

# Introduction to Meshing

Tiziano Ghisu

# Outline

- Perché abbiamo bisogno di una griglia?
- Geometria
- Tipi di elementi
- Tipi di griglie
- Linee guida

# Perché abbiamo bisogno di una griglia?

- La griglia:
  - Identifica le celle o gli elementi in cui risolvere le equazioni di conservazione
  - E' una rappresentazione discreta della geometria e del problema
  - Ci consente di applicare le condizioni al contorno
- La griglia ha un impatto considerevole su:
  - Accuratezza della soluzione
  - Velocità di convergenza (o anche mancanza di convergenza)
  - Tempo di calcolo
- È necessaria una buona mesh per arrivare a una buona soluzione
  - Densità di elementi
  - Rapporto lunghezza/volume di celle adiacenti
  - Skewness.
  - Tipo di celle (Tet vs. hex)
  - Boundary layer mesh.
  - Mesh refinement through adaption.

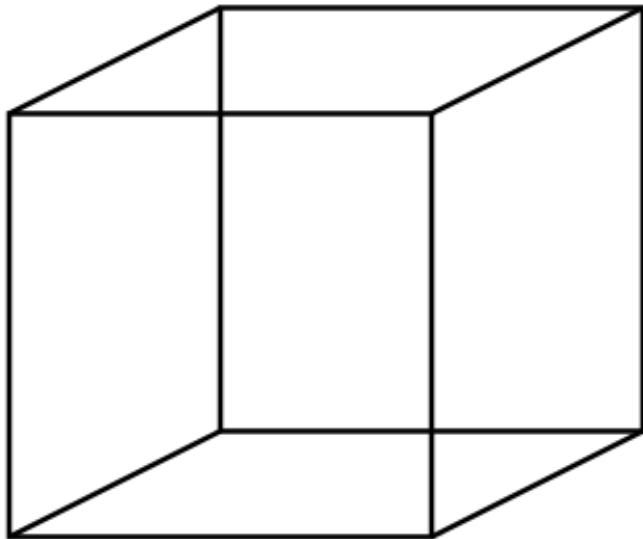
# Geometria

- La geometria e' il punto di partenza di qualunque problema
- Può consistere in volumi, faces (superfici), edges (curve) e vertices (punti).

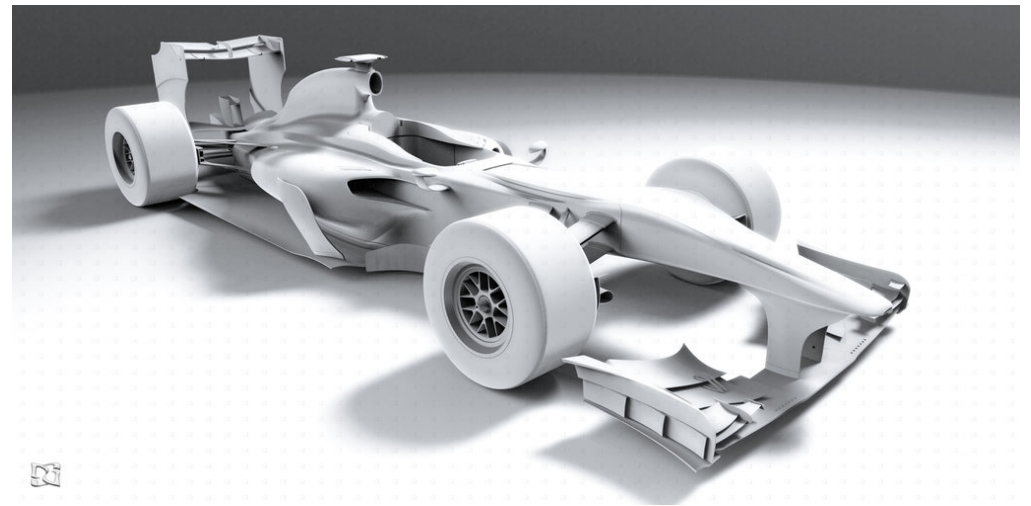
# Geometria

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- Può consistere in volumi, superfici, curve e punti.

può essere semplice

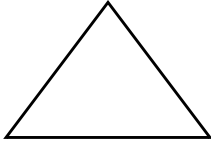

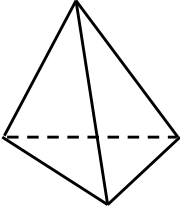
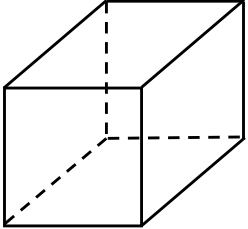
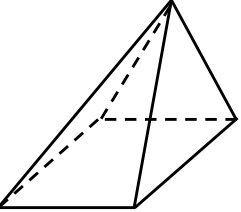
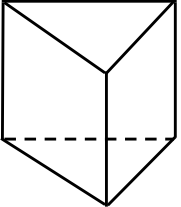
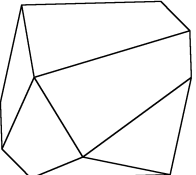


