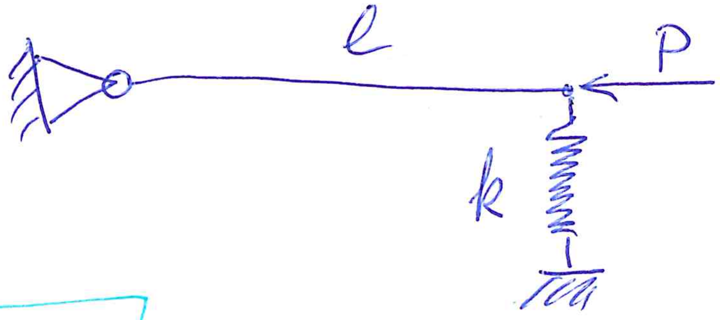
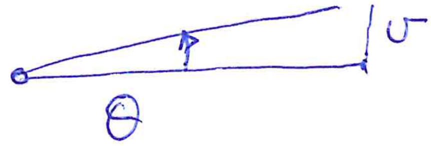
Esempio



$\boxed{P_c - ?}$

1) il criterio energetico:

$$\Pi = U + V$$



$$U = \frac{1}{2} k v^2 , \quad v = l \sin\theta , \quad \theta \sim 0$$

$$v \approx l\theta$$

$$V = -P\Delta , \quad \Delta = l - l\cos\theta$$

$$\theta \sim 0 \quad \Delta \sim l\left(1 - 1 + \frac{\theta^2}{2} + \dots\right)$$

$$\Delta \approx l\frac{\theta^2}{2}$$

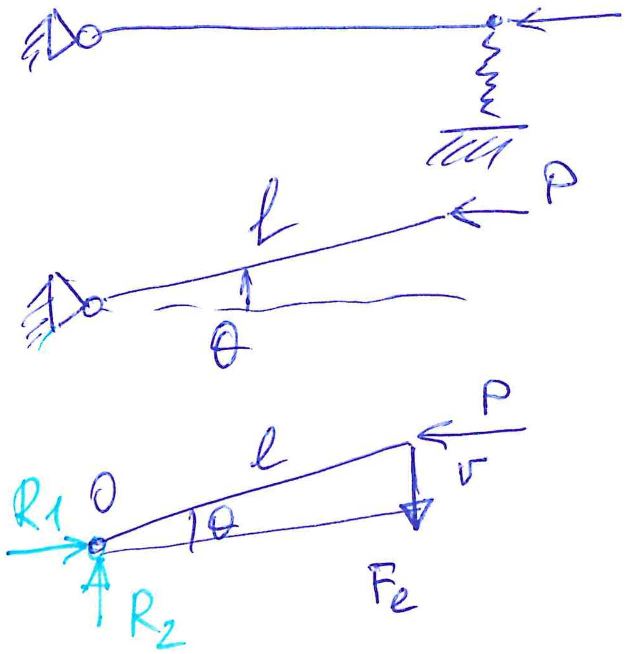
$$\frac{d\Pi}{d\theta} = 0 \Rightarrow \frac{d\Pi}{d\theta} = kl^2\theta - Pl\theta = 0$$

