Il criterio dinamico.

$$EI\,v^{(4)} + P v'' = p(x)$$

$$v = v(x)$$

$$v = v(x,t) \qquad p \rightarrow -m\ddot{v}$$

$$\ddot{v} = \frac{\partial^2 v}{\partial t^2} \quad , \quad m = \frac{M}{l}$$

M – massa

l

$$EI\,v^{(4)} + P v'' + m\ddot{v} = 0$$

$$v^{(4)} + \underbrace{\frac{P}{EI}}_{\mu^2} v'' + \underbrace{\frac{m}{EI}}_{\rho} \ddot{v} = 0$$

$$\mu^2 = \frac{P}{EI} \quad , \quad \rho = \frac{m}{EI}$$

$$V^{(4)} = V'''' = V^{IV}$$

$$v^{(4)} + \mu^2 v'' + \rho\ddot{v} = 0$$

Posto: $v(x,t) = V(x)\,T(t)$

$$V^{(4)}T + \mu^2 V'' T + \rho V \ddot{T} = 0$$

diviso VT $\Rightarrow$ $$\frac{V^{(4)} + \mu^2 V''}{V} + \rho\frac{\ddot{T}}{T} = 0$$

$$\underbrace{\frac{V^{(4)}(x) + \mu^2 V''(x)}{V(x)}}_{(x)} = -\rho \underbrace{\frac{\ddot{T}(t)}{T(t)}}_{(t)}$$

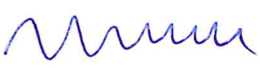
$$\underline{f(x) = g(t)} \quad ?$$

$$f(x) = \underbrace{\text{cost}}_{\lambda} = g(t)$$

$$\Rightarrow \quad -\rho \frac{\ddot{T}}{T} = \lambda$$

$$\rho \ddot{T} + \lambda T = 0$$

$$\ddot{T} + \frac{\lambda}{\rho} T = 0$$

$T(t) \sim$  $\Rightarrow \lambda > 0$

$$\frac{\lambda}{\rho} = \omega^2$$

$$T(t) = A \cos\omega t + B \sin\omega t \, , \quad A, B \sim \text{cost}.$$

$$= C \sin(\omega t + \varphi)$$

$$T(t) = C \sin(\omega t + \varphi) \quad \circ \quad \underline{T \sim e^{i\omega t}}$$

$$\frac{V^{(4)} + \mu^2 V''}{V} = \lambda$$

$$\boxed{V^{(4)} + \mu^2 V'' - \lambda V = 0}$$

$$V = V_0 e^{\alpha x}$$

$$\underline{\alpha^4 + \mu^2 \alpha^2 - \lambda = 0} \quad | \rightarrow \quad \underline{\alpha - ?}$$

$$\beta = \alpha^2 \rightarrow \quad \beta^2 + \mu^2 \beta - \lambda = 0$$

$$\Rightarrow \beta_{\pm} = \frac{-\mu^2 \pm \sqrt{\mu^4 + 4\lambda}}{2} \qquad \mu^2 = \frac{P}{EI}$$

$$\underline{\mu^4 + 4\lambda > 0} \qquad \lambda = ?$$

$$\lambda > 0$$

$$\beta_+ = \sqrt{\frac{\mu^4}{4} + \lambda} - \frac{\mu^2}{2} > 0$$

$$\beta_- = -\sqrt{\frac{\mu^4}{4} + \lambda} - \frac{\mu^2}{2} < 0$$

$\alpha^2 = \beta \Rightarrow$ 4 radici :

$$\alpha_{1,2} = \pm \underbrace{\sqrt{\sqrt{\frac{\mu^4}{4} + \lambda} - \frac{\mu^2}{2}}}_{S_1} = \pm S_1$$

$$\alpha_{3,4} = \pm i \underbrace{\sqrt{\sqrt{\frac{\mu^4}{4} + \lambda} + \frac{\mu^2}{2}}}_{S_2} = \pm i S_2$$

