

REVIEW

LOSSES AND EFFICIENCIES IN AXIAL-FLOW TURBINES

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Summary—Calculations of efficiencies of axial-flow steam turbines have been based for many years upon experimentally determined velocity coefficients. A great amount of uncorrelated data is available. Efficiency calculations for axial-flow gas turbines have been based on loss coefficients obtained from two-dimensional cascade tests, and some correlation of this data has been attempted, notably by Ainley and Mathieson¹. The present paper attempts to bring together the two sets of data and the differing nomenclature and approaches of the steam-turbine designer and the gas-turbine designer. Areas are indicated where further research would be useful.

1.0 NOTATION

P	force
ρ	density
A	area
c	velocity
c_x	axial velocity
c_θ	tangential velocity
w	relative velocity
w_θ	relative tangential velocity
η	efficiency
η_n	nozzle efficiency
m	mass flow rate
ϕ	velocity coefficient (stators)
ψ	velocity coefficient (rotors)
p	pressure
p_0	stagnation pressure
h	enthalpy
h_0	stagnation enthalpy
U	blade speed
ξ	“enthalpy-loss” coefficient
Y	loss coefficient based on stagnation-pressure loss
s	entropy
C_L	lift coefficient
C_{L_u}	tangential-force coefficient
C_D	drag coefficient
α	air angle (absolute)

β	air angle (relative)
α'	blade angle (absolute)
β'	blade angle (relative)
T	temperature
R	reaction
v_{is}	blade-speed ratio (based on isentropic velocity)
v	blade-speed ratio (based on nozzle velocity)
π	blade-loading coefficient
ΔW	work output
μ	viscosity
a	velocity of sound
S	blade spacing
L	blade chord
o	blade throat
b	blade axial-chord
H	blade height
t	blade thickness
δ	blade clearance
δ_1	boundary-layer thickness at entry
A.R. = H/b	aspect ratio
Re	Reynolds number
γ	specific-heat ratio
\bar{R}	gas constant
M	Mach number
D_h	hydraulic mean diameter
ϵ	deflexion

Subscripts and superscripts

1	upstream of nozzle
2	between nozzle and rotor
3	downstream of rotor
TS	total-to-static
TT	total-to-total
p	profile
s	secondary
c	clearance
m	mean
' , "	referring to loss-coefficient correlation
*	“nominal”
is	isentropic
e	trailing edge

2.0 INTRODUCTION

MUCH of the early information on flows in steam-turbine nozzles and blades has been brought together in the works of Stodola² and Kearton³. In these books the values of leaving-angles and losses are presented for a wide variety

of impulse stators (both nozzles and blades) and impulse rotors (or buckets), and for Parsons (50 per cent) reaction blading.

In the gas turbine the sharp distinction between impulse and reaction (50 per cent) disappears, for the reaction varies along the blade, which is usually twisted from root to tip. The reaction may approach zero (impulse) at the root and exceed 50 per cent at the tip. Gas-turbine designers have used tests of blade aerofoils in cascades to provide data on leaving-angles and losses.

Experimental data on losses obtained from static tests on steam nozzles have usually been presented in the form of a velocity coefficient, the ratio of the "observed" velocity to the so-called "isentropic steam velocity"—the velocity obtained in a hypothetical isentropic expansion to the exhaust back pressure. The "observed" velocity is usually based on a force measurement (the impulsive force of the jet on a plate, or the reactive force on the nozzle row) and a flow measurement. The force is

$$P = \int \rho c^2 dA$$

and the flow rate is

$$m = \int \rho c dA$$

The "observed" velocity $\bar{c} = \frac{P}{m} = \frac{\int \rho c^2 dA}{\int \rho c dA}$

and is the velocity of a uniform stream having the same mass flow and the same momentum as the actual stream. The velocity coefficient is

$$\phi = \frac{\bar{c}}{c_{is}} = \frac{\int \rho c^2 dA}{c_{is} \int \rho c dA}$$

Kearton³ and Keenan⁴ have pointed out that the energy of such a uniform stream ($\frac{1}{2}\rho\bar{c}^3 A$) is not the same as the energy of the actual stream ($\int \frac{1}{2}\rho c^3 dA$). The nozzle efficiency in three-dimensional flow

$$\eta_{n_2} = \frac{\int \rho c^3 dA}{c_{is}^2 \int \rho c dA}$$

is therefore not the square of the velocity coefficient in a three-dimensional flow. Kearton quotes experiments by Hodkinson and Devey⁵ in which the following values of the discrepancy between $\sqrt{\eta_{n_2}}$ and ϕ were obtained:

ϕ	1	0.98	0.96	0.94
$\frac{\sqrt{(\eta_{n_2})} - \phi}{\phi} \%$	0	0.3	0.6	0.9

Oakden⁶ pointed out that in a "strip theory" of the flow through a turbine stage (in which the axial velocity profile remains unaltered across the rotor) the work delivered by the stage is the same as that delivered by a stage with uniform nozzle velocity \bar{c} . In fact, the variation of work along the blade will alter the velocity profile considerably.

Experimental data on losses obtained from gas-turbine cascades are usually presented in the form of a mean stagnation pressure loss across the cascade. (Wirt⁷ and Ackeret *et al.*⁸ were among the first to test turbine blading in a wind tunnel, and the latter pointed out the limitations of using such data in steam turbines. These limitations are discussed later.) The mean stagnation pressures are based on flow averages,

$$\bar{p}_0 = \frac{\int \rho c_x p_0 dA}{\int \rho c_x dA}$$

A stagnation pressure loss, based on such averaged stagnation pressures, is the loss of a uniform stream with the same mass flow and overall losses as those observed in the experiment.

While the early steam-turbine designers recognized the existence of secondary losses and clearance losses at root and tip, the nature of the experimental data available to them (an overall force measurement, which is related to a mean velocity coefficient) meant that they were not able to split down the losses into separate components. The gas-turbine designers, and later steam-turbine designers, have had more information on which to base correlations of the separate components of the losses.

A third method of obtaining design data on losses in turbines is to assemble information on the overall efficiencies of a wide variety of turbines and to calculate back to the individual blade-row component losses. This has been attempted by Soderberg⁹, and it is his correlation, as used by Stenning¹⁰, which is used as a basis for comparison in this paper.

Differences of approach also exist in the statements and definitions of efficiency and in the relations between measured losses and the efficiency. These are due largely to the use of the concept of stagnation enthalpy and pressure by the gas-turbine designer, instead of the use of "carry-over" kinetic energy.

This paper attempts to re-define the efficiencies and relate the differing terms and concepts. Further, an attempt is made to bring together data on losses known to the author. Inevitably there will be omissions in this attempt, for much industrial data remain unpublished, and any further information which would increase the scope and usefulness of this work would be welcome.

3.0 TURBINE EFFICIENCY

Definitions and analyses of turbine efficiency are presented below, using the concepts of stagnation enthalpy, pressure and temperature. All flows are assumed to be adiabatic, and the stagnation enthalpy remains constant across

a stationary row. Across a moving row, the “absolute” stagnation enthalpy drop is equal to the shaft-work output.

3.1. *The enthalpy–entropy diagram for a turbine stage*

The stagnation enthalpy relative to the moving row remains constant across that row. This may be illustrated by reference to Fig. 1, which shows a set of velocity triangles for a turbine in which the streamlines may move radially. (The angles are measured from the axial direction, and the sign convention is such that angles and tangential velocities are taken as positive as drawn.) The work output per unit-flow rate is

$$h_{02} - h_{03} = U_2 c_{\theta_2} + U_3 c_{\theta_3}, \tag{1}$$

where

$$\left. \begin{aligned} h_{02} &= h_2 + c_2^2/2 \\ h_{03} &= h_3 + c_3^2/2 \end{aligned} \right\} \tag{2}$$

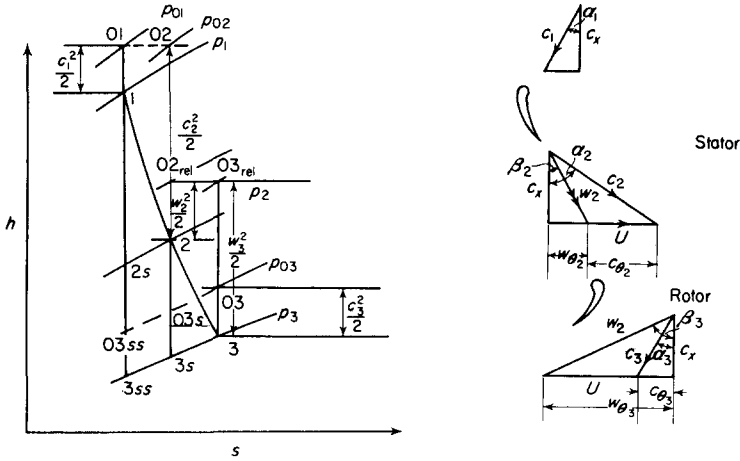


FIG. 1. Notation.