... o complessa



# Tipi di celle

Esistono molti diversi tipi di celle/elementi. La scelta dipende dal problema e dalle caratteristiche del solutore

– 2D:		triangle (“ <b>tri</b> ”)		2D prism ( <b>quadrilateral</b> or “ <b>quad</b> ”)
– 3D:		tetrahedron (“ <b>tet</b> ”)		prism with quadrilateral base ( <b>hexahedron</b> or “ <b>hex</b> ”)
		pyramid		prism with triangular base ( <b>wedge</b> )
		arbitrary polyhedron		

# Un po' di terminologia

Cella (cell) = Volume di controllo in cui suddividiamo il dominio

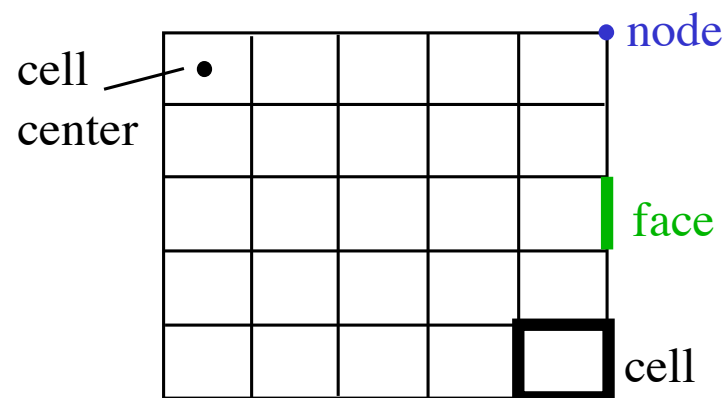
Nodo (node) = punto della griglia

Centro cella (cell centre) = Centro della cella

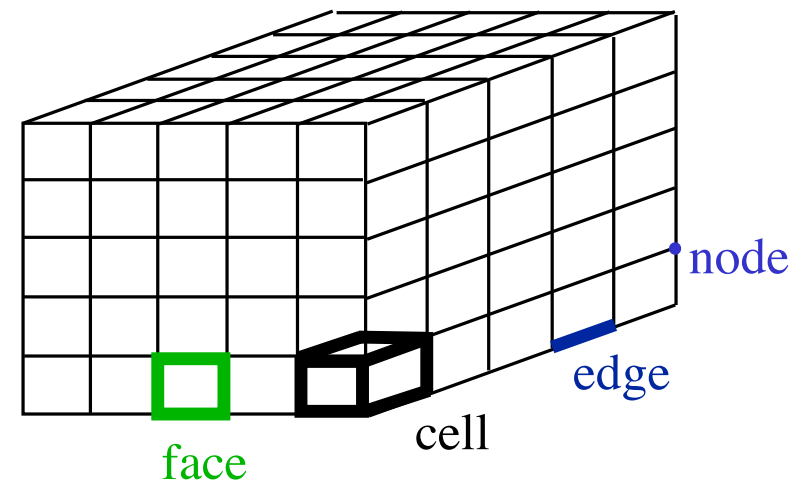
Spigolo (edge) = contorno (boundary) di una faccia