$\Rightarrow$ I. $\theta = 0$ – triviale / banale

II $\theta \neq 0 \quad kl^2 - Pl = 0 \Rightarrow \underline{P = kl}$

$$\boxed{P_c = kl}$$

2) il criterio statico



$\uparrow \quad R_2 = F_e$

$\rightarrow \quad R_1 = P$

$\circlearrowright$

$M_O = 0$

$P v = F_e \, l \cos\theta$

$\theta \sim 0$

$v = l\theta$

$\cos\theta \sim 1$

$P l \theta = F_e l$

$F_e = k v = k l \theta$

$$P l \theta = k l^2 \theta$$

l'equazione di equilibrio

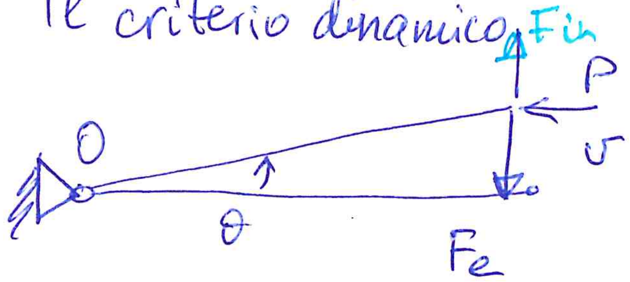
$$(Pl - kl^2)\theta = 0$$

$\Rightarrow$ I. $\theta = 0$

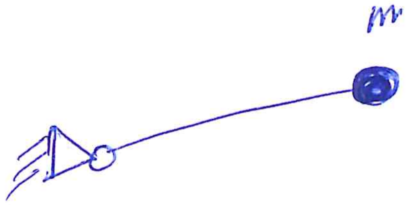
II. $Pl = kl^2 \Rightarrow P = kl$

$$\boxed{P = P_c = kl}$$

3) Il criterio dinamico



F_{in} - forza di inerzia



$\boxed{\text{massa ?}}$ massa m

d'Alambert =>

$\boxed{M_0 = 0} \Rightarrow P l \theta - F_e l - \underbrace{m \ddot{v} l}_{F_{in} \cdot l} = 0$

$$P l \theta - k l^2 \theta - m l^2 \ddot{\theta} = 0$$

$$m l \ddot{\theta} = (P - k)\theta$$

$$\theta = \theta_0 e^{\lambda t} \qquad \boxed{\lambda - ?}$$

$$m l \lambda^2 \theta_0 = (P - kl)\theta_0$$

$$\theta_0 = 0 \Rightarrow \theta = 0$$

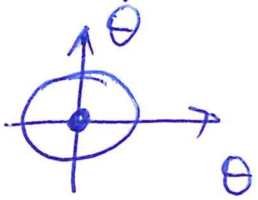
$$m l \lambda^2 = P - kl$$

$$\boxed{\lambda = \pm \sqrt{\frac{P - kl}{ml}}}$$

$P < kl$, $P < P_c$

$$\lambda = \pm i \sqrt{\frac{k - P}{ml}}$$

$$\theta = a \cos \lambda t + b \sin \lambda t$$



$P > kl \quad (P > P_c)$

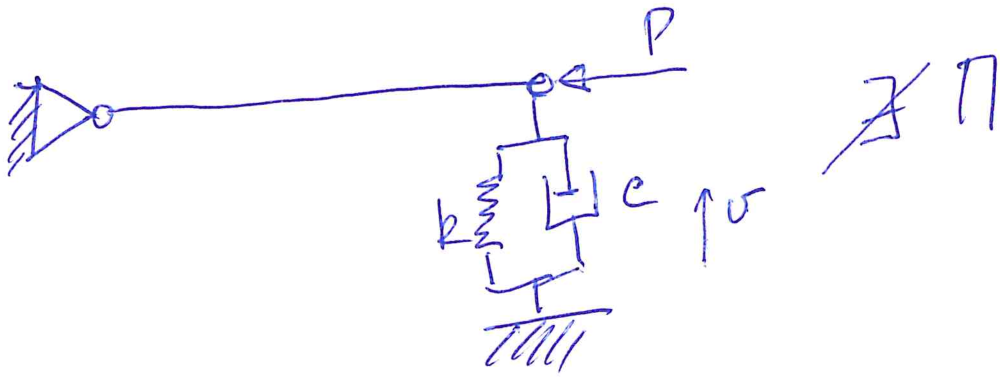
$\lambda = \pm \sqrt{\frac{P-k}{ml}}$ $\quad \underline{\underline{\mathrm{Re}\,\lambda}} \neq 0$

$\theta(t) = \underline{\underline{a e^{\lambda t}}} + b e^{-\lambda t}$

<u>instabilità</u>

$\Rightarrow \boxed{P_c = kl}$

lo sforzamento



il criterio dinamico

$F_v = c\dot{v}$

$F_e \Rightarrow F_e + F_v$

$$\underbrace{ml^2\ddot{\theta}}_{M_{in}} = \underbrace{Pl\theta}_{M_P} - \underbrace{kl^2\theta}_{M_e} - \underbrace{cl\dot{\theta}}_{M_{viscosite}}$$

$\theta = \theta_0 e^{\lambda t}$

$$\underbrace{ml^2}_{a}\lambda^2 + \underbrace{cl}_{b}\lambda + \underbrace{(kl-P)}_{c}l = 0$$

$\lambda - ?$

$ax^2 + bx + c = 0$

$$x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-c \pm \sqrt{c^2 - 4ml(kl-P)}}{2ml}$$

$$\boxed{\lambda = -\frac{c}{2ml} \pm \sqrt{\left(\frac{c}{2ml}\right)^2 - \frac{kl-P}{ml}}} \Rightarrow \underline{Re\,\lambda - ?}$$