If the relative stagnation enthalpies are defined as

$$\left. \begin{aligned} h_{02rel} &= h_2 + w_2^2/2 \\ h_{03rel} &= h_3 + w_3^2/2 \end{aligned} \right\} \tag{3}$$

then

$$h_{02rel} - h_{03rel} = U_2 c_{\theta_2} + U_3 c_{\theta_3} + \frac{w_2^2}{2} - \frac{w_3^2}{2} - \frac{c_2^2}{2} + \frac{c_3^2}{2} \tag{4}$$

From the velocity triangles

$$w^2 = c_x^2 + w_\theta^2,$$

$$c^2 = c_x^2 + c_\theta^2,$$

$$w^2 - c^2 = w_\theta^2 - c_\theta^2,$$

and

$$\begin{aligned} w_2^2 - c_2^2 &= (c_{\theta_2} - U_2)^2 - c_{\theta_2}^2 \\ &= U_2^2 - 2U_2 c_{\theta_2}, \end{aligned} \tag{5}$$

$$w_3^2 - c_3^2 = U_3^2 + 2U_3 c_{\theta_3}$$

and

$$h_{02_{rel}} - h_{03_{rel}} = U_2 c_{\theta_2} + U_3 c_{\theta_3} + \frac{U_2^2}{2} - U_2 c_{\theta_2} - \frac{U_3^2}{2} - U_3 c_{\theta_3} = U_2^2/2 - U_3^2/2 \quad (6)$$

If the radial shift is small, as in an axial turbine, then $U_2 = U_3$ and $h_{02_{rel}} = h_{03_{rel}}$.

The enthalpy-entropy diagram for a stage may now be drawn and is illustrated, in Fig. 2, for the stage with velocity triangles as shown in Fig. 1. The stagnation enthalpy remains constant across the nozzle or stationary row ($h_{01} = h_{02}$), although the stagnation-pressure may drop. The stagnation-enthalpy relative to the moving row remains constant, although the relative stagnation-pressure (the pressure that would be obtained after complete isentropic diffusion of the relative velocity) will drop.

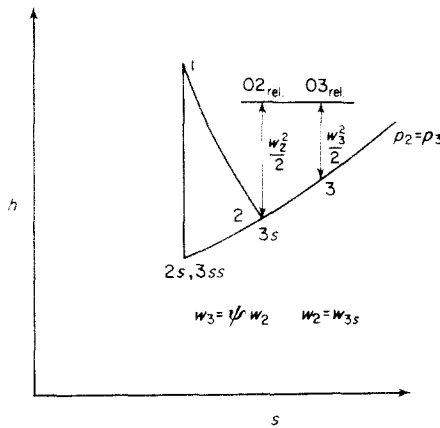


FIG. 2. h - s diagram (impulse).

3.2. Irreversibility and losses

The irreversibilities in the flow through stators and rotors may be expressed in a number of ways:

(1) The velocity coefficients (ϕ, ψ for stator and rotor respectively) which are defined as the ratio of the actual velocity to the velocity that would be obtained in isentropic flow to the same back-pressure

$$\phi = \frac{c_2}{c_{2s}} = \sqrt{\left(\frac{h_{01} - h_2}{h_{01} - h_{2s}}\right)}, \quad \psi = \frac{w_3}{w_{3s}} = \sqrt{\left(\frac{h_{02_{rel}} - h_3}{h_{02_{rel}} - h_{3s}}\right)} \quad (7)$$

(2) The nozzle efficiency (η_n) which may be defined in three ways (Kearton³):

(i) $\eta_{n_1} = \frac{\text{The gain in kinetic energy}}{\text{The gain in kinetic energy in isentropic expansion to the same back pressure}} = \frac{h_1 - h_2}{h_1 - h_{2s}} \quad (8)$

(ii) $\eta_{n_2} = \frac{\text{Final kinetic energy}}{\text{Maximum kinetic energy obtained in an isentropic expansion to the same back pressure}} = \frac{h_{01} - h_2}{h_{01} - h_{2s}} \quad (9)$

$$\begin{aligned}
 \text{(iii) } \eta_{ns} &= \frac{\text{Final kinetic energy—“carry-over” kinetic energy}}{\text{The gain in kinetic energy obtained in an isentropic expansion to the same back pressure}} \\
 &= \frac{(h_{01} - h_2) - \eta_c(h_{01} - h_1)}{(h_1 - h_{2s})} \quad (10)
 \end{aligned}$$

where η_c is the efficiency of conversion of the inlet kinetic energy.

(3) The “enthalpy-loss”—the difference between the static enthalpies h_2 and h_{2s} , and h_3 and h_{3s} —and the “enthalpy-loss” coefficients

$$\begin{aligned}
 \xi_N &= \frac{h_2 - h_{2s}}{\frac{1}{2}c_2^2} = \frac{h_2 - h_{2s}}{h_{01} - h_2} \\
 \xi_R &= \frac{h_3 - h_{3s}}{\frac{1}{2}w_3^2} = \frac{h_3 - h_{3s}}{h_{02\text{rel}} - h_3} \quad (11)
 \end{aligned}$$

(4) The increase in entropy

$$s_2 - s_1 = s_2 - s_{2s}$$

$$s_3 - s_2 = s_3 - s_{3s}$$

(5) The stagnation losses

$$Y_p = \frac{p_{01} - p_{02}}{\frac{1}{2}\rho_2 c_2^2}, \quad \frac{p_{02\text{rel}} - p_{03\text{rel}}}{\frac{1}{2}\rho_3 w_3^2}$$

(6) The blade-drag coefficients

$$C_D = \frac{\text{Drag}}{\frac{1}{2}\rho_2 c_2^2} \quad \text{or} \quad \frac{\text{Drag}}{\frac{1}{2}\rho_3 w_3^2} \quad (12)$$

These various forms of expressing the irreversibility may be related as follows:

$$\eta_{n_2} = \phi^2 \quad (13)$$

$$\xi_N = \frac{c_{2s}^2 - c_2^2}{c_2^2} = \frac{1}{\phi^2} - 1 \quad (14)$$

$$\phi = \frac{1}{\sqrt{(1 + \xi_N)}} \quad (15)$$

$$s_2 - s_{2s} \simeq \frac{h_2 - h_{2s}}{T_2} = \frac{\xi_N c_2^2}{2T_2} = \frac{\xi_N \gamma R M_2^2}{2} \quad (16)$$

$$\frac{p_{01}}{p_{02}} = \frac{p_{01}}{p_{2s}} \frac{p_2}{p_{02}} = \left(\frac{T_{01}}{T_{2s}}\right)^{\gamma/(\gamma-1)} \left(\frac{T_2}{T_{02}}\right)^{\gamma/(\gamma-1)} \doteq \frac{1}{1 - \frac{(h_2 - h_{2s})\gamma}{C_p T_2(\gamma-1)}} \quad (17)$$

$$\frac{\Delta p_0}{p_{01}} = \frac{\gamma(h_2 - h_{2s})}{C_p T_2(\gamma-1)} = \frac{\xi_N M_2^2 \gamma}{2} \quad (18)$$

$$Y = \frac{\Delta p_0}{\frac{1}{2}\rho_2 c_2^2} = \xi_N \left[1 + \frac{\gamma-1}{2} M_2^2\right]^{\gamma/(\gamma-1)} \quad (19)$$

$$\doteq \xi_N \left[1 + \frac{\gamma M_2^2}{2}\right] \quad \text{for } M_2 < 1 \quad (19a)$$

$$\doteq \xi_N, \quad \text{for incompressible flow} \quad (19b)$$

$$C_D = \frac{\Delta p_0}{\frac{1}{2}\rho_2 c_2^2} \frac{S \cos^3 \alpha_m}{L \cos^2 \alpha_2} \doteq \xi_N \frac{S \cos^3 \alpha_m}{L \cos^2 \alpha_2}, \quad \text{for incompressible flow} \quad (20)$$

where

$$\alpha_m = \tan^{-1} \left(\frac{\tan \alpha_1 + \tan \alpha_2}{2} \right)$$

Here the different forms of expressing the losses in the stator row have been related by assuming the fluid to be a perfect gas. Similar relationships in terms of relative properties may be obtained for the rotor. These loss coefficients are for an imaginary, uniform, two-dimensional flow having the same flow rate and overall losses as the actual stream. The differences between η_{m_2} and ϕ^2 in a three-dimensional flow, for example, are ignored.