Faccia (face) = contorno (boundary) di una cella

Zona (zone) = gruppo di nodi, facce e celle



2D computational grid

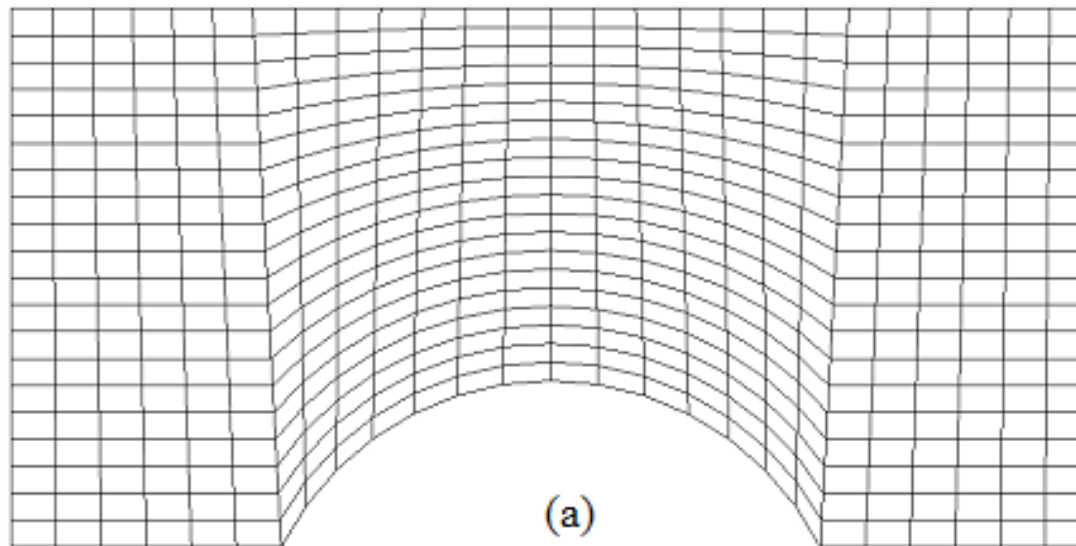


3D computational grid

# Tipi di griglia

## Griglie strutturate (blocco singolo)

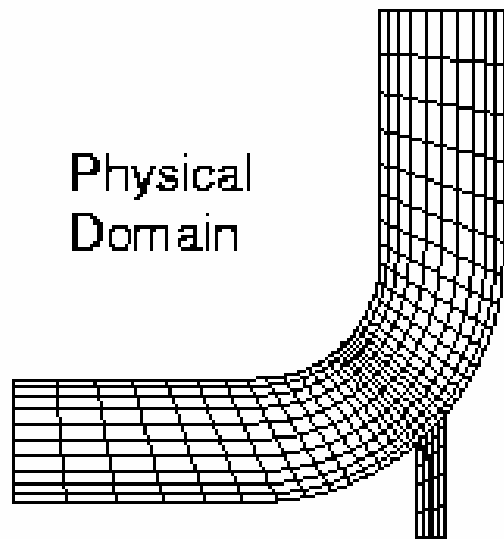
- Ogni cella e' identificata da una terna  $(i,j,k)$
- Le linee griglia devono attraversare tutto il dominio
- Chiaramente le griglie strutturate possono essere utilizzate solo per geometrie semplici



# Tipi di griglia

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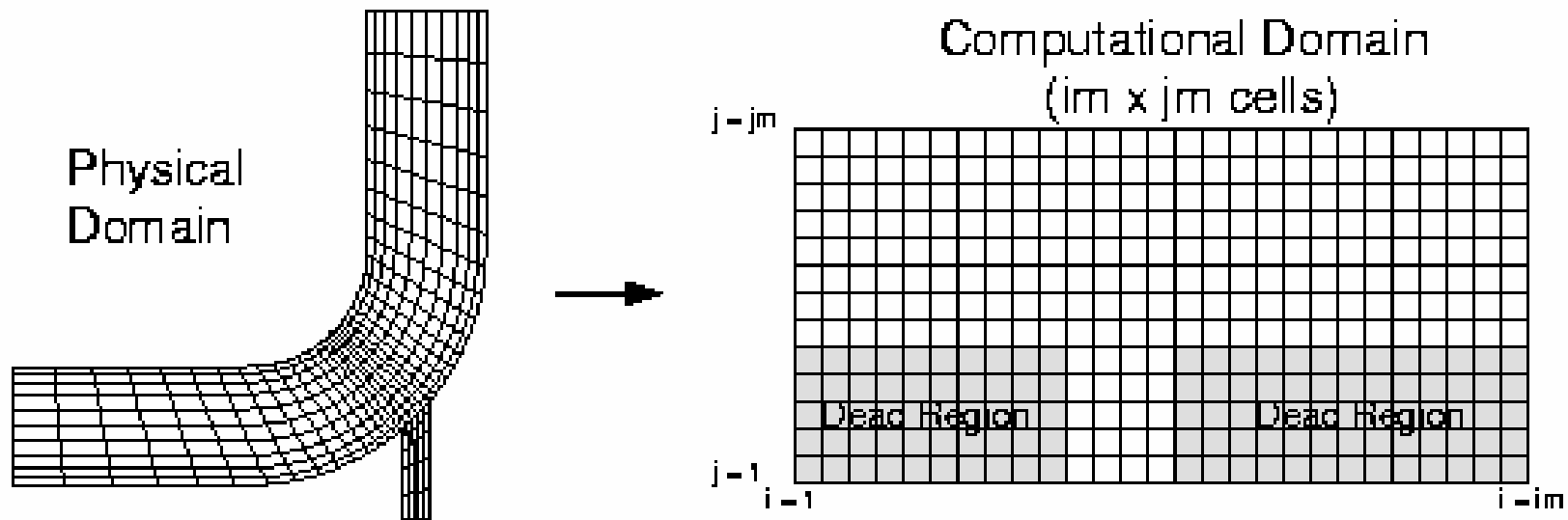
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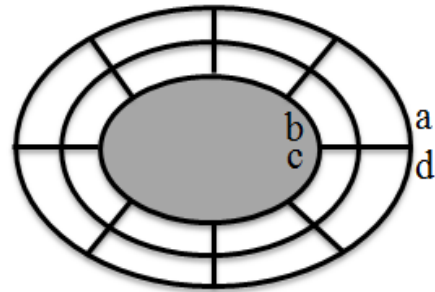
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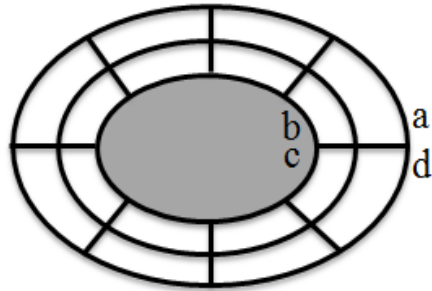
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# Tipi di griglia

