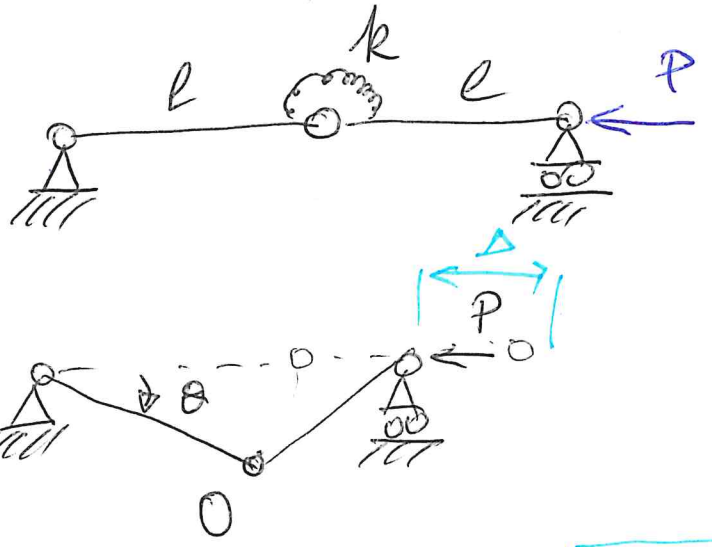
Esempio 1



Il criterio statico.

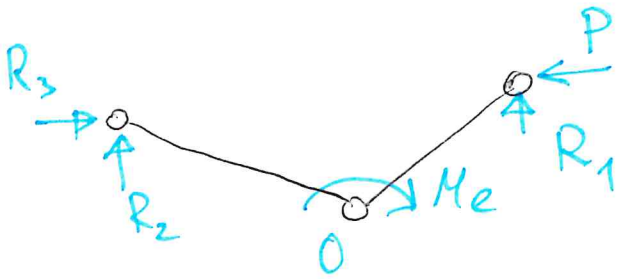
$$\sum_i F_i = 0 \qquad \sum_i M_{O_i} = 0$$

θ – un grado di libertà

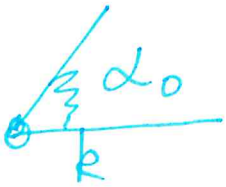
$R_1 + R_2 = 0$

$R_1 = R_2 = 0$

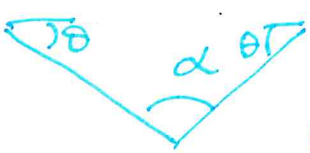
$R_3 = P$



M_O : M_e – ?



$M \sim k(\alpha - \alpha_0)$



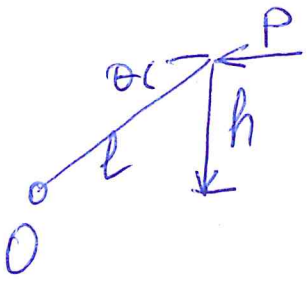
$\alpha = \pi - 2\theta$

$\alpha - \alpha_0 = \pi - 2\theta - \pi = -2\theta$

$\Rightarrow$ (2) !

M_O :