3.3. Definitions of efficiency

The definition of the stage efficiency depends on the application. In a multi-stage machine the efficiency is best defined as a "total-to-total" efficiency,

$$\eta_{TT} = \frac{\text{Actual work output}}{\text{Maximum adiabatic work output in operation to the same back pressure}} = \frac{h_{01} - h_{03}}{h_{01} - h_{03_{ss}}} \quad (21)$$

[The difference between $h_{03_{ss}}$ (at p_{03}, s_1 as indicated on Fig. 1) and $(h_{3_{ss}} + c_{3_{ss}}^2/2)$ is usually neglected. In isentropic expansion to the pressure p_3 , the density ($\rho_{3_{ss}}$) and therefore the velocity ($c_{3_{ss}}$) leaving the stage would be slightly different from ρ_3, c_3 , and the stagnation-pressure would not be p_{03} .]

This efficiency may be expressed in terms of the work done and the loss coefficients for stator and rotor, ξ_N and ξ_R , (Hawthorne¹¹):

$$\eta_{TT} = \left[1 + \frac{\left(\frac{\xi_N c_2^2}{2} + \frac{T_3}{T_2} \frac{\xi_R w_3^2}{2} \right)}{h_{01} - h_{03}} \right]^{-1} \simeq \left[1 + \frac{(\xi_N c_2^2 + \xi_R w_3^2)}{2(h_{01} - h_{03})} \right]^{-1} \quad (22)$$

Further, the loss coefficients are for the complete flow process through each row. The separation of the losses into a "carry-over" loss and a blade loss is artificial, for the boundary-layer behaviour within the nozzle is strongly dependent upon the entry flow.

If the exhaust kinetic energy is not used (e.g. as in a single-stage turbine without an exhaust diffuser) then the efficiency should be defined as a total-to-static efficiency, the ratio of the actual work to the isentropic work obtained in expansion to the exhaust back-pressure at zero-leaving velocity, for this is the maximum work that could be obtained in adiabatic flow.

Thus

$$\eta_{TS} = \frac{h_{01} - h_{03}}{h_{01} - h_{3_{ss}}} = \left[1 + \frac{\left(\frac{\xi_N c_2^2}{2} + \frac{T_3}{T_2} \xi_R \frac{w_3^2}{2} + \frac{T_3}{T_2} \frac{c_3^2}{2} \right)}{h_{01} - h_{03}} \right]^{-1} \quad (23)$$

These separate concepts of turbine efficiency are not new, for both Stodola² and Kearton³ present efficiency curves on the two bases.

The concept of blading or diagram efficiency is useful only in the case where the exhaust kinetic energy is not used.

The diagram efficiency η_D is defined as

$$\eta_D = \frac{\text{Work output}}{\text{Kinetic energy available to the blades}} \quad (24)$$

If the second definition of nozzle efficiency is used, then the diagram efficiency may be related to the total-to-static efficiency for an impulse-turbine stage (a stage with zero pressure drop across the moving blades—Fig. 2)

$$\begin{aligned} \eta_{TS} &= \frac{h_{01} - h_{03}}{h_{01} - h_{3ss}} \\ &= \frac{h_{01} - h_{03}}{h_{01} - h_2} \times \frac{h_{01} - h_2}{h_{01} - h_{3ss}} \\ &= \eta_D \times \eta_n, \end{aligned} \quad (25)$$

The blading efficiency for the impulse stage can be related to the basic loss coefficients described above

$$\eta_D = \frac{h_{01} - h_{03}}{h_{01} - h_2} = \left[1 + \left\{ \frac{(\xi_R w_3^2)/2 + c_3^2/2}{(h_{01} - h_{03})} \right\} \right]^{-1} \quad (26)$$

4.0 REACTION

The classical definition of reaction is related to the pressure drop across moving and stationary rows. The impulse condition, in which the force on the moving blades is impulsive and results from change in tangential momentum across the blades, is then defined as one of zero reaction—there is no pressure drop across the moving blades.

The author considers that it is more useful to define reaction, which is effectively only a statement on blading geometry, as the ratio of the static enthalpy-drop through the moving blades to the static enthalpy-drop through the stage, i.e.

$$R = \frac{h_2 - h_3}{h_1 - h_3} \quad (27)$$

If the velocity leaving the stage (c_3) is the same as that entering the stage (c_1), then

$$\begin{aligned} R &= \frac{h_2 - h_3}{h_{01} - h_{03}} \\ &= \frac{w_3^2 - w_2^2}{2U(c_{\theta_2} + c_{\theta_3})} \end{aligned} \quad (28)$$

and if the axial velocity remains constant through the stage

$$\begin{aligned} R &= \frac{w_{\theta_3}^2 - w_{\theta_2}^2}{2U(c_{\theta_2} + c_{\theta_3})} \\ &= \frac{w_{\theta_3} - w_{\theta_2}}{2U} \\ &= \frac{c_x}{2U} (\tan \beta_3 - \tan \beta_2) \\ &= \frac{1}{2} + \frac{c_x}{2U} (\tan \beta_3 - \tan \alpha_2) \end{aligned} \quad (29)$$

If $\beta_3 = \beta_2$ the reaction is zero, and if $\beta_3 = \alpha_2$ the reaction is 50 per cent.

The (h, s) diagram for a stage of zero reaction defined on this basis is shown in Fig. 3. There is a pressure drop across the moving row, and the stage is not truly impulse. The (h, s) diagram for an "impulse" stage of zero pressure-drop is shown in Fig. 2. There is an enthalpy increase across the moving row, and the stage is strictly one of negative reaction on the above definition of reaction.

It should be noted that in a reversible stage the definitions of reaction, based on pressure- and enthalpy-drop, are identical, but only by basing the definition on enthalpy-drop is it possible to relate the reaction directly to the blading angles and the flow coefficient c_p/U' .

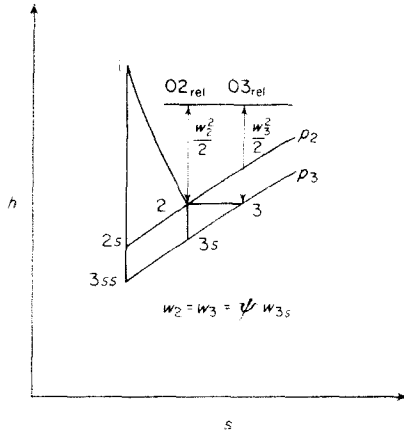


FIG. 3. $h-s$ diagram (zero reaction).

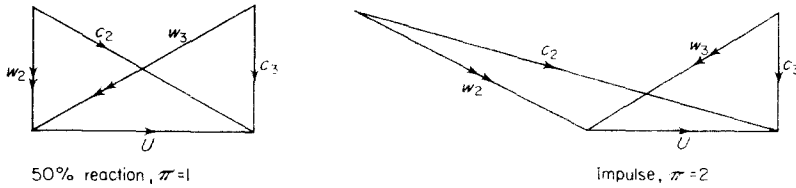


FIG. 4. Velocity triangles for maximum η_{TS} .

5.0 THE BLADE-SPEED RATIO

The blade-speed ratio is widely used by steam-turbine designers. The definition is taken as the ratio of blade speed to the velocity that would be obtained in isentropic expansion through the stage pressure-drop,

$$v_{is} = \frac{U}{c_{is}} = \frac{U}{\sqrt{(h_1 - h_{3ss})}} \tag{30}$$

or sometimes as the ratio of blade speed to nozzle-leaving velocity $v = U/c_2$. These parameters are widely used in choosing the operating condition of a turbine.

The maximum total-to-static efficiency of a reversible turbine is obtained when the leaving velocity is axial. For example, Fig. 4 shows the velocity triangles for zero and 50 per cent reaction designs with axial leaving velocity.

The work done in the two cases is $2U^2$ and U^2 respectively. Assuming there is no change in kinetic energy across the stage, the static enthalpy-drops are $2U^2$ and U^2 and

$$v_{is} = \frac{U}{\sqrt{4U^2}} = \frac{1}{2} \text{ (impulse)}$$

$$v_{is} = \frac{U}{\sqrt{2U^2}} = \frac{1}{\sqrt{2}} \text{ (50 per cent reaction)}$$

These well-known results have been modified to give the velocity ratios for maximum total-to-static efficiency, using nozzle- and blade-velocity coefficients which are assumed to be constant (e.g. Kearton³ pp. 180, 213). Essentially, such analyses give the optimum reaction for a given work loading on the stage, but only if the total-to-static efficiency is the criterion of performance.