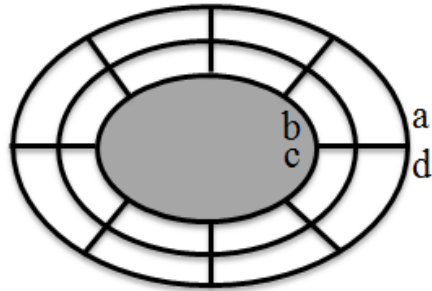


# Tipi di griglia

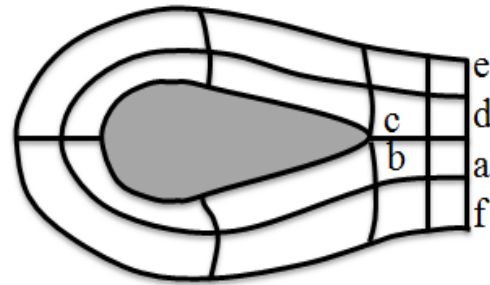


O grid

# Tipi di griglia

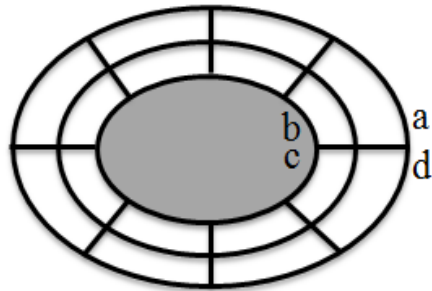


O grid

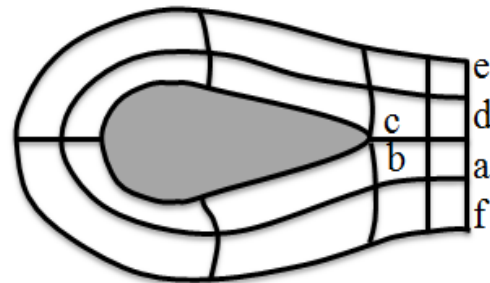


C grid

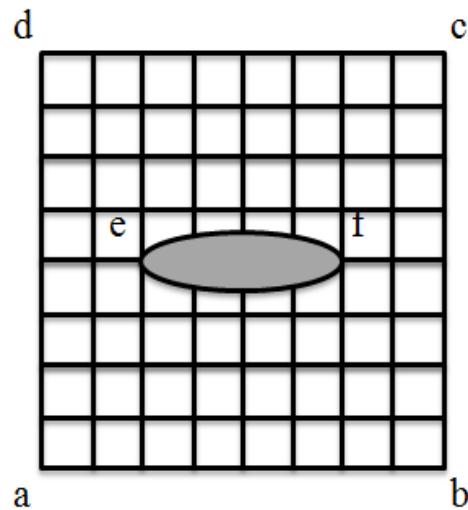
# Tipi di griglia



O grid



C grid

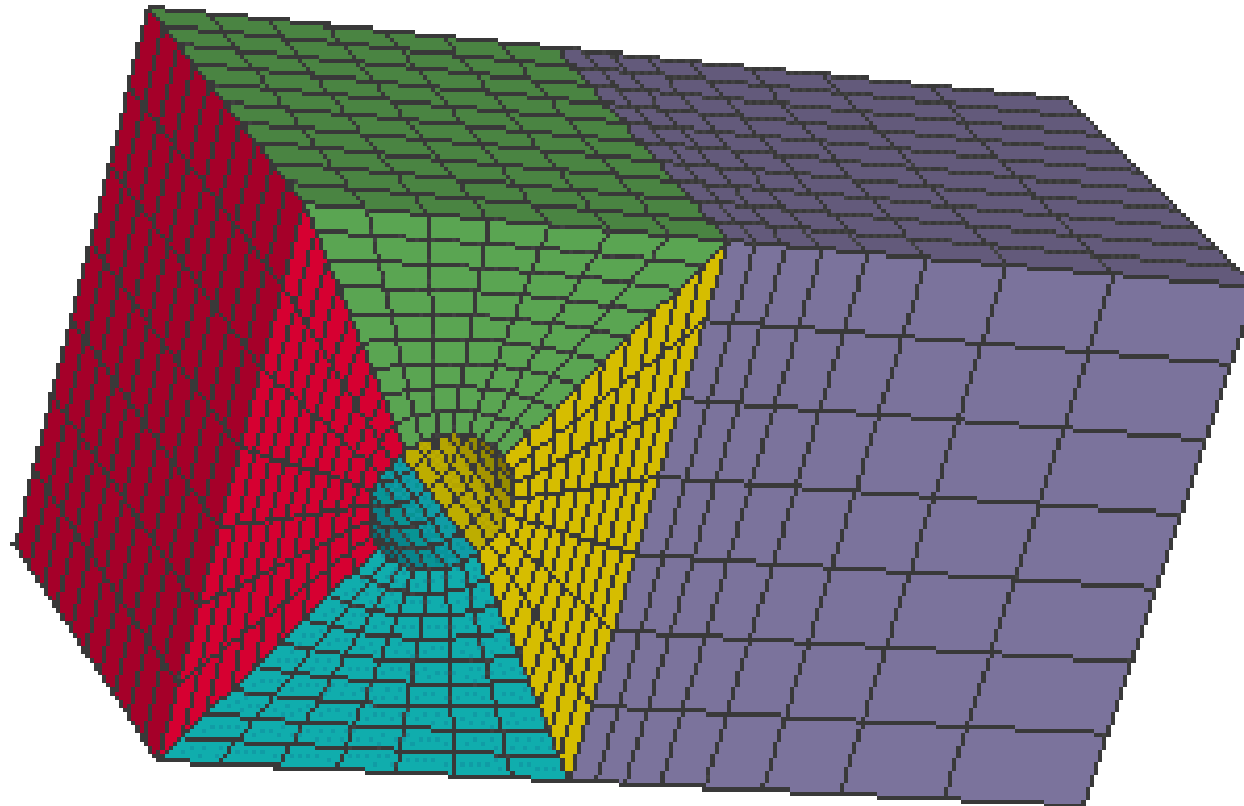


H grid

# Tipi di griglia

## Griglie strutturate (multi-blocco)

- Ogni cella e' identificata da una terna  $i,j,k$  all'interno di ogni blocco
- I blocchi possono essere connessi in modo arbitrario (o quasi)
- Sono più flessibili delle griglie strutturate a singolo blocco, ma presentano delle limitazioni