$$M_e = k\,2\theta$$



$$M_P = P\,h$$

$$l \sin\theta = h$$

$$\Rightarrow M_P = P\,l\sin\theta$$

$\Rightarrow$ l'equilibrio $\quad \Sigma M_{O_i} = 0$

$$M_e = P\,l\sin\theta$$

$$2k\theta - P\,l\sin\theta = 0$$

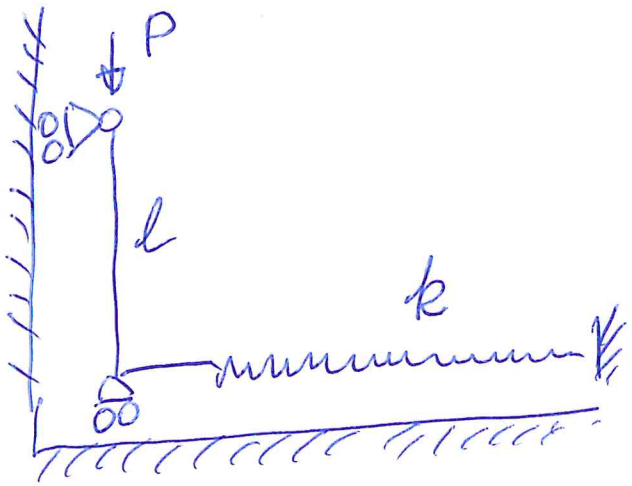
$\Rightarrow$ (I) $\quad \theta = 0$

(II) $\quad P = \dfrac{2k}{l}\,\dfrac{\theta}{\sin\theta}$

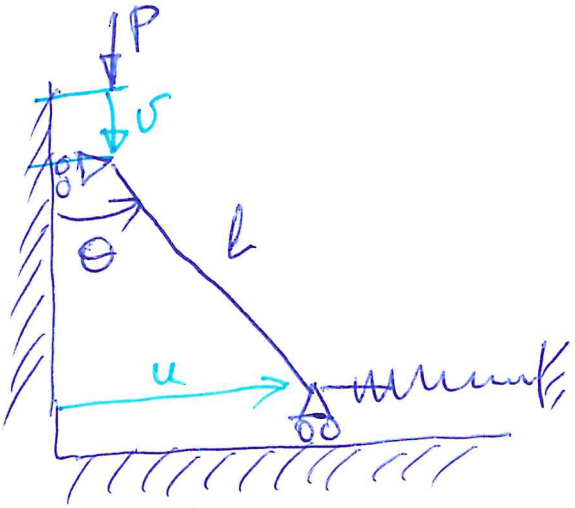
$\theta \sim 0$ $\quad$ ~~dll~~ $\quad \boxed{P_c = \dfrac{2k}{l}}$

Esempio 2

$P_c = ?$



θ – ! nostra coordinata



(1) Il criterio energetico

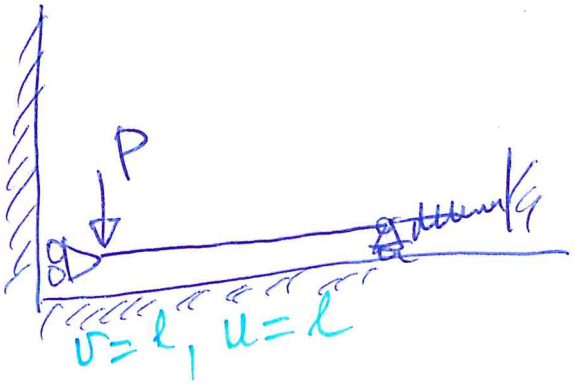
$$\Pi = U + V$$

$$U = \frac{k}{2} u^2$$

$$V = -Pv$$

$$u = l \sin\theta$$

$$v = l - l\cos\theta$$



$$\Pi = \frac{1}{2} k u^2 - Pv = \frac{1}{2} k (l\sin\theta)^2 - Pl(1-\cos\theta)$$

a) $$\frac{d\Pi}{d\theta} = \frac{1}{2} k l^2 2\sin\theta\cos\theta - Pl\sin\theta$$

$$\frac{d\Pi}{d\theta} = \frac{1}{2} k\ell^2 2\sin\theta\cos\theta - P\ell\sin\theta$$