A further point may be made concerning the use of the velocity ratio. Not only are the usual plots of efficiency against the velocity ratio (e.g. Keenan¹² p. 156) for total-to-static efficiency, but also they are *design*-efficiency plots. They do not represent the off-design performance of turbines designed for the conditions of maximum efficiency. For example, one of the plots of total-to-static efficiency given later in Fig. 17 is for a zero-reaction stage, and all points on the curve represent possible designs for zero reaction. But a turbine stage designed at $(v) = 0.5$ would be required to operate at reactions other than zero off-design, and the curve does not represent its off-design performance.

In the majority of turbine applications it is the "total-to-total" efficiency which is important, and the variation of efficiency with reaction at a given stage-loading is not too critical (see Section 7).

The gas-turbine designer usually uses a blade-loading criterion

$$\pi = \frac{\Delta W}{U^2} = \frac{h_{01} - h_{03}}{U^2} \doteq \frac{h_1 - h_3}{U^2} = \eta_{TT}/2v_{is}^2 \quad (31)$$

or

$$\begin{aligned} \pi &= 1/(2v^2) \text{ for an impulse stage} \\ &= 1/v^2 \text{ for a 50 per cent reaction stage} \end{aligned}$$

6.0 CORRELATION OF LOSS COEFFICIENTS

For incompressible flow through a row of blades the stagnation pressure loss might be expected to be a function of several parameters,

$$\Delta p_0 = f(\text{blade geometry, } S, L, H, \rho, t, c_2, \mu, \delta, \text{ entry-boundary-layer parameters and turbulence level),}$$

$$\text{or } \frac{\Delta p_0}{\frac{1}{2}\rho c_2^2} = f(Re, \text{ blade geometry, } S/L, t/L, H/L, \delta/L, \text{ entry-boundary-layer parameters and turbulence level).} \quad (32)$$

In compressible flow of a perfect gas the "enthalpy loss" ($h_2 - h_{2s}$) will be a function of the same parameters plus others allowing for changes in density:

$$\Delta h_s = f(\text{blade geometry, } S, L, H, t, \rho_2, c_2, \mu, \delta, \gamma, \bar{R}, a_2, \text{entry-boundary-layer parameters and turbulence level}),$$

$$\text{or } \frac{\Delta h_s}{\frac{1}{2}c_2^2} = f(Re, \text{blade geometry, } S/L, t/L, H/L, \delta/L, M_2, \gamma, \bar{M}, \text{entry-boundary-layer parameters and turbulence level}). \quad (33)$$

In early work the Reynolds number was based on the exit velocity and the hydraulic mean diameter at the throat section,

$$D_h = 2HS \cos \alpha_2 / (S \cos \alpha_2 + H)$$

This implies that the flow is similar to a pipe flow, and that a Reynolds number defined in such a way might be used to correlate losses, eliminating parameters such as H/L from the correlation. In fact, the losses are boundary-layer phenomena and at least two parameters, such as $Re = \rho c_2 L / \mu$ and H/L , must be used. In this paper the Reynolds number based on the hydraulic mean diameter has been retained, but only because most data on the variation of losses with Re is presented in this way.

6.1. Soderberg's correlation

Soderberg⁹ has correlated the losses on a basis of space-chord ratio, Reynolds number, aspect ratio, thickness ratio and blading geometry.

Soderberg uses the work of Zweifel¹³ to obtain the optimum space-chord ratio for a given change of direction through a cascade. Zweifel suggests that the lift coefficient based on the tangential loading

$$C_{L_t} = (2S/b) (\tan \alpha_2 - \tan \alpha_1) \cos^2 \alpha_2 \quad (34)$$

should be approximately 0.85. The optimum space-chord ratio is obtained from this equation, for given gas flow angles α_1 and α_2 .

For turbine rows operating at this design tangential-lift coefficient (at a Reynolds number of 10^5 and with an aspect ratio (H/b) of 3 : 1), the "nominal" loss-coefficient is ξ^* and is related to gas deflexion $\epsilon = \alpha_1 + \alpha_2$ alone by the relation plotted in Fig. 5. For turbine rows operating at zero incidence, the basis of Soderberg's correlation, the gas deflexion is little different from the blading deflexion ($\alpha_1^1 + \alpha_2^1$), since the deviations ($\alpha_2 - \alpha_2^1$) are, in general, small (see sub-section 6.9).

For aspect ratios other than 3 : 1

$$\xi' = (1 + \xi^*) \left(0.975 + 0.075 \frac{b}{H} \right) - 1 \quad (35)$$

and for Reynolds numbers other than 10^5

$$\xi'' = \left(\frac{10^5}{Re} \right)^{\frac{1}{4}} \xi' \quad (36)$$

Soderberg's correlation implies that the effect of profile shape is limited. Fig. 5 shows how the basic $\xi^* - \epsilon^*$ correlation is affected by thickness-chord ratio, but no effect of trailing-edge thickness is included. Further, it is implied that the degree of reaction (or stagger) is unimportant, so long as the optimum space-chord ratio is chosen.

A correction on the efficiency for clearance losses is included by multiplying the final calculated stage-efficiency by the ratio of "blade" area to total area, including leakage space.

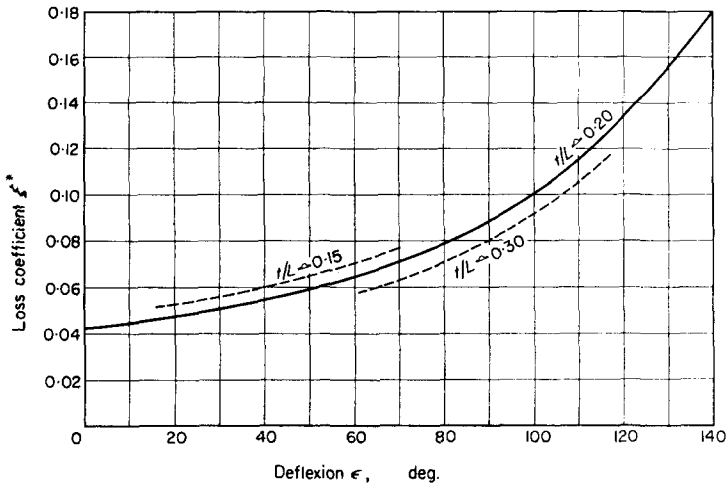


FIG. 5. Soderberg's correlation.

Soderberg's correlation is somewhat oversimplified because the effect of reaction, the Mach number and the non-dimensional parameters of the fluid (specific-heat ratio and molecular weight) have been neglected. Further, the aspect ratio alone cannot be expected to be the only important parameter in the secondary loss-correction, for the entry-boundary layer and the blade geometry must be vital parameters.

However, it is understood that the correlation gives turbine efficiencies to within 3 per cent over a wide range of Reynolds number and aspect ratio, and the method followed here is to compare it with other data.

Hawthorne¹¹ refers briefly to a correlation similar to Soderberg's. Apparently an analytical simplification of Soderberg's work, the correlation may be written

$$\xi_p = 0.025 \left[1 + \left(\frac{\epsilon}{90} \right)^2 \right] \quad (37)$$

$$\xi = \xi_p \left[1 + 3.2 \frac{b}{H} \right] \quad (38)$$

This implies that the secondary loss ($\xi - \xi_p$) is equal to the profile loss (ξ_p) at an aspect ratio of 3.2, for all reactions and deflexions. The analytical expression fits Soderberg's curve well at low deflexion but is inaccurate for high-deflexion impulse-blading.

6.2. *Ainley's correlation*

Ainley and Mathieson¹ first correlate the profile drag. Figs. 6 and 7 show the profile drag for nozzles and impulse-blading at various values of the outlet angle and various space-chord ratios. For blading of the same space-chord ratio but between impulse and reaction the profile loss is given by

$$Y_{p(i=0)} = Y_{p(\alpha_1=0)} + (\alpha_1/\alpha_2)^2 (Y_{p(\alpha_1=\alpha_2)} - Y_{p(\alpha_1=0)}) \left(\frac{t/L}{0.2} \right)^{\alpha_1/\alpha_2} \quad (39)$$

where the correlation for thickness-chord ratios away from $t/L = 0.2$ is also included.

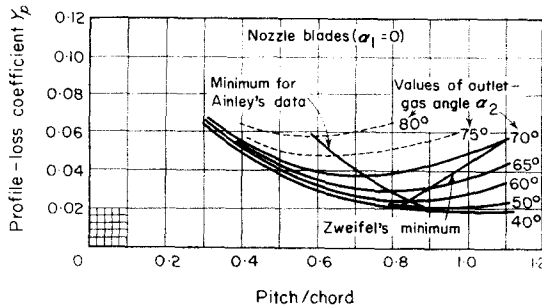


FIG. 6. Profile loss for conventional section-blades at zero incidence ($t/L = 20$ per cent; $Re = 2 \times 10^5$; $M < 0.6$) (Ainley and Mathieson).