$V \sim e^{\alpha x}$

1) α – reale $\quad e^{\pm \alpha x} \quad$ sinh cosh

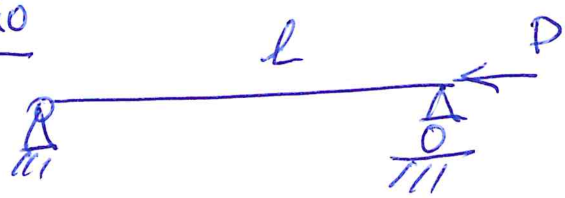
$$\sinh x = \frac{e^{x} - e^{-x}}{2}, \quad \cosh = \frac{e^{x} + e^{-x}}{2}$$

2) α – imaginarie $\rightarrow$ cos $\quad$ sin

$$\boxed{V(x) = C_1 \cosh S_1 x + C_2 \sinh S_1 x + C_3 \cos S_2 x + C_4 \sin S_2 x}$$

$C_1, \dots C_4$ – ?

§ Esempio



$$\begin{cases} V(0) = 0 \\ V''(0) = 0 \end{cases} \qquad \begin{rcases} V(\ell) = 0 \\ V''(\ell) = 0 \end{rcases}$$

$$V(0) = C_1 + C_3 = 0$$

$$V''(0) = C_1 S_1^2 - C_3 S_2^2 = 0$$

$\sin'' = -\sin$

$\cos'' = -\cos$

$\sinh'' = \sinh$

$\cosh'' = \cosh$

$$c_1 + c_3 = 0$$

$$s_1^2 c_1 - s_2^2 c_3 = 0$$

$$\underbrace{\begin{pmatrix} 1 & 1 \\ s_1^2 & -s_2^2 \end{pmatrix}}_{A} \begin{pmatrix} c_1 \\ c_3 \end{pmatrix} = 0$$

$$\det A = 0 \qquad \begin{vmatrix} 1 & 1 \\ s_1^2 & -s_2^2 \end{vmatrix} = -s_2^2 - s_1^2 \neq 0$$

$$\Rightarrow \boxed{c_1 = c_3 = 0}$$

$x = l$ $\qquad V(x) = c_2 \sinh s_1 x + c_4 \sin s_2 x$

$$V(l) = c_2 \sinh s_1 l + c_4 \sin s_2 l = 0$$

$$V''(l) = c_2 s_1^2 \sinh s_1 l - c_4 s_2^2 \sin s_2 l = 0$$

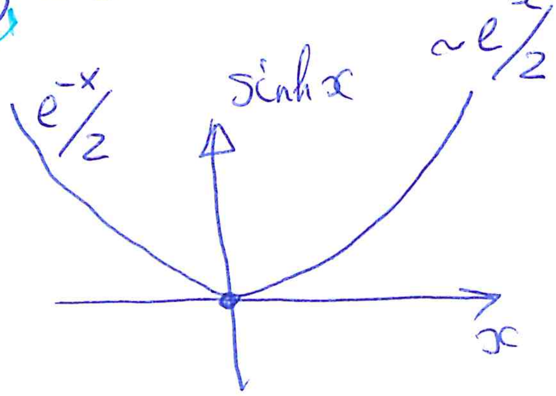
$$\underbrace{\begin{pmatrix} \sinh s_1 l & \sin s_2 l \\ s_1^2 \sinh s_1 l & -s_2^2 \sin s_2 l \end{pmatrix}}_{B} \begin{pmatrix} c_2 \\ c_4 \end{pmatrix} = 0$$

$\det B = 0$

$$-\sinh s_1 l \; s_2^2 \sin s_2 l - \sin s_2 l \; s_1^2 \sinh s_1 l = 0$$

$$-\sin s_2 l \left[s_2^2 \sinh s_1 l + s_1^2 \sinh s_1 l \right] = 0$$

$$-\sin s_2 l \; \underbrace{\sinh s_1 l}_{\neq 0} \; \underbrace{\left(s_1^2 + s_2^2 \right)}_{\neq 0} = 0$$



$$\Rightarrow \boxed{\sin s_2 l = 0}$$

$$s_2 l = \pi n \quad , n = 1, 2 \ldots$$

$$s_2 = \frac{\pi n}{l} \qquad s_2^2 = \frac{\mu^2}{2} + \sqrt{\frac{\mu^4}{4} + \lambda}$$