1) $P < kl$ $\quad (P < P_c)$ $\qquad \lambda_\pm = -\frac{c}{2ml} \pm \sqrt{\left(\frac{c}{2ml}\right)^2 - \frac{kl-P}{ml}}$

a) $\underline{\mathrm{Re}\,\lambda < 0}$

Perchè : $\underline{c > 0}$, $\quad \sqrt{\left(\frac{c}{2ml}\right)^2 - \underbrace{(\quad)}_{>0}} < \frac{c}{2ml}$

$\Rightarrow$ stabilità

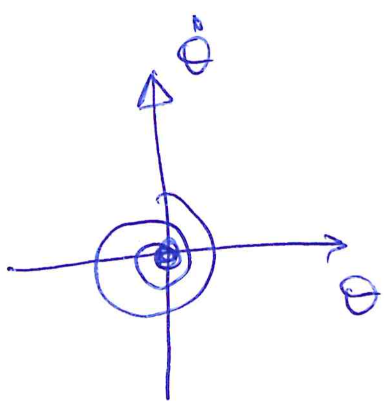
2) $P > kl$ $\quad (P > P_c)$

$$\left(\frac{c}{2ml}\right)^2 - \frac{kl-P}{ml} = \left(\frac{c}{2ml}\right)^2 + \frac{P-kl}{ml} > 0$$

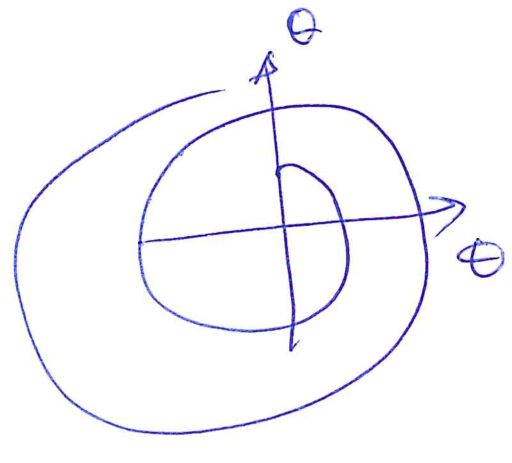
$$> \frac{c^2}{4(ml)^2}$$

$\left.\begin{array}{l} \lambda_+ > 0 \\ \lambda_- < 0 \end{array}\right| \Rightarrow$ instabilità

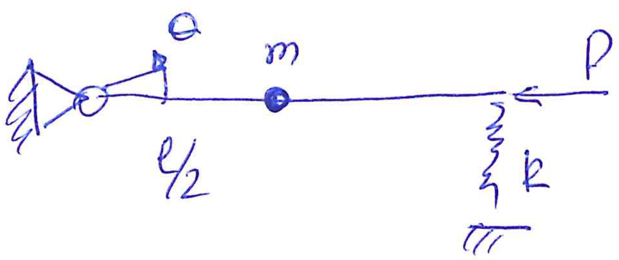
stabilità



instabilità



distribuzione di massa



$$F_{in} = -m\ddot{v}_m \quad , \quad v_m = \frac{\ell}{2}\theta$$

$$M_{in} = \frac{\ell}{2} F_{in}$$

$$\Rightarrow \quad \frac{m\ell^2\ddot{\theta}}{4} = (P\ell - k\ell^2)\theta$$

differenza!