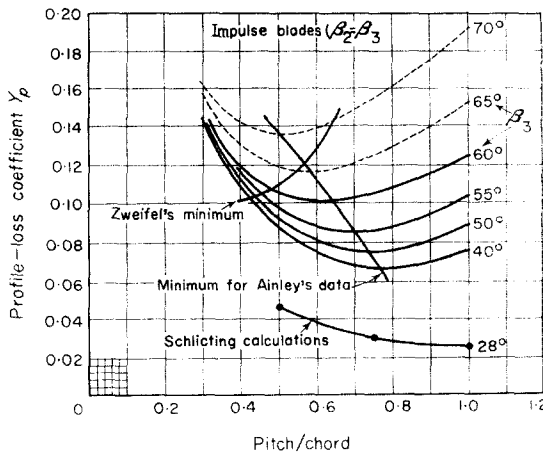


FIG. 7. Profile loss for conventional section-blades at zero incidence ($t/L = 20$ per cent; $Re = 2 \times 10^5$; $M < 0.6$) (Ainley and Mathieson).

Ainley correlates secondary and clearance losses ($Y_s + Y_c$) on the basis of lift coefficient,

$$Y_s + Y_c = \left(\lambda + B \frac{\delta}{H} \right) [C_L / (S/L)]^2 \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m} \quad (40)$$

where (i) λ is a function of

$$\frac{(A_2/A_1)^2}{1 + (\text{inner dia./outer dia.})}$$

(Fig. 8), A_1 and A_2 being the areas normal to the entering and leaving flow, and (ii) B is 0.5 for a radial clearance and 0.25 for a shroud clearance.

Ainley's data are for a mean Reynolds number (based on blade chord) of 2×10^5 . He suggests that the stage losses $(1 - \eta_{TT})$ vary as $Re^{-1/5}$ down to Reynolds numbers of 5×10^4 .

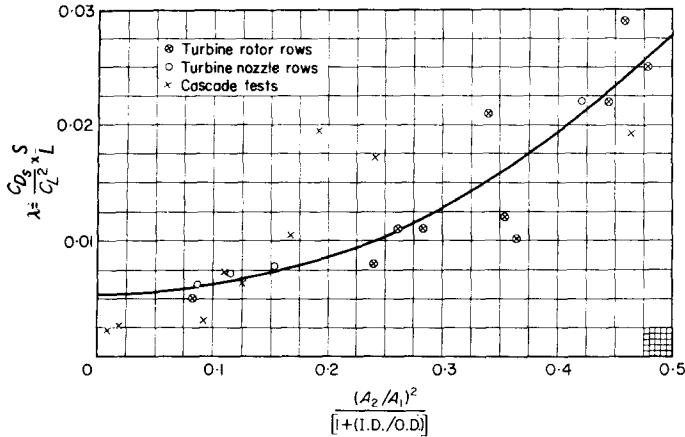


FIG. 8. Secondary losses in turbine blade-rows (Ainley and Mathieson).

6.3. Comparison of the data of Ainley and Soderberg

A first comparison is made between the data of Ainley and Soderberg's correlation. This is shown in Fig. 9, in which Soderberg's loss coefficients for $Re = 10^5$ and an aspect ratio of 3 : 1 are shown compared with the minimum profile losses picked off Figs. 6 and 7.

On Figs. 6 and 7 are shown the calculated space-chord ratios for minimum loss as estimated from Zweifel's¹³ formula for the tangential-lift coefficient. The profile losses given by Ainley for these space-chord ratios are also plotted in Fig. 9.

Also shown is the Soderberg loss-correlation corrected for infinite aspect ratio, and to a Reynolds number of 2×10^5 . It should be noted that Ainley's data are for a mean Reynolds number of 2×10^5 , based on the blade chord. At large aspect ratios the hydraulic mean diameter becomes

$$2S \cos \alpha_2 = [2(S/L) \cos \alpha_2] L$$

The factor $2(S/L) \cos \alpha_2$ varies from approximately 0.2 to 1.4 for the minima of Ainley's data.

It is clear that the Zweifel¹³ relation for the optimum space-chord ratio is at variance with Ainley's cascade data, although the trends of increased loss with deflexion are similar. The minimum losses for impulse and nozzle blades of the same deflexion (80°) are of the same order, but it would be unwise to conclude from this one result that, for the chosen optimum spacing, the reaction has no effect on the losses.

It should be pointed out again that the Soderberg loss coefficient is essentially a “compressible” irreversibility estimate being based on high-speed-turbine data while Ainley’s cascade data are for Mach number less than 0.6. Other evidence (see below) suggests that for the aerofoil-type cascades there is little variation in loss coefficient with Mach numbers in the range 0.6–1.0.

A comparison of the secondary and other losses cannot be made generally, because of the difference in the method of correlation, but two examples given by Ainley are calculated on the Soderberg basis and the results compared. The Reynolds number based on the hydraulic mean diameter is 1.8×10^5 in each example, if the Reynolds number based on the blade chord is 2×10^5 . A comparison is made on the basis of both Reynolds numbers being 2×10^5 , and for zero clearance.

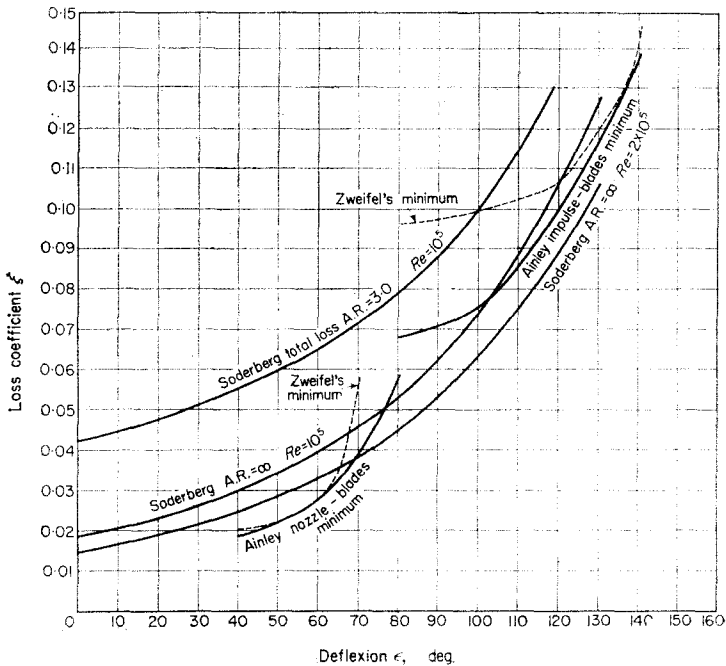


FIG. 9. Comparison between the correlations of Ainley and Soderberg.

Example 1. Nozzle row

$$\alpha_1 = 0, \quad \alpha_2 = 63.5, \quad S/L = 0.739, \quad t/L = 0.20,$$

$$\text{aspect ratio} = H/(L \cos 32^\circ) = 1.5.$$

Ainley gives

$$Y_p = \xi_p = 0.0288,$$

$$Y_s = \xi_s = 0.0295,$$

$$Y = 0.0583.$$

Soderberg gives ξ^* as 0.064 for a Reynolds number of 10^5 and an aspect ratio of 3 : 1. For an infinite aspect ratio, ξ_p^* at this Reynolds number would be 0.039 and, at a Reynolds number of 2.0×10^5 , $\xi_p = 0.033$. The aspect-ratio

correction for an aspect ratio of 1.5 at $Re = 2 \times 10^5$ gives the overall loss coefficient (ξ) as 0.0765, of which $0.0765 - 0.033 = 0.0435$ may be assumed to be secondary loss (ξ_s), if the aspect-ratio correction is assumed to be valid up to large aspect ratios.

The total loss is greater on the Soderberg correlation, although the division between secondary and profile loss is approximately the same.

Example 2. Rotor row

$$\beta_2 = 36^\circ, \quad \beta_3 = 48.6,$$

$$S/L = 0.749, \quad \beta_2/\beta_3 = 0.074, \quad t/L = 0.15.$$

Ainley gives

$$Y_p = \xi_p = 0.041,$$

$$Y_s = \xi_s = 0.091.$$

Soderberg gives

	<i>Re</i>	A.R.
$\xi^* = 0.074$	10^5	3 : 1
$\xi_p^* = 0.049$	10^5	∞
$\xi_p = 0.041$	2×10^5	∞
$\xi = 0.084$	2×10^5	1.5
$\xi_s = 0.043$	2×10^5	1.5

The profile losses are comparable, but the Soderberg correlation gives a very small secondary loss for this impulse-type blade.

