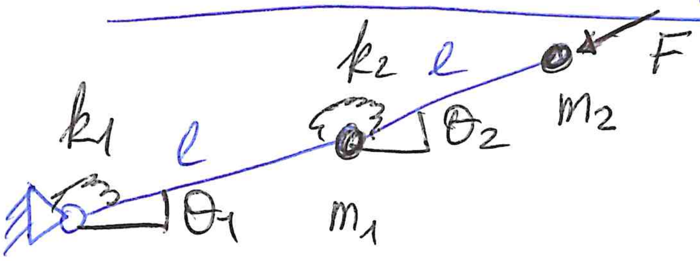
# La colonna di Ziegler




$$\boxed{F_c - ?}$$

$M$ = massa totale

$M = m_1 + m_2$

$m_1 = \delta m, \quad m_2 = m$

$M = m_1 + m_2 = \delta m + m =$

$= (1+\delta) m$

$$\Rightarrow \boxed{m = \frac{M}{1+\delta}}$$

## distribuzione di massa

$\begin{cases} m_1 = 2m \\ m_2 = m \end{cases} \quad F_c = 2{,}09 \dfrac{k}{l}$

$\begin{cases} m_1 = m \\ m_2 = m \end{cases} \quad F_c = 2 \dfrac{k}{l}$

$\begin{cases} m_1 = m \\ m_2 = 2m \end{cases} \quad F_c = 2{,}04 \dfrac{k}{l}$

$$\begin{cases} \delta = 9 \Rightarrow m_1 = 9m = 9\dfrac{M}{10} = 0.9\,M \\ \qquad m_2 = m = 0.1\,M \\ \delta = 99 \quad m_1 = 99\dfrac{M}{1+99} = 0.99\,M, \quad m_2 = 0.01\,M \\ \delta = \dfrac{1}{9} \quad m_1 = \dfrac{1}{9}\dfrac{M}{1+1/9} = \dfrac{1}{9}\dfrac{9}{10}M = 0.1\,M \end{cases}$$

$m_2 = \dfrac{9}{10} M \quad \Rightarrow \quad \boxed{\delta \in (0, +\infty)}$

$\underline{F_c = F_c(\delta)}$ ? $\quad F_c \in (\,.\ ?\ .\,)$

$$M = \begin{bmatrix} m_1 + m_2 & m_2 \\ m_2 & m_2 \end{bmatrix} = m \begin{bmatrix} 1+\delta & 1 \\ 1 & 1 \end{bmatrix}$$

$$S = M\lambda^2 + (K_e + \mu K_g) \qquad \boxed{\mu = \frac{F\ell}{k}}$$

$$\det S = 0$$

$$K_e = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}, \quad K_g = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 1+\delta & 1 \\ 1 & 1 \end{bmatrix}\lambda^2 + \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} +$$

$$+ \mu \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} (1+\delta)\lambda^2 + 2 - \mu & \lambda^2 - 1 + \mu \\ \lambda^2 - 1 & \lambda^2 + 1 \end{bmatrix}$$

$$\det S = \left[(1+\delta)\lambda^2 + 2 - \mu\right]\left[\lambda^2+1\right] -$$

$$-\left[\lambda^2-1\right]\left[\lambda^2-1+\mu\right] =$$

$$= (1+\delta)\lambda^4 + \underline{(2-\mu)\lambda^2} + \underline{(1+\delta)\lambda^2} + 2 - \mu$$

$$- \left[\lambda^4 - \lambda^2 + \lambda^2(-1+\mu) + 1 - \mu\right] =$$

$$= \underline{(1+\delta)\lambda^4} + \underline{(2-\mu+1+\delta)\lambda^2} + 2 - \mu$$

$$\underline{-\lambda^4} + \underline{\lambda^2} + \underline{\lambda^2(1-\mu)} - 1 + \mu =$$

$$= \lambda^4\left[\cancel{1}+\delta\cancel{-1}\right] + \lambda^2\left[2-\mu+1+\delta+1+1-\mu\right]$$

$$+ 2 \cancel{-\mu} - 1 \cancel{+\mu} =$$

$$= \delta\lambda^4 + \lambda^2\left[5+\delta-2\mu\right] + 1 = 0$$

$$\lambda^2_{1,2} = \frac{-(5+\delta-2\mu) \pm \sqrt{(5+\delta-2\mu)^2 - 4\delta}}{2\delta}$$

$$\Delta = (5+\delta-2\mu)^2 - 4\delta$$

$\mu:\quad \Delta = 0$

$$\Delta = (5+\delta-2\mu)^2 - 4\delta = 0$$

$$(5+\delta-2\mu)^2 = 4\delta \qquad \underline{\delta > 0}$$

$$5+\delta-2\mu = 2\sqrt{\delta}$$

$$2\mu = 5+\delta-2\sqrt{\delta}$$

$$\mu = \frac{5}{2}+\frac{\delta}{2}-\sqrt{\delta}$$

$$\Rightarrow \boxed{F_c = \mu_c \frac{k}{l}}$$

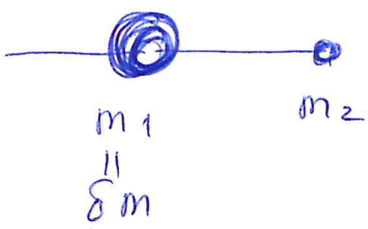
$$\delta = 9: \quad \mu_c = \frac{5}{2}+\frac{9}{2}-\sqrt{9} = $$
$$= \frac{14}{2}-3 = 7-3 = 4$$

$$\underline{\delta=9: \quad F_c = 4\frac{k}{l}}$$

$$\delta = 100: \quad \mu_c = \frac{100}{2}+\frac{5}{2}-\sqrt{100} =$$
$$= 50+\frac{5}{2}-10 = \underline{\underline{42.5}}$$

$$\delta \to \infty \qquad \underline{F_c \to \infty}$$

$\delta \to \infty$



$$\mu = \mu(\delta) = \frac{5}{2} + \frac{\delta}{2} - \sqrt{\delta}$$

$$F_c = \mu(\delta)\frac{k}{l}$$

$$\mu(0) = \frac{5}{2} = 2.5 \qquad \mu(9) = 4$$

$$\mu(\delta = \infty) = \infty \qquad \mu(99) = 42.5$$

$$\mu' = \frac{1}{2} - \frac{1}{2}\frac{1}{\sqrt{\delta}}$$

$$\mu' = 0 \ ? \qquad \frac{1}{2} - \frac{1}{2}\frac{1}{\sqrt{\delta}} = 0 \Rightarrow$$

$$1 - \frac{1}{\sqrt{\delta}} = 0 \qquad \sqrt{\delta} = 1$$

$$\Rightarrow \boxed{\delta = 1}$$

$$\mu = \mu_{min} : \delta = 1 \qquad \mu(1) = \frac{5}{2} + \frac{1}{2} - 1 = 2$$

$$m_1 = m_2$$