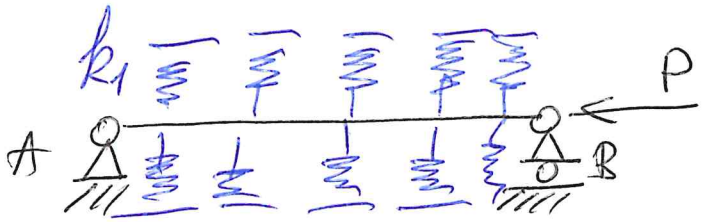
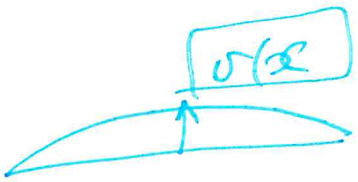
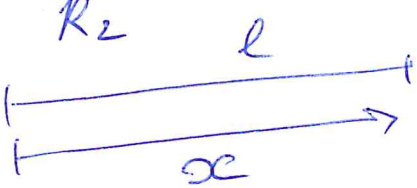
I. **Esempio 1**







$$v(0) = v''(0) = 0$$
$$v(\ell) = v''(\ell) = 0$$

$$EI\,v^{(4)} + P v'' + \underbrace{(k_1 + k_2)}_{k} v = 0$$

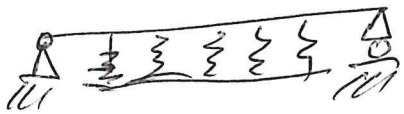
$$v = v_0 \sin \frac{\pi n}{\ell} x$$

$$\left[EI \left(\frac{\pi n}{\ell}\right)^4 - P \left(\frac{\pi n}{\ell}\right)^2 + (k_1 + k_2) \right] v_0 \sin \frac{\pi n}{\ell} x = 0$$

$$\Rightarrow EI \left(\frac{\pi n}{\ell}\right)^4 - P \left(\frac{\pi n}{\ell}\right)^2 + k = 0$$

$$P_n = \frac{(\pi n)^2}{\ell^2} EI + \frac{k \ell^2}{\pi n^2}$$

$$\rightarrow \min_n P_n = \underline{\underline{P_c}}$$

$\Rightarrow$ 

III

Posto $w = v_1 + v_2$

$(1) + (2) \Rightarrow$

$$EI\left[v_1^{(4)} + v_2^{(4)}\right] + \frac{P}{2}\left(v_1'' + v_2''\right) = 0$$

$$EI\, w^{(4)} + \frac{P}{2} w'' = 0$$

$$w = W_0 \sin\frac{\pi n}{\ell} x$$

$$w(0) = w''(0) = 0, \quad w(\ell) = w''(\ell) = 0$$

$\Rightarrow$

$$\frac{P}{2} = EI\frac{\pi^2 n^2}{\ell^2}$$

$$P_c = P_1 = 2EI\frac{\pi^2}{\ell^2}$$

$(n=1)$

$$P_c = \frac{2\pi^2}{\ell^2} EI$$

IV

Posto $u = v_1 - v_2$

$(\cdot) - (\ )$ =>

$$EI\left[v_1^{(4)} - v_2^{(4)}\right] + \frac{P}{2}\left[v_1'' - v_2''\right] + 2k\,(v_1 - v_2) = 0$$

$$EI\,u^{(4)} + \frac{P}{2}u'' + 2ku = 0$$

u : $u(0) = 0 = u''(0)$

$u(\ell) = 0 = u''(\ell)$

$$u = U_0 \sin\frac{\pi n}{\ell}x \quad , \quad n = 1, 2 \dots$$

$$EI\left(\frac{\pi n}{\ell}\right)^4 + \frac{P}{2}\left(\frac{\pi n}{\ell}\right)^2 + 2k = 0$$

$$P_n = 2\left(\frac{\pi n}{\ell}\right)^2 EI + \frac{4k\ell^2}{(\pi n)^2}$$

II modi :

1) w

2) u

barelling

$$P_c = \frac{2\pi^2}{\ell^2}EI$$