λ – ?

$$\sqrt{\frac{\mu^4}{4} + \lambda} = s_2^2 - \frac{\mu^2}{2}$$

$$\frac{\mu^4}{4} + \lambda = \left(s_2^2 - \frac{\mu^2}{2} \right)^2$$

$$\lambda = -\frac{\mu^4}{4} + \left(s_2^2 - \frac{\mu^2}{2} \right)^2 =$$

$$= \cancel{-\frac{\mu^4}{4}} + s_2^4 - \cancel{2} s_2^2 \frac{\mu^2}{\cancel{2}} + \cancel{\frac{\mu^4}{4}} =$$

$$= s_2^2 \left(s_2^2 - \mu^2 \right)$$

$$\lambda = s_2^2(s_2^2 - \mu^2) \quad , \quad \boxed{s_2 = \frac{\pi n}{\ell}}$$

$$\lambda = \frac{\pi^2 n^2}{\ell^2}\left(\frac{\pi^2 n^2}{\ell^2} - \frac{P}{EI}\right) \qquad \mu^2 = \frac{P}{EI}$$

$$\lambda = \frac{\pi^2 n^2}{\ell^2} \cdot \frac{\pi^2 n^2}{\ell^2}\left(1 - \frac{P}{EI}\,\frac{\ell^2}{\pi^2 n^2}\right)$$

$$^{\shortparallel} P_n$$

$$\boxed{P_n = \frac{\pi^2 n^2}{\ell^2} EI}$$

$$\lambda = \frac{\pi^4 n^4}{\ell^4}\left(1 - \frac{P}{P_n}\right)$$

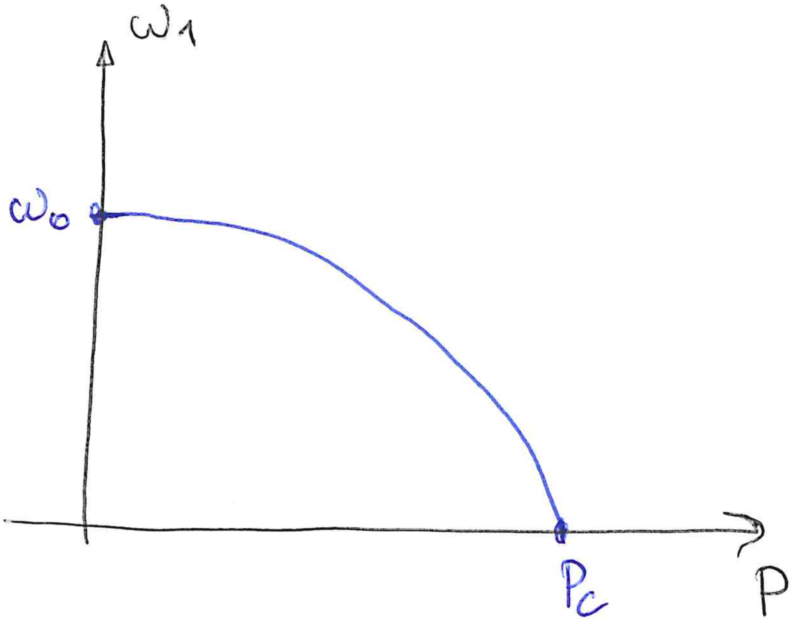
$$v(x,t) = \bar{V}(x)\,T(t) =$$

$$\boxed{v = \left[C_2 \sinh s_1 x + C_4 \sin s_2 x\right] \sin(\omega t + \varphi)}$$