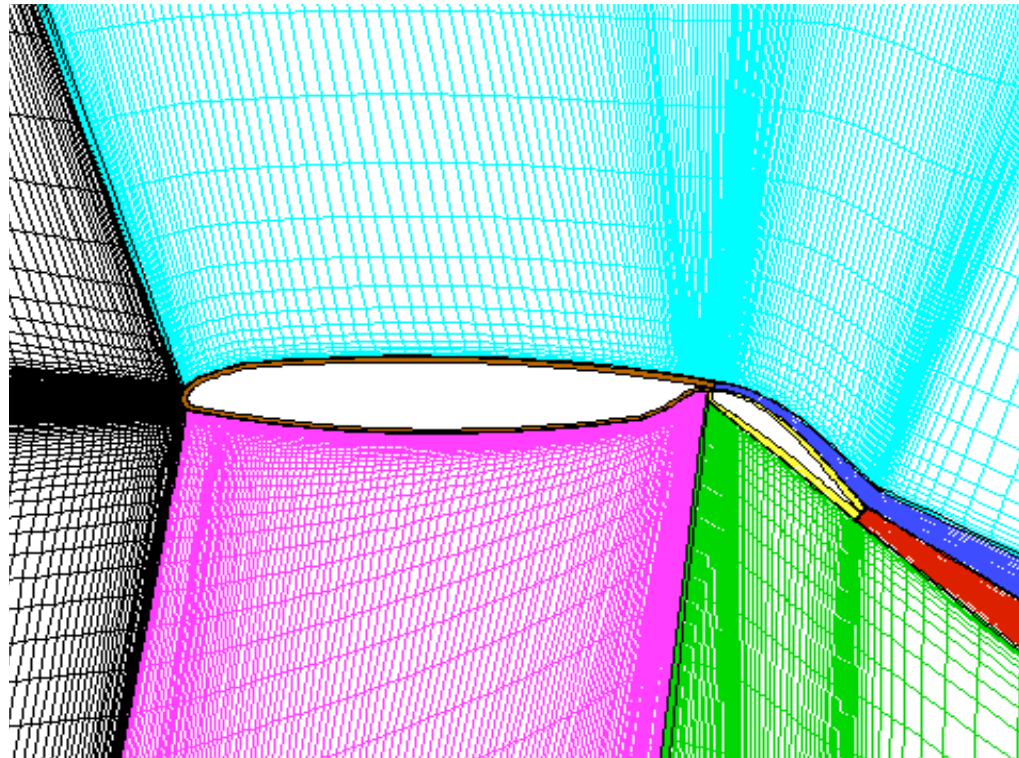


# Tipi di griglia

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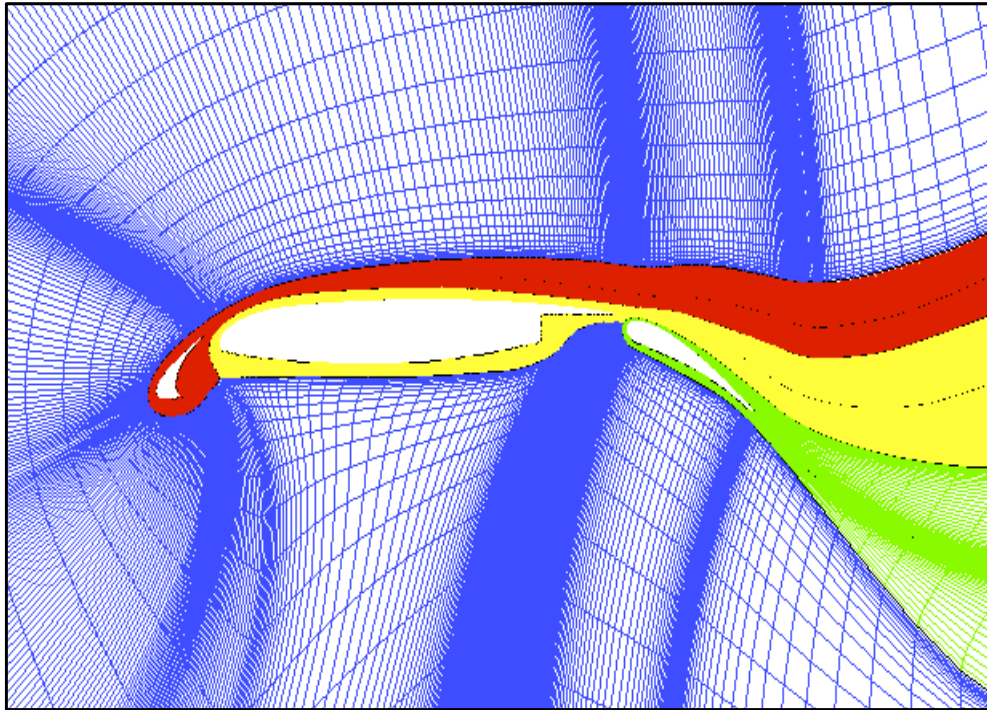