$$= \frac{4}{2} k\ell^2 \sin\theta\cos\theta - P\ell\sin\theta$$

<u>l'equilibrio</u> : $\frac{d\Pi}{d\theta} = 0$

$$\ell\sin\theta\,(k\ell\cos\theta - P) = 0$$

$$\Rightarrow \quad \sin\theta = 0 \quad (I)$$

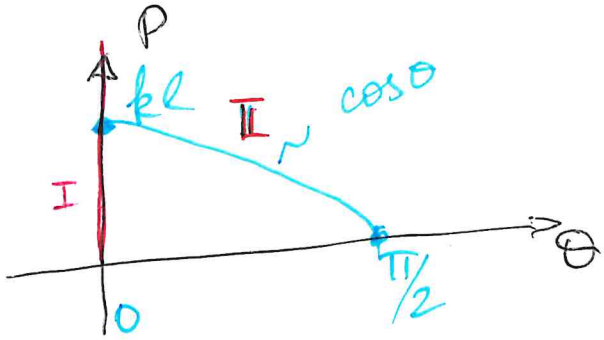
$$P = k\ell\cos\theta \quad (II)$$

(I) $\sin\theta = 0$ $\quad \theta = 0, \quad \theta = \pm\pi n$

<u>$0 \le \theta \le \pi/2$</u>

$\Rightarrow$ <u>$\theta = 0$</u>

(II) $P = k\ell\cos\theta$,



b) $\Rightarrow \quad \frac{d^2\Pi}{d\theta^2} - ?$

$$\frac{d\Pi}{d\theta} = kl^2 \sin\theta\cos\theta - Pl\sin\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\frac{d\Pi}{d\theta} = \frac{1}{2}kl^2\sin 2\theta - Pl\sin\theta$$

$$\frac{d^2\Pi}{d\theta^2} = kl^2\cos 2\theta - Pl\cos\theta$$

$\frac{d^2\Pi}{d\theta^2} \gtrless 0$?

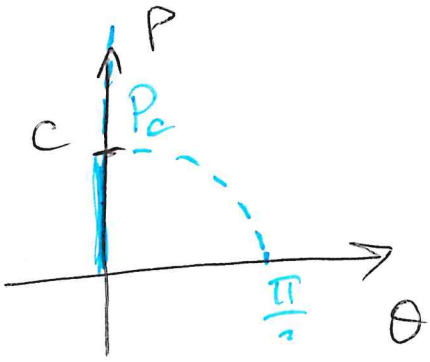
(I) $\theta = 0$

$$\frac{d^2\Pi}{d\theta^2}^{\circ} = kl^2 - Pl$$

$P < kl \rightarrow \frac{d^2\Pi}{d\theta^2} > 0 \rightarrow$ stabile

$P > kl \rightarrow \frac{d^2\Pi}{d\theta^2} < 0 \rightarrow$ instabile

P_c: $\frac{d^2\Pi}{d\theta^2} = 0 \Rightarrow \boxed{P_c = kl}$



(II) $P = kl\cos\theta$

$$\frac{d^2\Pi}{d\theta^2} = kl^2\cos 2\theta - \underbrace{(kl\cos\theta)}_{P}l\cos\theta =$$

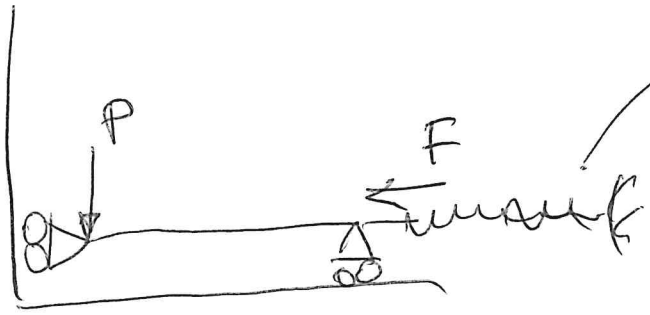
$$= kl^2\cos 2\theta - kl^2\cos^2\theta = kl^2[\cos 2\theta - \cos^2\theta]$$

$$= kl^2[\cos^2\theta - \sin^2\theta - \cos^2\theta] =$$

$$= -kl^2\sin^2\theta \leq 0$$

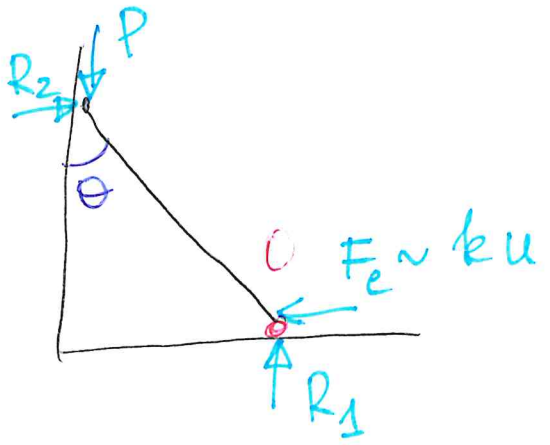
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