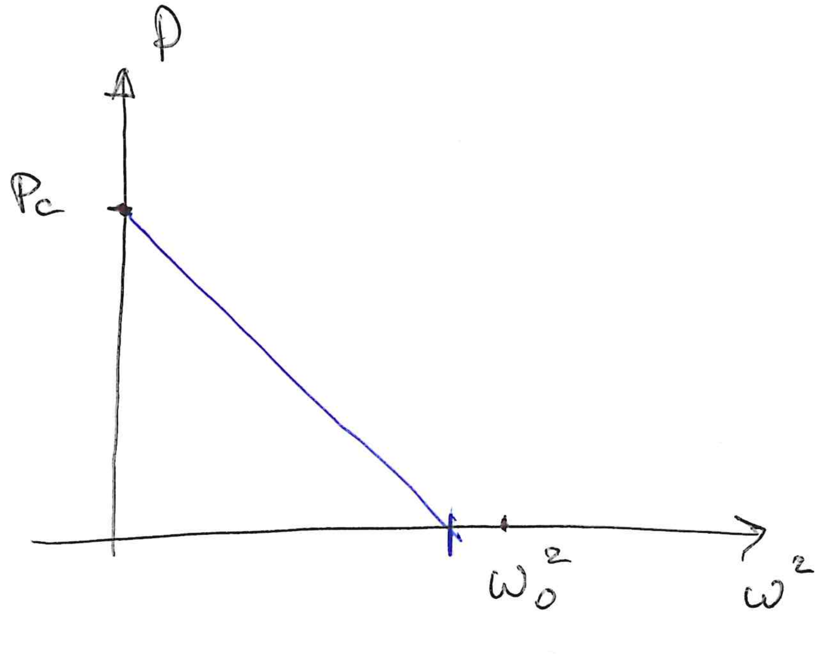
$$\omega = \sqrt{\frac{\lambda}{\rho}}$$

$$\omega = \omega_n = \frac{\pi^2 n^2}{\ell^2}\sqrt{1 - \frac{P}{P_n}}$$

$$\boxed{\omega_1 = \frac{\pi^2}{\ell^2}\sqrt{1 - \frac{P}{P_c}}}$$

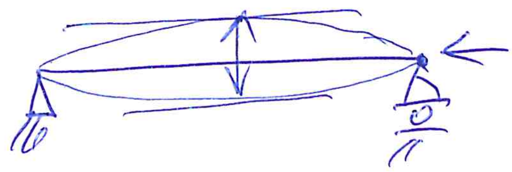


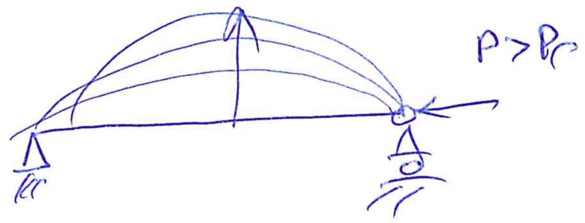
$\omega_0 = \frac{\pi^2}{\ell^2}$ dove $P=0$



1) $P < P_c$ $\qquad v \sim \sin \omega t$

2) $P > P_c$ $\qquad v \sim e^{\gamma t}$

1) 

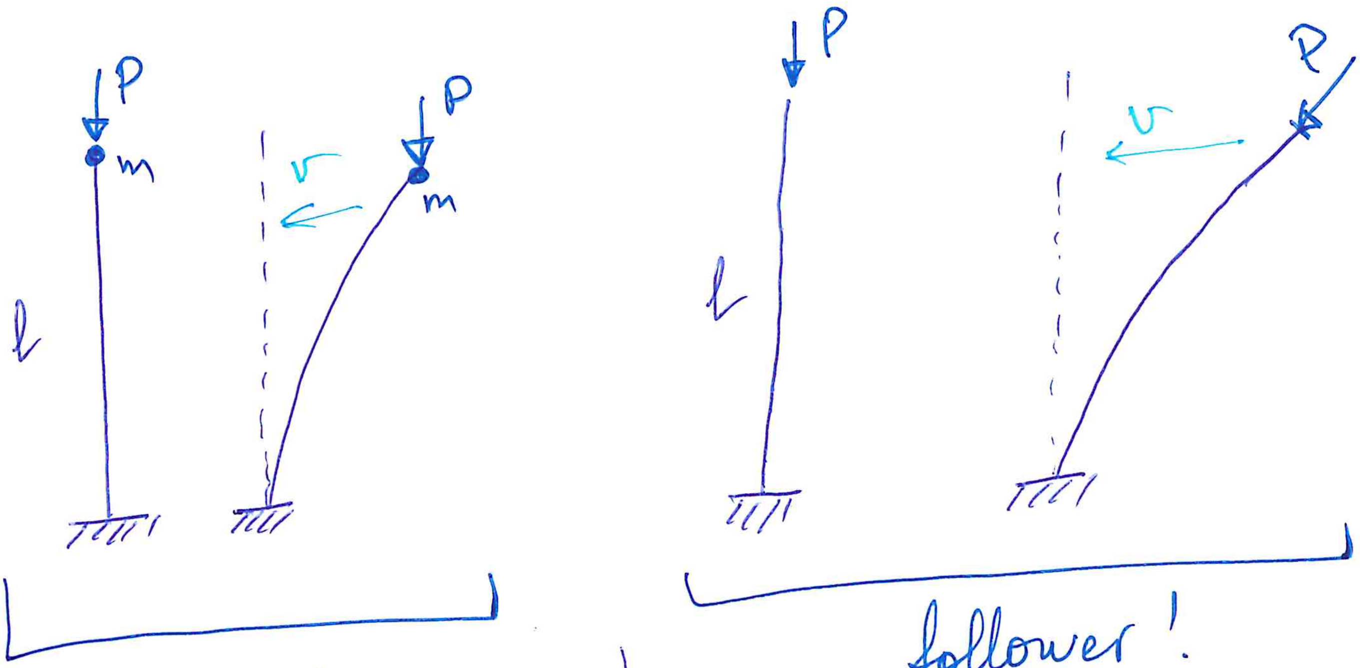
2) 