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**airfoil with flap**

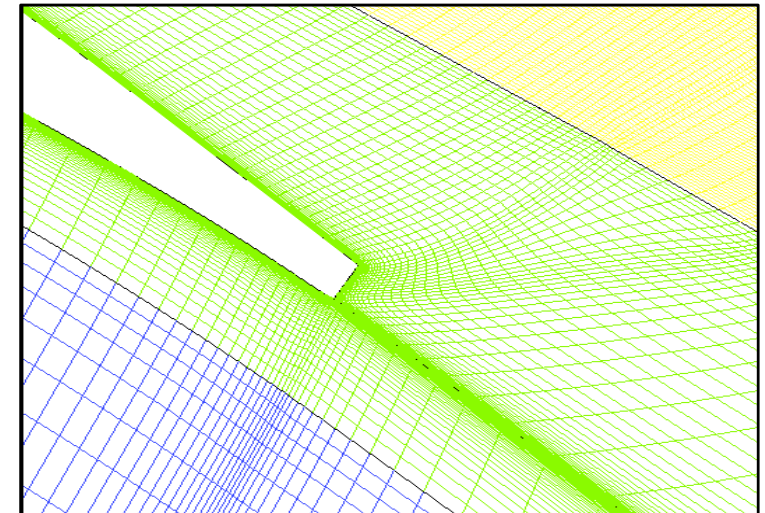
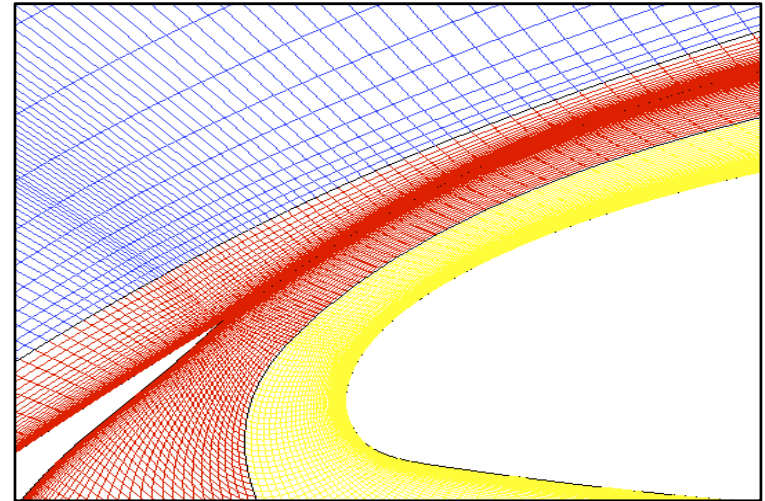


