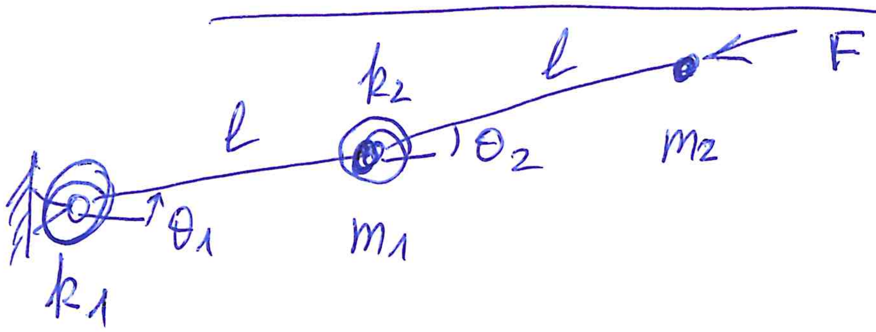
La colonna di Ziegler



F_c - ? 1) $F_c \sim m_1, m_2$

Ziegler : $m_1 = 2m$, $m_2 = m$

$$\begin{cases} (m_1+m_2)\ell^2\ddot{\theta}_1 + m_2\ell^2\ddot{\theta}_2 + M_1 - M_2 + F\ell(\theta_2-\theta_1) = 0 \\ m_2\ell^2\ddot{\theta}_1 + m_2\ell^2\ddot{\theta}_2 + M_2 = 0 \end{cases}$$

$$\underbrace{\ell^2\begin{bmatrix} m_1+m_2 & m_2 \\ m_2 & m_2 \end{bmatrix}}_{M}\begin{pmatrix}\ddot{\theta}_1 \\ \ddot{\theta}_2\end{pmatrix} + \left(\underbrace{\begin{pmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix}}_{K_e} + F\ell\underbrace{\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}}_{K_g}\right)\begin{pmatrix}\theta_1 \\ \theta_2\end{pmatrix} = 0$$

$$M\ddot{q} + (K_e + K_g \cdot \mu)q = 0$$

$$\mu = \frac{F\ell}{k}$$

1) $k_1 = k_2 = k$

2) $m_1 = m_2 = m$

$$l^2 m \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} +$$

$$+ \left(k \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} + Fl \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \right) \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = 0$$

$$q = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \sim q_0 e^{\lambda t}$$

$$\left\{ \underbrace{l^2 m \lambda^2 \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} + k \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} + Fl \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}}_{S} \right\} q_0 = 0$$

$\underline{\det S = 0}$

$$\tilde{t} = t \sqrt{\frac{k}{ml^2}} \quad , \quad \mu = \frac{Fl}{k}$$

$$\Rightarrow \underline{\left| \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \lambda^2 + \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} + \mu \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} \right| = 0}$$

$$S = \begin{bmatrix} 2\lambda^2 + 2 - \mu & \lambda^2 - 1 + \mu \\ \lambda^2 - 1 & \lambda^2 + 1 \end{bmatrix}$$

$$\det S = (2\lambda^2 + 2 - \mu)(\lambda^2 + 1) - (\lambda^2 - 1)(\lambda^2 - 1 + \mu) = 0$$

Posto $\lambda^2 = z$

$$(2z + 2 - \mu)(z + 1) - (z - 1)(z - 1 + \mu)$$

$$= 2z^2 + 2z - \mu z + 2z + 2 - \mu - [z^2 - z + \mu z - z + 1 - \mu] =$$

$$= z^2 + z(2 - \mu + 2 + 1 - \mu + 1) + 2 - \mu - 1 + \mu = 0$$

$$\boxed{z^2 + z(6 - 2\mu) + 1 = 0}$$

$$\Delta = (6-2\mu)^2 - 4 = 0$$

$$(6-2\mu)^2 = 4$$

$$6-2\mu = 2$$

$$2\mu = 4 \, , \quad \underline{\underline{\mu = 2}}$$

$$\boxed{\mu_c = 2}$$

$$\mu = \frac{F\ell}{k}$$

$$\boxed{F_c = 2\,\frac{k}{\ell}}$$

$$F = \mu\,\frac{k}{\ell}$$

$$\boxed{m_1 = m_2 = m \Rightarrow F_c = 2\,\frac{k}{\ell}}$$

$$\boxed{m_1 = 2m \, , \, m_2 = m \quad F_c = 2.09\,\frac{k}{\ell}}$$

Per sistemi nonconservativi come la colonna di Ziegler la forze crit. dipenda ~~di massa~~ ~~di~~ di distribuzione di massa