$$\theta = \theta_0 e^{\lambda t}$$

$$\frac{m\ell^2\lambda^2}{4} = (P - k\ell)\ell$$

$$\lambda = \pm\sqrt{\frac{4(P-k\ell)}{m\ell}}$$

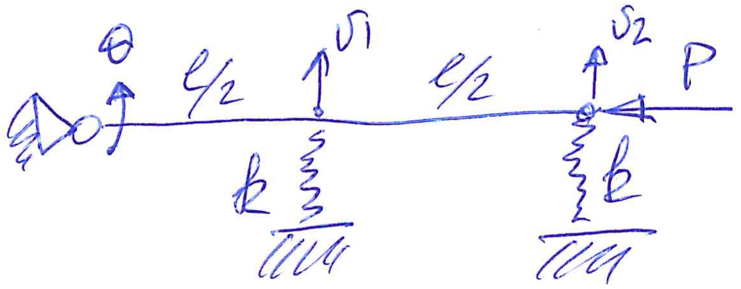
$\Rightarrow \quad Re\,\lambda - ?$

$P < k\ell \qquad Re\,\lambda = 0$

$P > k\ell \qquad Re\,\lambda \neq 0$

$$\boxed{Re\,\lambda > 0}$$

Esempio 2



$\boxed{P_c - ?}$

$$\Pi = U + V,$$

$$V = -P\Delta = \underline{-P(l - l\cos\theta)}$$

$$U = \frac{1}{2}k v_1^2 + \frac{1}{2}k v_2^2$$

$$\theta \sim 0, \quad v_1 = \frac{l}{2}\theta, \quad v_2 = l\theta$$

$$U = \frac{1}{2}k\frac{l^2}{4}\theta^2 + \frac{1}{2}k l^2\theta^2 =$$

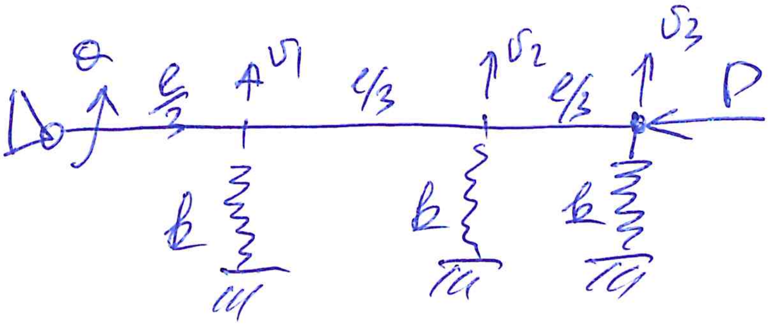
$$= \frac{1}{2}k\left(l^2 + \frac{l^2}{4}\right)\theta^2$$

$$\frac{d\Pi}{d\theta} = kl^2\left(1 + \frac{1}{4}\right)\theta - Pl\theta = 0$$

I) $\theta = 0$

II) $kl^2\frac{5}{4} - Pl = 0 \implies \boxed{P_c = \frac{5}{4}kl}$

Esempio 3



$$U = \frac{1}{2} k v_1^2 + \frac{1}{2} k v_2^2 + \frac{1}{2} k v_3^2 =$$

$$= \frac{1}{2} k \left(\frac{\ell}{3}\theta\right)^2 + \frac{1}{2} k \left(\frac{2\ell}{3}\right)^2 \theta^2 + \frac{1}{2} k \ell \theta^2 =$$

$$= \frac{1}{2} k \ell^2 \left[\frac{1}{9} + \frac{4}{9} + 1 \right] \theta^2$$

$$\frac{1+4+9}{9} = \frac{14}{9}$$

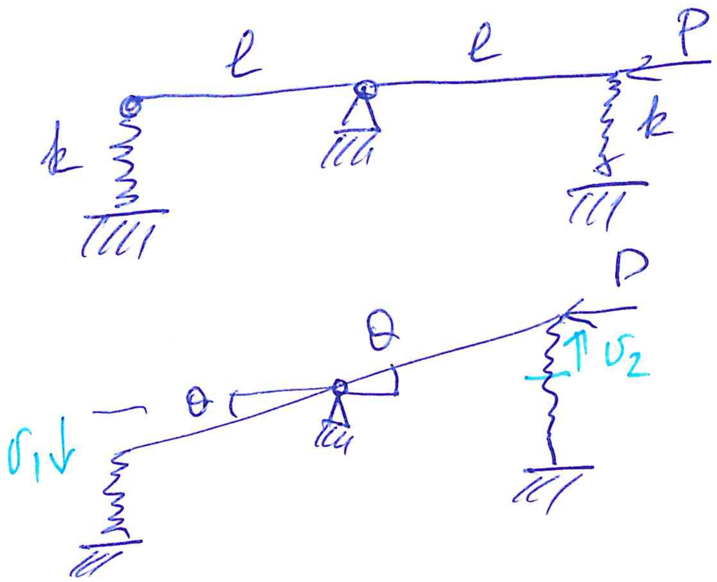
$$U = \frac{1}{2} k \ell^2 \frac{14}{9} \theta^2$$

$$\Pi = U + V \quad , \quad V = -P(\ell - \ell\cos\theta) \sim P\ell\frac{\theta^2}{2}$$

$$\frac{d\Pi}{d\theta} = \frac{1}{2} k \ell \frac{14}{9} \cdot 2\theta - P\ell\theta = 0$$

$$\boxed{P_c = \frac{14}{9} k \ell}$$

Esempio 4



$$\Pi = U + V \quad , \quad V = -P(l - l\cos\theta)$$

$$\theta \sim 0$$

$$V = -Pl\frac{\theta^2}{2}$$

$$U = \frac{1}{2}kv_1^2 + \frac{1}{2}kv_2^2 = \frac{1}{2}k(l\theta)^2 + \frac{1}{2}k(l\theta)^2$$