Il criterio dinamico

$\omega = 0$ $\qquad P = P_c$

$$\boxed{\text{Il cr. din.} = \text{il criterio statico}}$$

forze follower



conservativo

$$EI v'''' + P v'' = 0 \; / -m\ddot{v}$$

$$\left.\begin{aligned} EI v''' + \underline{P v'} &= 0 \\ EI v'' &= 0 \end{aligned}\right\} x = l$$

$$v(0) = 0 = v'(0); \; x = 0$$

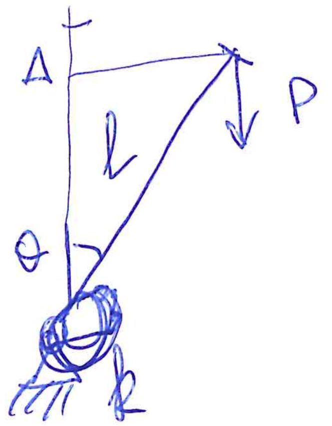
follower!

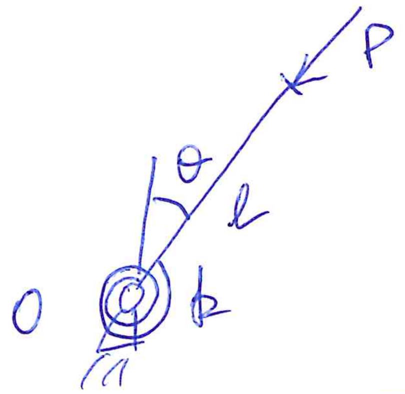
$$EI v'''' + P v'' = 0 \; / -m\ddot{v}$$

$$\left.\begin{aligned} EI v''' &= 0 \\ EI v'' &= 0 \end{aligned}\right\} x = l$$

$$v(0) = 0 = v'(0); \; x = l$$

Esempio





1) ~~k~~ ETP :

$$\Pi = \frac{1}{2} k\theta^2 - P\Delta$$

$$\Delta = l - l\cos\theta$$

$$\Pi = \frac{1}{2} k\theta^2 - Pl(1-\cos\theta)$$

$$\Pi' = k\theta - Pl\sin\theta = 0$$

$$P_c = \frac{k}{l}$$

2) Il criterio statico

$$k\theta = Pl\sin\theta$$

$$P_c = \cancel{P}\,\frac{k}{l}$$

① ETP:

$$\Pi = \frac{1}{2} k\theta^2$$

$$\Pi' = k\theta \Rightarrow \theta = 0 \,!$$

2) Il criterio statico

$$M_0 = 0$$

$$M_e = k\theta = 0$$

$$\boxed{\theta = 0}$$