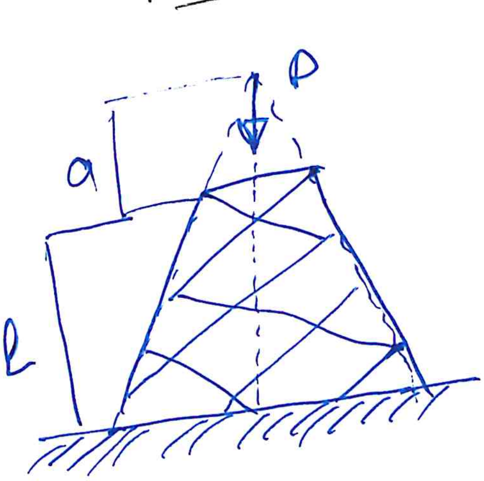
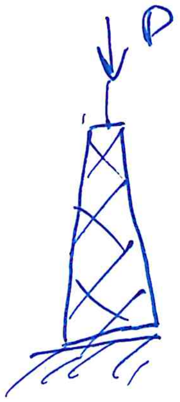
Aste a sezione variabile

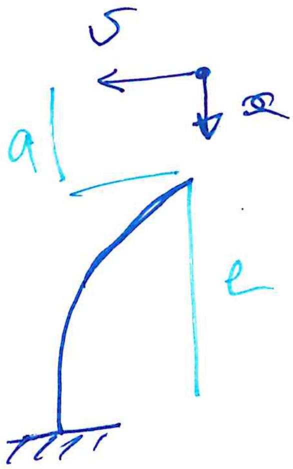
$EI \qquad I = I(x)$

$$\left[E I(x) v''\right]'' + P v'' = 0$$

$$E I(x) v'' + P v = 0$$







le cond. al contorno $\boxed{\begin{array}{l} v(a) = 0 \\ v'(a+l) = 0 \end{array}}$

$$E I(x) v'' + P v = 0$$

$$I(x) = I_0 \left(\frac{x}{a}\right)^n, \quad n = 2$$

$$\Rightarrow \quad E I_0 \frac{x^2}{a^2} v'' + P v = 0$$

$$\frac{EI_0}{a^2} x^2 v'' + Pv = 0$$

$$z: \quad \frac{x}{a} = e^z$$

$$\frac{d}{dx} v = \frac{dv}{dz} \frac{dz}{dx}$$

$$\frac{dz}{dx} - ? \qquad \frac{dz}{dx} = \left(\frac{dx}{dz}\right)^{-1} = (ae^z)^{-1} = x^{-1}$$

$$\frac{dx}{dz} = \frac{d}{dz}(ae^z) = ae^z$$

$$\Rightarrow \boxed{\frac{dv}{dx} = \frac{1}{x} \frac{dv}{dz}}$$

$$\frac{d^2 v}{dx^2} = \frac{d}{dx} \frac{dv}{dx} = \frac{d}{dx} \left[\frac{1}{x} \frac{dv}{dz}\right] =$$

$$= \frac{1}{x} \frac{d}{dz} \left[\frac{1}{ae^z} \frac{dv}{dz}\right] =$$

$$= \frac{1}{x} \left[-\frac{1}{ae^z} \frac{dv}{dz} + \frac{1}{ae^z} \frac{d^2 v}{dz^2}\right] =$$

$$= \frac{1}{x} \left[-\frac{1}{x} \frac{dv}{dz} + \frac{1}{x} \frac{d^2 v}{dz^2}\right] =$$

$$= \frac{1}{x^2} \frac{d^2 v}{dz^2} - \frac{1}{x^2} \frac{dv}{dz}$$

$$\frac{EI_0}{a^2}\cancel{x^2}\left[\frac{1}{\cancel{x^2}}\frac{d^2v}{dz^2}-\frac{1}{\cancel{x^2}}\frac{dv}{dz}\right]+Pv=0$$

$$\frac{EI_0}{a^2}\left[v''-v'\right]+Pv=0$$

dove $\quad ()'=\frac{d}{dz}$

$$v''-v'+\frac{Pa^2}{EI_0}v=0$$

$\Rightarrow \quad \underline{v=Ve^{\lambda z}}$

$$\lambda^2-\lambda+\frac{Pa^2}{EI_0}=0$$

$$\lambda_\pm=\frac{1\pm\sqrt{1-4\frac{Pa^2}{EI_0}}}{2}$$

$\Rightarrow \quad \boxed{\frac{4Pa^2}{EI_0}-1>0}$

$$\lambda_\pm=\frac{1}{2}\pm i\beta\,,\quad \beta=\sqrt{\frac{Pa^2}{EI_0}-\frac{1}{4}}$$