# Tipi di griglia

## Griglie strutturate (multi-blocco)



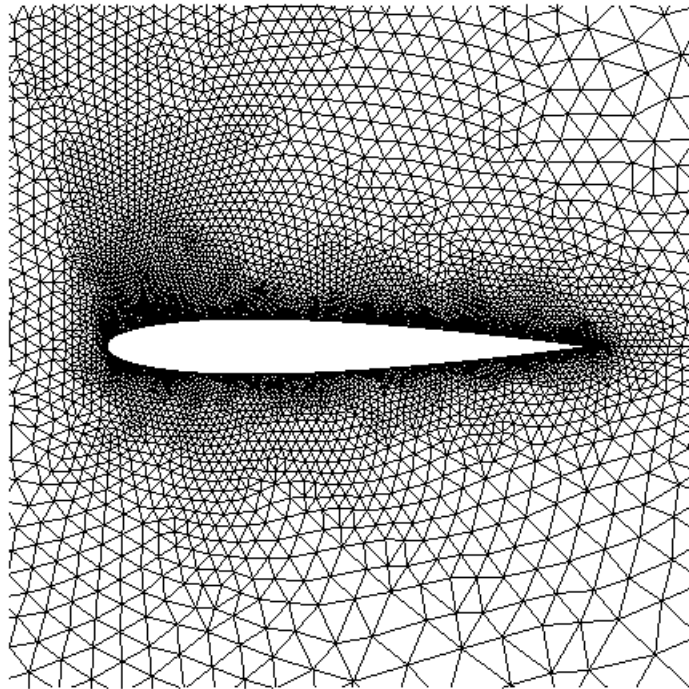
**airfoil with flap and slats**



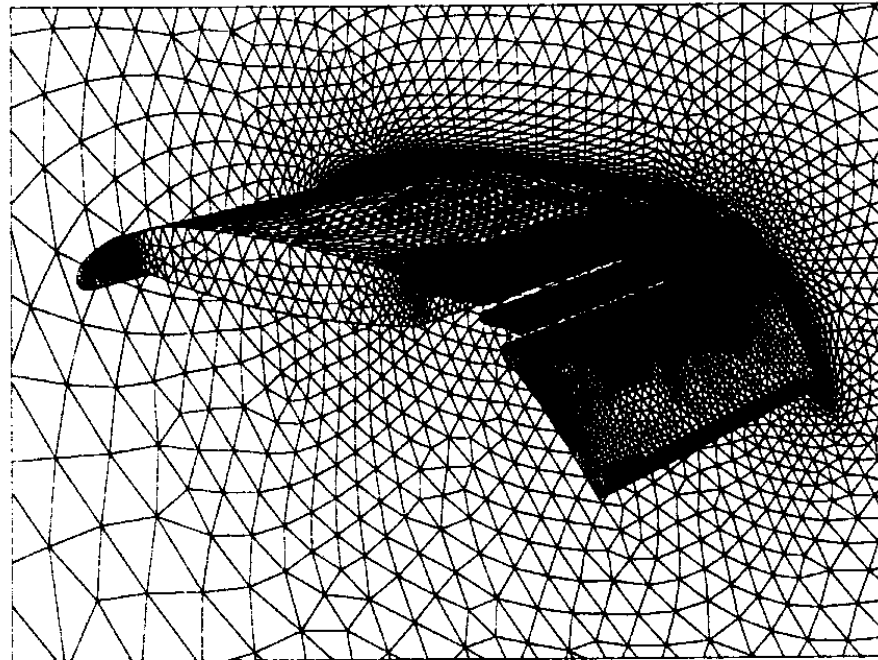
# Tipi di griglia

## Griglie non-strutturate

- Le celle sono organizzate in modo arbitrario (non e' definita da una terna  $(i,j,k)$ ) e non ci sono limiti nel modo in cui le celle sono organizzate
- E' presente un incremento della richiesta di memoria e di tempo di calcolo per griglie non strutturate (memory and CPU overheads)