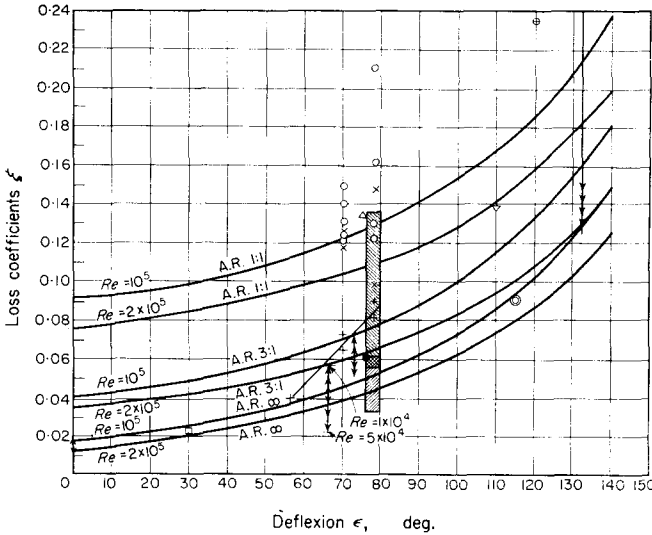


FIG. 10. Steam-turbine data.

6.4. *Steam-turbine data*

There are numerous experimental results obtained from tests of steam-turbine nozzles and blade rows. In general, the data are difficult to compare with that of Soderberg and Ainley, for the testing was not done with reference to Reynolds number or aspect ratio as important parameters.

TABLE for Fig. 10

Experimenter	Aspect ratio	S/L	Symbol	Re (based on hydraulic diameter)	Remarks
Kraft Steam Nozzles Research Committee	0.33-1.0 of order	0.55-0.80 of order	⊙ (First series of tests) × (Second series of tests increased radius of curvature)	$1.4-4.5 \times 10^5$ of order $2-2.6 \times 10^5$	Loss coefficients appear to be generally low Consistent with Ainley and Soderberg
	?	0.46	●	10^5	Range of Reynolds number and blade surfaces. Consistent with Ainley and Soderberg
Nozzles Ackeret <i>et al.</i> Guy Faltin Scholz Stodola	of order unity	of order 0.4-0.7	∇	of order $2-2.6 \times 10^5$	Losses appear to be generally low and variation with deflexion to be too large
	0.8	0.5	⊙	$8 \cdot 10^4$ (chord)	
	∞	1.0	∇	$2-3 \times 10^5$	Consistent with Ainley and Soderberg
	of order 1.0	?	∇	Losses appear to be low	
Impulse blades Kearton Brown Boveri	1.5	0.5	∇	$1.6-2.4 \times 10^5$	Consistent with Ainley and Soderberg
	?	?	∇	?	Consistent with Ainley and Soderberg Loss coefficients vary greatly due to variation of incidence. Minimum loss-coefficient consistent with Ainley and Soderberg at small positive reaction. Large variation with Mach number
Nozzles and blades, model turbines Faltin Brilling Rateau Christiein	0.28	0.6	∇	?	Results not shown. Losses very high for reasons given in text
	?	?	∇	$1 \times 10^5-5 \times 10^4$	Losses appear to be generally low
Straight nozzles Steam Nozzles Research Committee	—	1.0	∇	10^6 (chord)	Boundary layer assumed fully turbulent
	—	—	∇	?	Circular and square straight nozzles

The presentation adopted here is to convert the velocity coefficients to loss coefficients and to plot the results on a graph of ξ^* against ϵ^* (Fig. 10) together with a grid based on Soderberg's correlation. Where the Reynolds number and aspect ratio are known they are stated in the associated table. In comparing results the following points should be borne in mind:

(i) Most of the steam results are based on force measurements; the velocity coefficient is essentially a measure of the total irreversibility, and the loss coefficients calculated are total-loss coefficients.

(ii) The steam results are usually presented in the form of velocity coefficient against the isentropic exit-velocity (e.g. Fig. 11, which is a typical result from Kraft's¹⁵ paper). Most of the results for "convergent" blade rows designed for subsonic and transonic operation show that the velocity coefficient is constant over a wide range,† from exit Mach numbers of the order of 0.6 up to well beyond the sonic speed. They indicate that it is possible, for subsonic and transonic operation of convergent rows, to ignore the effect of Mach number in equation (33). The velocity coefficients presented in Fig. 10 are mean values taken over the range $0.6 < M_2 < 1.0$. There is some disagreement on the variation in velocity coefficient at lower Mach number (Kearton³ p. 160).

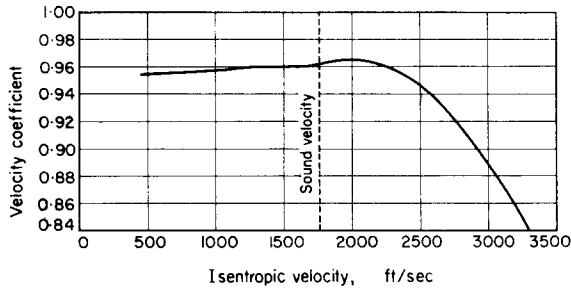


FIG. 11. Nozzle velocity coefficient (Kraft).

Some of the very early data (e.g. that of Christlein¹⁶ and Faltin¹⁷) show considerable variations with Mach number, presumably because of the bluff profile shapes. In these cases the maximum-velocity coefficient is used.

(iii) Equation (33) shows that the compressible-loss coefficient is dependent upon the fluid molecular weight and the specific-heat ratio. This dependence is usually ignored, for results with air at low velocities ($\gamma = 1.4$, $\bar{M} \doteq 28.8$) are used in steam turbines ($\gamma \doteq 1.3$, $\bar{M} = 18$) and in gas turbines ($\gamma \doteq 1.33$, $\bar{M} \doteq 29$ at high temperatures).

(iv) Most of the steam-turbine cascade or "lattice" data were obtained with three or four blades in the cascade. It is unlikely that uniform conditions along the span were established in such tests. Further, some of the blade shapes have large thickness-chord ratios and trailing-edge thicknesses when judged by modern standards.

† Equation (15) shows that if the velocity coefficient is constant, then the "compressible" loss coefficient ξ is constant. But equation (19) shows that the coefficient Y , based on stagnation pressure loss, will not be constant but will vary with the Mach number.

The main sources of data are (i) the work of the Steam Nozzles Research Committee¹⁸ and that of Metropolitan Vickers¹⁹, both reported extensively by Kearton, (ii) the Swiss work reported by Stodola², (iii) the work of Dollin²⁰ at Parsons, and (iv) the work of Kraft¹⁵ at the General Electric Company of America.

The latter is the most comprehensive but also the most difficult to interpret and to correlate. For example, tests by Kraft which are stated to show the variation of loss with aspect ratio, in fact show the variation with aspect ratio and Reynolds number combined, for as the aspect ratio was changed, so too was the Reynolds number.

Further results of Ackeret *et al.*⁸, Kearton³, Scholz²¹ and Faltin¹⁷ are shown.

Data obtained by Rateau²², Christlein¹⁶ and Briling²³ on impulse-type blades are open to some criticism and are not plotted. Rateau's blades had very large trailing-edge thickness and Briling and Christlein worked with a large difference in height between nozzles and blades.

In general, it appears that for nozzle-type rows the data of Soderberg, Ainley and the Steam Nozzle Research Committee are reasonably consistent. For the impulse type blades the data of Soderberg, Ainley, Brown Boveri¹³ and Kearton are similarly consistent, although there is less experimental data available.

Some of Kraft's experiments at low aspect ratios are comparable with the nozzle data listed above, but most of his results, together with those of Guy¹⁹ (correlated by Kearton and reproduced in Fig. 10) and those of Dollin²⁰ at low Reynolds number (1×10^4 – 5×10^4) give considerably lower losses than the main body of the data.

The minimum of the impulse data of Faltin¹⁷ appears consistent with results of other workers, but the losses obtained by Christlein, Briling and Rateau are high, probably for the reasons given above.

6.5. *Effect of Reynolds number*

Workers who have studied the effect of Reynolds number on losses include Ackeret *et al.*⁸, Dollin²⁰, Kearton³ and Armstrong²⁴. Cheshire²⁵ also gives data on the effect of Reynolds number. Some losses are shown in Fig. 12, together with the Soderberg's Reynolds-number correlation, which is a correction for the *total* loss. Both Armstrong's and Ackeret's cascade results are for profile loss alone; the results of Dollin and Kearton are for the total loss.