$$v(z)=C_1e^{\lambda_+z}+C_2e^{\lambda_-z}$$

$\underline{z=\ln\frac{x}{a}} \leftarrow \frac{x}{a}=e^z$

$$e^{\lambda_+ z} = e^{\frac{1}{2}z} \cdot e^{i\beta z}$$

$$e^{\frac{1}{2}z} = e^{\frac{1}{2}\ln\frac{x}{a}} = \sqrt{\frac{x}{a}}$$

$$e^{i\beta z} = \cos\beta z + i\sin\beta z$$

$$= \cos\left(\beta \ln\frac{x}{a}\right) + i\sin\left(\beta\ln\frac{x}{a}\right)$$

$$\Rightarrow v(x) = \sqrt{\frac{x}{a}}\left[c_1\cos\left(\beta\ln\frac{x}{a}\right) + c_2\sin\left(\beta\ln\frac{x}{a}\right)\right]$$

$$v(a) = 0 \Rightarrow$$

$$v(a) = \sqrt{\frac{a}{a}}\left[c_1 \cdot 1 + c_2 \cdot 0\right] = 0$$

$$\Rightarrow \boxed{c_1 = 0}$$

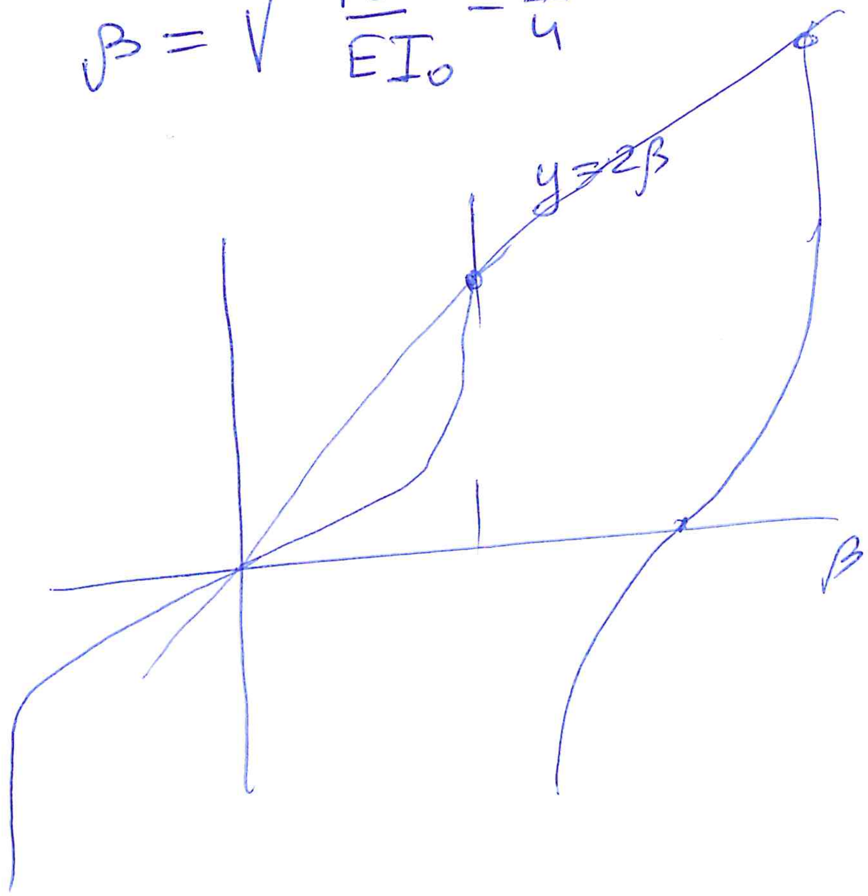
$$v'(a+l) = 0 \ -?$$

$$v' = \left[\sqrt{\frac{x}{a}}\sin\left(\beta\ln\frac{x}{a}\right)\right]' = \ldots$$

$$v'(a+l) = \frac{1}{2}\frac{1}{\sqrt{a}}\frac{1}{\sqrt{a+l}}\sin\left(\beta\ln\frac{a+l}{a}\right) + \sqrt{\frac{a+l}{e}}\cos\left(\beta\ln\frac{a+l}{a}\right)\beta\frac{a}{a+l}\frac{1}{a} = 0$$

$$\tan\left(\beta \ln \frac{a+l}{a}\right) + 2\beta = 0$$

$$\beta = \sqrt{\frac{Pa^2}{EI_0} - \frac{1}{4}}$$



$\beta_c \rightarrow P$

$I_2 = I(a+l)$

$$\boxed{P_c = \frac{m\,E I_2}{l^2}}$$