III

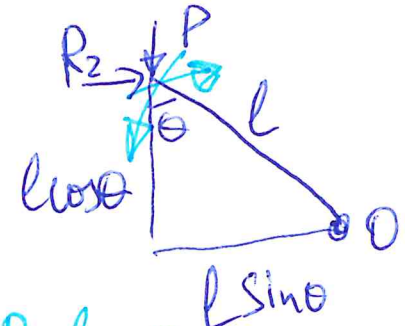


$\Rightarrow$ Il criterio statico.

$\theta \in [0, \frac{\pi}{2}]$



$$\begin{cases} \sum_i F_i = 0 \\ \sum_i M_i = 0 \end{cases}$$



$\rightarrow$ $R_2 = F_e$

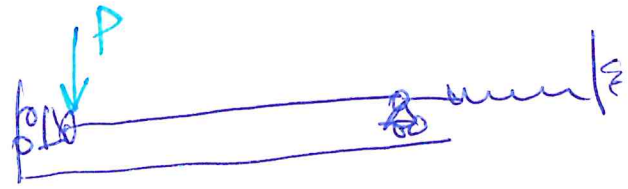
$\uparrow$ $R_1 = P$

P.O $M_O = 0 \Leftrightarrow P\ell\sin\theta = R_2\ell\cos\theta$

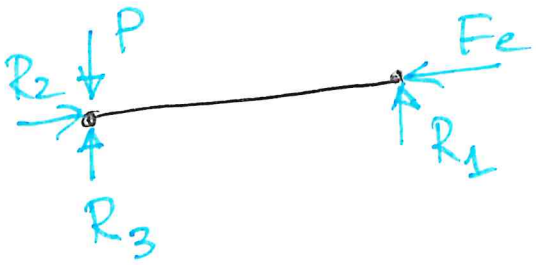
$$P\ell\sin\theta = F_e\ell\cos\theta = ku\ell\cos\theta = kl^2\sin\theta\cos\theta$$

($u = \ell\sin\theta$)

$$\boxed{P\ell\sin\theta = kl^2\sin\theta\cos\theta}$$



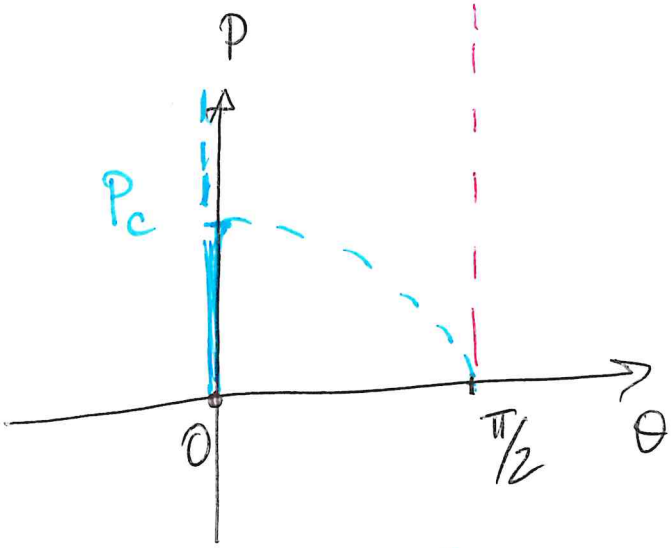
l'equilibrio :



$\Rightarrow R_1 = 0 \qquad \Rightarrow \theta = \frac{\pi}{2} \quad \forall P$

$R_3 = P$

$R_2 = Fe$



$P_c = kl$

$\theta = \frac{\pi}{2} \Rightarrow \Pi = \frac{1}{2} k l^2 - Pl$

$\frac{d\Pi}{d\theta} = 0$

$\frac{d^2\Pi}{d\theta^2} = 0$