_2 + m_1\ddot{v}_B l + y_B l - X_B l \theta_1 = 0}$

$v_B,\ v_c - ?$




$$v_B = l\sin\theta_1 = l\theta_1$$

$$v_c = l\sin\theta_1 + l\sin\theta_2 = l(\theta_1+\theta_2)$$

$$M_1 = k_1\theta_1\ ,\quad M_2 = k_2(\theta_2-\theta_1)$$

$$\begin{cases} M_2 + m_2\ddot{v}_c l - Pl\theta_2 = 0 \\ M_1 - M_2 + m_1\ddot{v}_B l + y_B l - X_B l\theta_1 = 0 \end{cases}$$

① $$\boxed{k_2(\theta_2-\theta_1) + m_2 l^2(\ddot{\theta}_1+\ddot{\theta}_2) - Pl\theta_2 = 0}$$

$$k_1\theta_1 - k_2(\theta_2-\theta_1) + m_1 l^2\ddot{\theta}_1 +$$
$$+ \left[F\theta_2 + m_2 l(\ddot{\theta}_1+\ddot{\theta}_2)\right]l -$$
$$-(F+P)l\theta_1 = 0$$

② $$\boxed{\begin{aligned}&(m_1 l^2 + m_2 l^2)\ddot{\theta}_1 + m_2 l^2\ddot{\theta}_2 + (k_1+k_2)\theta_1 \\ &\quad -(F+P)l\theta_1 - k_2\theta_2 + Fl\theta_2 = 0\end{aligned}}$$

$$
\left\{
\begin{aligned}
&\textcircled{2} \leftarrow \quad m_2 \ell^2 (\ddot{\theta}_1 + \ddot{\theta}_2) + k_2(\theta_2 - \theta_1) - P\ell\theta_2 = 0 \\
&\textcircled{1} \leftarrow \quad (m_1 + m_2)\ell^2 \ddot{\theta}_1 + m_2 \ell^2 \ddot{\theta}_2 + (k_1 + k_2)\theta_1 \\
&\qquad\qquad - k_2\theta_2 - (F+P)\ell\theta_1 + F\ell\theta_2 = 0
\end{aligned}
\right.
$$

$$\vec{q} = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \qquad M\ddot{q} + (K_e + \underbrace{(\cdot)}_{F,P} K_g) q = 0$$

$$M = \begin{bmatrix} m_2 \ell^2 & m_2 \ell^2 \\ (m_1 + m_2)\ell^2 & m_2 \ell^2 \end{bmatrix} = \ell^2 m \begin{bmatrix} 1 & 1 \\ 1+\delta & 1 \end{bmatrix}$$

$$m_1 = \delta m, \quad m_2 = m$$

$$K_e = \begin{bmatrix} -k_2 & k_2 \\ k_1 + k_2 & -k_2 \end{bmatrix}$$

$$K_g = \begin{bmatrix} 0 & -P\ell \\ -(F+P)\ell & F\ell \end{bmatrix}$$

$$M = ml^2 \begin{bmatrix} 1+\delta & 1 \\ 1 & 1 \end{bmatrix}$$

$$K_e = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} = k \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\boxed{k_1 = k_2 = k}$$

$$K_g = l \begin{bmatrix} -F-P & F \\ 0 & -P \end{bmatrix}$$

$$F, P \Rightarrow F = \alpha P$$

$$S = \begin{bmatrix} 1+\delta & 1 \\ 1 & 1 \end{bmatrix} \lambda^2 + \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} +$$

$$+ \mu \begin{bmatrix} -\alpha-1 & \alpha \\ -0 & 1 \end{bmatrix}$$

$S$: $\underline{\underline{\det S = 0}}$

$$S = \begin{bmatrix} (1+\delta)\lambda^2 + 2 + \mu(-\alpha-1) & \lambda^2 - 1 + \alpha\mu \\ \lambda^2 - 1 \cancel{-\mu} & \lambda^2 + 1 + \mu \end{bmatrix}$$

# Il criterio statico $P \neq 0,\ F = 0$ ⑥

$$\begin{cases} 2k\theta_1 - P\ell\theta_1 - k_2\theta_2 = 0 \\ k(\theta_2 - \theta_1) - P\ell\theta_2 = 0 \end{cases}$$

$$S = \begin{pmatrix} 2k - P\ell & -k \\ -k & k - P\ell \end{pmatrix}$$

$S \cdot \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = 0$ – l'equazione equilibrio

$$(2k - P\ell)(k - P\ell) - k^2 = 0$$

$$k^2 - 3kP\ell + (P\ell)^2 = 0$$

$$P\ell = \frac{3k \pm \sqrt{(3k)^2 - 4k^2}}{2}$$

$$= \frac{3k \pm k\sqrt{5}}{2}$$

$$\boxed{P_c = \begin{cases} 0{,}382\ \dfrac{k}{\ell} \\ 2.62\ k/\ell \end{cases}}$$

Il criterio dinamico

$F \neq 0, P \neq 0$

$$S = \begin{bmatrix} (1+\delta)\lambda^2 + 2 + \mu(-\alpha-1) & \lambda^2 - 1 + \alpha\mu \\ \lambda^2 - 1 & \lambda^2 + 1 + \mu \end{bmatrix}$$

$\alpha : F = \alpha P, \quad \mu = \dfrac{P\ell}{k}, \quad m_1 = \delta m_2$

$$\boxed{\det S = 0}$$

1. $P = 0 \quad F_c = 2\dfrac{k}{\ell} \quad [m_1 = m_2,\ k_2 = k_1]$
2. $P = F \quad F_c = \{1.079,\ 1.74\}\ k/\ell$
3. $F = 0.5P, \quad P_c = \{1.6,\ 2\}\ k/\ell$
4. $F = 1.25P, \quad P_c = \{0.91,\ 1.6\}\ k/\ell$
5. $F = 2P, \quad P_c = \{0.687,\ 1.26\}\ k/\ell$