It is clear that the simple relation

$$\xi = \xi \left(\frac{10^5}{Re} \right)^{\frac{1}{2}}$$

is quite a good approximation at Reynolds numbers less than 10^5 , although Dollin's work shows a greater variation than this. However, it gives no indication of a "critical" Reynolds number above which the loss is substantially constant. The existence of such a critical Reynolds number (based on D_h) around 10^5 is implied in Kearton's data, and in Ackeret's work with smooth blades. Ackeret further showed a lower "transition" Reynolds number for

rougher blades. Only Cheshire's cascade tests show substantial variation above $Re = 10^5$.

Armstrong tested a cascade-section similar to a steam-turbine impulse blade of large deflexion. He found little variation in loss between $2.5 \times 10^5 < Re(\text{chord}) < 4 \times 10^5$, but a very large increase in Y_p (up to 0.5) at Reynolds numbers below 2×10^5 . The peculiar performance of this cascade may be related to the blade profile which was made up of circular arcs and straight lines, with a "transition" between arc and straight line half-way along the chord on the suction surface.

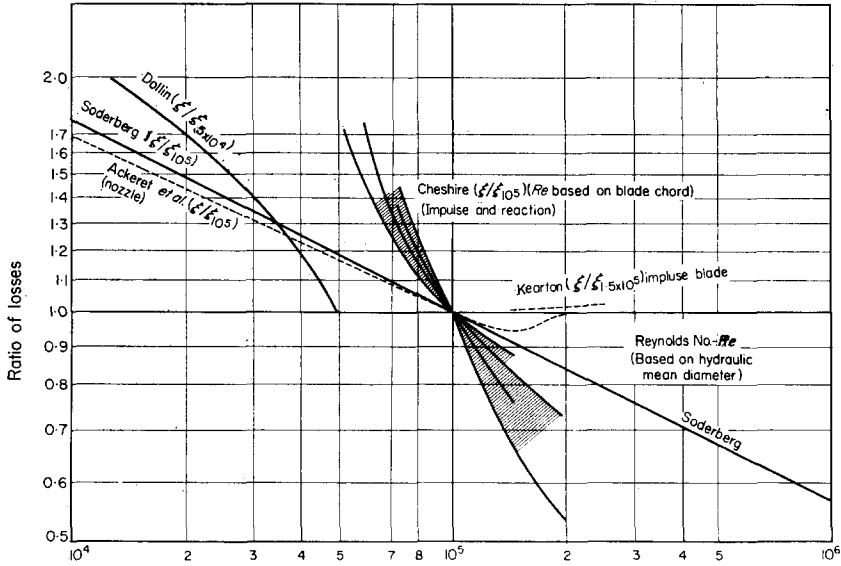


FIG. 12. Effect of Reynolds number on losses.

6.6. Calculation of the profile losses

Schlichting²⁶ has calculated the pressure distributions around turbine blades in cascade, and has estimated the growth of the boundary-layer momentum thickness over the blade surfaces. Results of such calculations for low-camber impulse blades ($\epsilon = 28^\circ$) are shown in Fig. 7 for varying space-chord ratio. The results appear consistent with Ainley's cascade data. The minimum, calculated, profile-loss coefficient is also shown on Fig. 10, and is of the order of that given by Soderberg for low-camber blades.

6.7. Secondary losses and the effect of aspect ratio

The secondary flow in turbine rows is an extremely complex phenomenon. Detailed experiments on the nature of the secondary flows in a nozzle have recently been performed by Senoo²⁷, and in an impulse-type blade by Armstrong²⁸.

Senoo suggests that the rapid acceleration in a nozzle row may laminarize the boundary layer and he is able to calculate the secondary velocities in the nozzle.

Armstrong has found very large secondary motions in the main stream associated with the growth of secondary vorticity, which may be predicted quite accurately. The vorticity in the wake region is made up of two components, a trailing-filament vorticity transmitted through the row from upstream, and a shed vorticity associated with change of lift along the blade. Large contra-rotating vortices are observed downstream of the cascade.

While the secondary motions can be predicted using inviscid analyses (Hawthorne and Armstrong²⁹) no attempt has been made to estimate the loss coefficients. Hawthorne³⁰ has shown that the drag coefficient associated with the induced velocities (i.e. the loss associated with viscous dissipation of these velocities) may be expressed as analytical functions of the following parameters for small deflexions

$$C_{Dx} = \frac{\epsilon^2 (S/L)^2}{H/L} f\left(\frac{\delta^1}{S \cos \alpha_2}\right)$$

where δ^1 is the inlet boundary-layer thickness. In practice the secondary losses are far greater than the losses associated with the dissipation of the induced velocities, but the expression gives a guide to an empirical correlation of losses.

Soderberg's correlation may be written, for a given Reynolds number,

$$1 + \xi = (1 + \xi^*) \left\{ 1 - 0.025 \left(1 - \frac{3b}{H} \right) \right\}$$

At infinite aspect ratio $\xi_\nu \doteq \xi^* - 0.025$

so that $\xi_s = \xi - \xi_\nu \doteq \frac{0.075}{H/b}$

In Ainley's correlation of secondary loss, the factor λ increases with reduced aspect-ratio, and the loss is proportional to the square of the lift coefficient

$$\left\{ \text{i.e. to } (\tan \alpha_1 + \tan \alpha_2)^2, Y_s = \xi_s = \frac{\lambda C_L^2 \cos^2 \alpha_2}{(S/L)^2 \cos^3 \alpha_m} \right\}$$

(The lack of dependence on deflexion in Soderberg's correlation is a weakness, and is illustrated in the examples calculated above.)

The predominant effect of aspect ratio is clear in Kraft's tests, which are replotted in Fig. 13. Most of the data is for nozzles with outlet angles in the range of $\alpha_2 = 76-80^\circ$, so no clue is given to the variation of secondary loss with deflexion. But the variation with aspect ratio is substantially as Soderberg predicts, although the total losses are much lower. The variations with Reynolds number also appear to be along the lines of Soderberg's predictions.†

† Mr. D. G. Ainley has informed the author that he and Mr. G. C. R. Mathieson analysed Kraft's results in an earlier paper, *An Examination of the Flow and Pressure Losses in Blade Rows in Axial Flow Turbines* A.R.C. R & M 2891 (1955). Ainley concludes that the increase in loss coefficient at low aspect ratio observed by Kraft (and Scholz) is not an aspect-ratio effect as such, but an effect due to changing the ratio δ_1/H .

Scholz²¹, on the basis of a series of tests with varying aspect ratio, has suggested that the overall loss-coefficient may be written

$$\xi = \xi_p + \frac{\xi_{\text{wall}}}{H/L}$$

This is basically the same as Soderberg's expression. The latter implies that ξ_{wall} is constant, and indeed, Scholz's experiments show that ξ_{wall} is constant over a large range of deflexions, for aspect ratios of 1, 3 and 5.

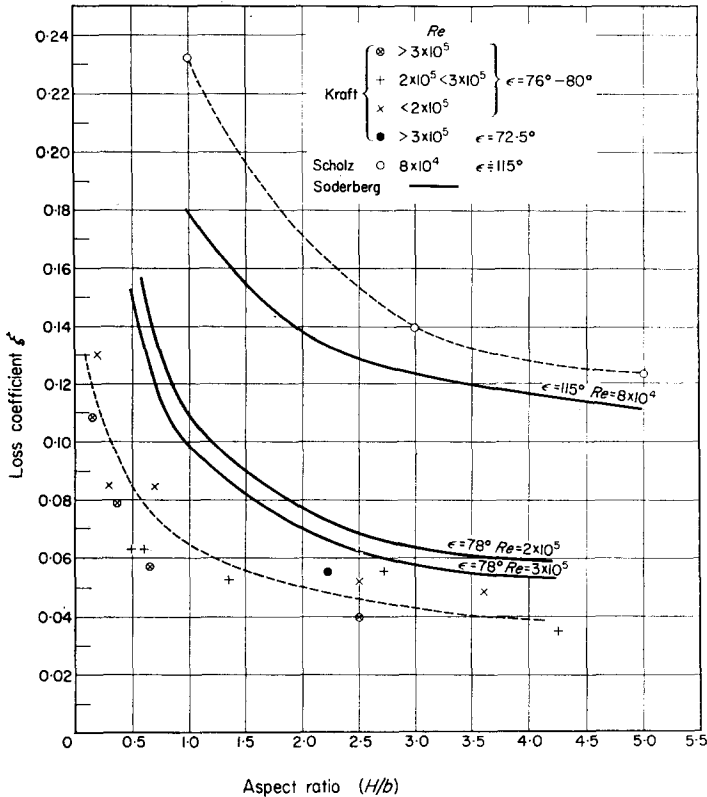


FIG. 13. Effect of aspect ratio on losses.