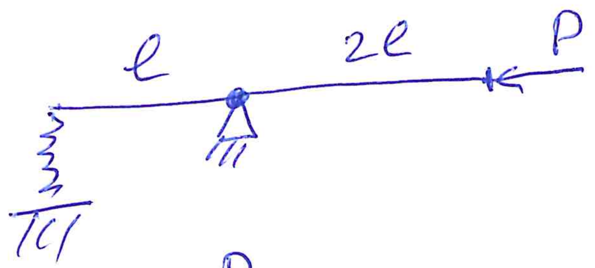
$$= \frac{1}{2}kl^2 \cdot 2\theta^2$$

$$\frac{d\Pi}{d\theta} = 2kl^2\theta - Pl\theta = 0$$

$$\Rightarrow \quad \text{I)}\ \theta = 0$$

$$\text{II)}\ \theta \neq 0 \qquad P_1 = P_c = 2kl$$

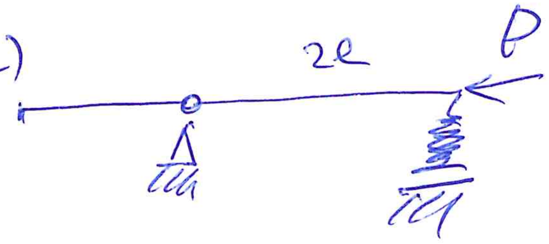
Esempio 5

1) 

P_{c1}

$$U = \frac{k}{2}(\ell\theta)^2$$

$$P_c = k\ell$$

2) 

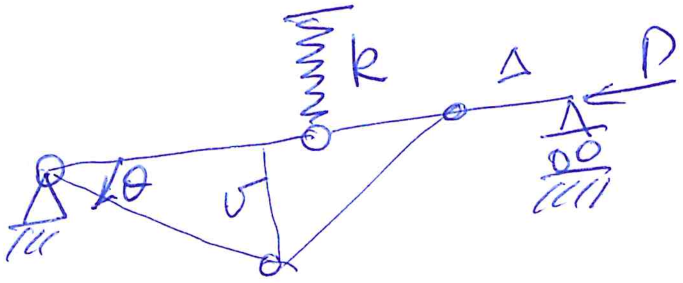
P_{c2}

$$U = \frac{k}{2}(2\ell\theta)^2$$

$$P_c = 4k\ell$$

Esempio 6

$\boxed{P_c - ?}$



$\theta \sim 0$

$$\Pi = U + V, \quad V = P(2\ell - 2\ell\cos\theta) = P2\ell\frac{\theta^2}{2}$$

$$v = \ell\sin\theta \sim \ell\theta$$

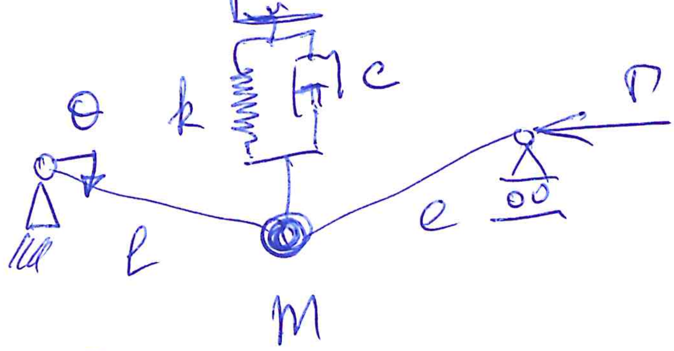
$$U = \frac{1}{2}kv^2 = \frac{1}{2}k\ell^2\theta^2$$

$$\Pi = \frac{1}{2}k\ell^2\theta^2 - P\ell\theta^2$$

$$0 = \frac{d\Pi}{d\theta} = k\ell^2\theta - 2P\ell\theta \Rightarrow$$

$$\boxed{P_c = \frac{k\ell}{2}}$$

Compiti a casa



$\boxed{\lambda - ?}$ $\theta(t)$