Scholz's results have been added to Figs. 10 and 13. Soderberg's prediction gives a greater loss than that observed by Scholz, particularly at the lowest aspect ratio, but a criticism of Scholz's work is that the loss coefficient is based upon area integration rather than mass-flow integration.

6.8. *Effect of incidence variation*

Variation of loss away from the zero-incidence condition is given by Ainley as a function of $(i - i_s)$ where i_s is the stalling incidence (this may be obtained from Ainley's paper). Soderberg's prediction is simpler (Fig. 14), the rate of variation with incidence being controlled by the thickness-chord ratio. Two of Ainley's cascade results are shown on the same graph for comparison.

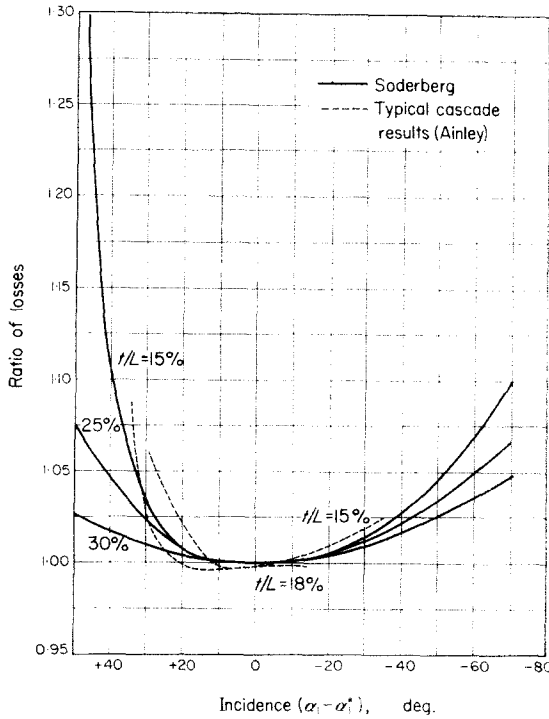


FIG. 14. Effect of incidence on losses.

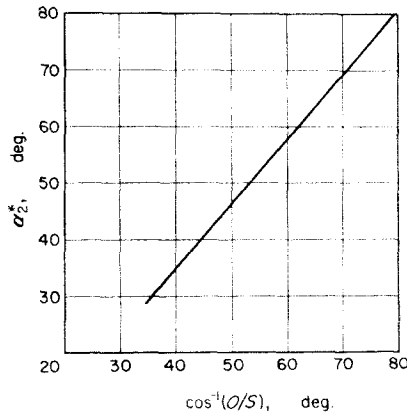


FIG. 15. Relationship between gas-outlet angles and $\cos^{-1}(O/S)$ for "straight-backed" blades operating at low Mach numbers ($M_2 < 0.5$, $Re = 2 \times 10^5$) (Ainley and Mathieson).

6.9. Effect of trailing-edge thickness

Ainley observed a substantial variation of loss with the trailing-edge thickness-space ratio as indicated on Fig. 15. Kraft made two tests with different thickness-space ratios of 0.95 per cent and 3.2 per cent, all other parameters remaining the same. The loss increased by 28 per cent in the latter case. Ainley's correlation suggests that the increase should be 16 per cent for these cascades.

6.10. Angle variation

For the subsonic operation ($0 < M_2 < 0.5$) of cascades such as those considered here, Ainley and Mathieson provide Fig. 16, showing the variation in outlet angle with $\cos^{-1}(o/S)$. They suggest that $\cos \alpha_2 = o/S$ at $M_2 = 1$, and that α_2 varies linearly between $M_2 = 0.5$ and $M_2 = 1.0$.

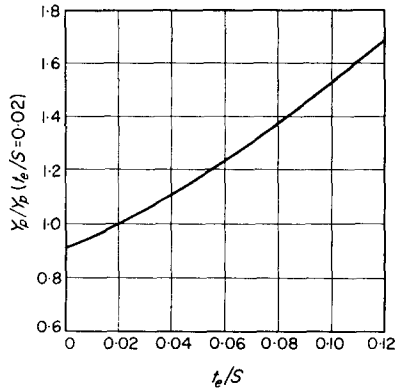


FIG. 16. Effect of trailing-edge thickness on blade-loss coefficients (Ainley and Mathieson).

6.11. Clearance loss

Soderberg's correlation, as used by Stenning¹⁰, suggests that a correction for clearance loss is obtained simply by multiplying the calculated efficiency for zero clearance by the ratio of ("blade" area)/("blade" area plus clearance space). Ainley's clearance loss is

$$Y_c = \frac{B\delta}{H} C_L \left(\frac{S}{L}\right)^2 \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m}$$

where B is 0.5 for a radial clearance and 0.25 for a shroud clearance.

7.0 DISCUSSION

The surprising conclusion from the assembly of loss data for turbine rows is that correlations such as that devised by Soderberg, on the basis of relatively few parameters, can be reasonably accurate. Some of the steam-cascade data is consistent with Ainley's and Soderberg's work, but the data of Guy¹⁹, Kraft¹⁵ and Dollin²⁰ appear to be optimistic. The following general conclusions emerge:

(i) Zweifel's¹³ criterion for optimum space-chord ratio is not consistent with Ainley's work, although large increases in loss are not involved with use of Zweifel's criterion.

(ii) The minimum profile-losses for impulse and reaction blades are strongly dependent upon overall gas deflexion. Soderberg and Ainley agree on this statement.

(iii) The weak effect of Mach number was established by early investigators, for subsonic-type nozzles and blades.

(iv) A Reynolds number dependence $\xi/\xi_{10^5} \propto Re^{-n}$, where $0.2 < n < 0.35$, appears to be substantiated by several results. It appears that there is less variation of ξ with Re for $Re > 10^5$.

(v) The secondary and clearance losses are difficult to correlate, and there is a considerable diversity in predictions based on Ainley's work and Soderberg's data. The predominant effect of aspect ratio is clearly indicated in Kraft's data and Scholz's data.†

Finally, it is of interest to use Soderberg's correlation to calculate the total-to-total and total-to-static efficiencies for turbine stages of given stage-loading ($\pi = \Delta W/U^2$) and varying reaction (Fig. 17). It is apparent that the simple analyses referred to in Section 5.0 give a valuable guide to the best reaction for a given loading when the total-to-static efficiency is important (when the exhaust kinetic energy is not used) but that the choice of reaction is not critical in most multi-stage-turbine applications, when total-to-total efficiency is the criterion which should be used.

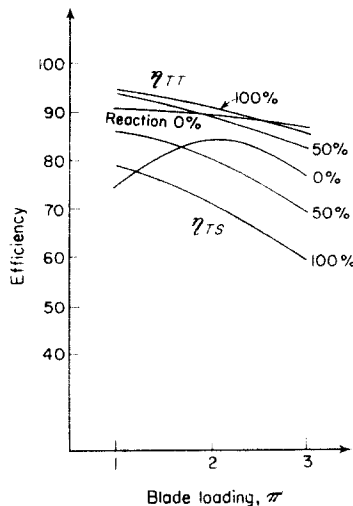


FIG. 17. Efficiency as a function of blade-loading and reaction.

8.0 RECOMMENDATIONS FOR RESEARCH

Data on profile losses and Reynolds-number effects below $Re = 10^5$ appear to be readily available, although there is some doubt about the variation of loss with Re in the $Re = 1-2 \times 10^5$ range. Further experiments to establish the Reynolds-number effect in this range would be useful. Cheshire's²⁵ and Armstrong's²⁴ results serve as a warning that losses may increase rapidly at Reynolds numbers below a critical value, for some blade profiles.

It is clear that the secondary losses require further investigation. The effect of profile shape is considered to be of little importance in the profile loss at high Reynolds numbers, owing to the high acceleration (although it must

† Stenning's experiments with axial-flow turbines of low aspect ratio indicate that Soderberg's prediction of secondary loss is pessimistic for aspect ratios less than unity.

be more significant in the low-reaction blading). However, it is probable that profile shape and even local twisting may produce drastic changes in the secondary loss. (Martin³¹, working with the author, has shown that the secondary losses in a low-stagger, low-deflexion compressor cascade may be substantially reduced by reducing the deflexion through the boundary-layer region.) The relation between the clearance losses and the secondary losses requires further investigation.

The most useful work that could be done would be to correlate interstage traverses on turbines with the data given here.

The old controversy, on the effect of the centrifuging of the surface boundary layer (an outward movement on most rotor blades and an inward movement on the stator blades) and the effect upon profile and secondary losses, remains to be settled.

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