**triangular mesh (2D)**

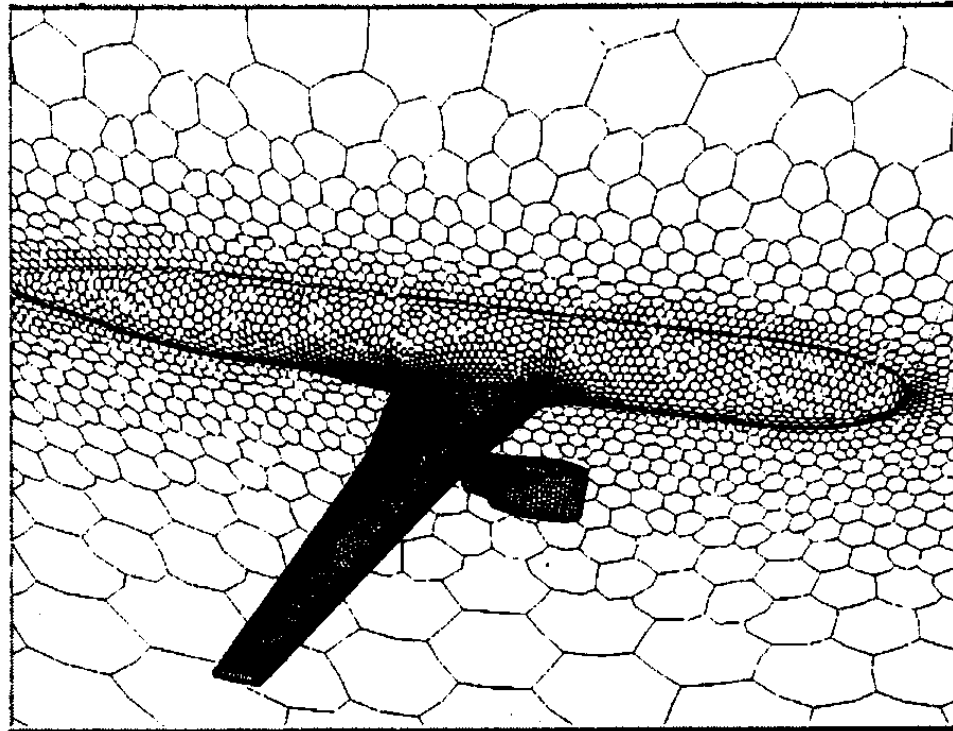


**tetrahedral mesh (3D)**

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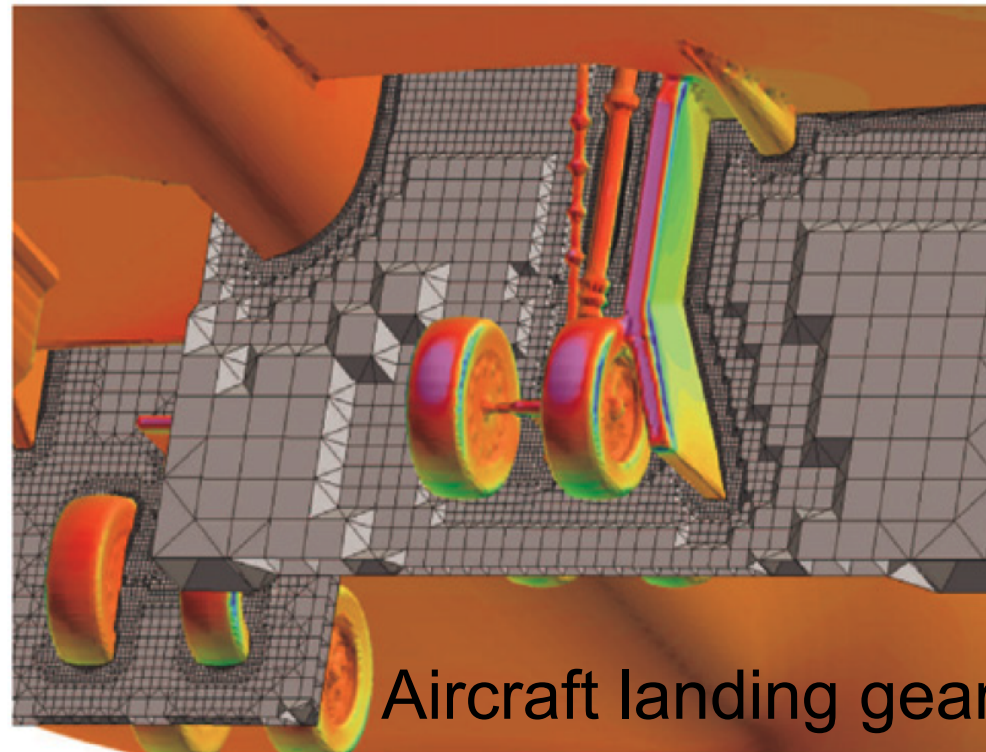


**polyhedral mesh (3D)**

# Tipi di griglia

## Griglie non-strutturate

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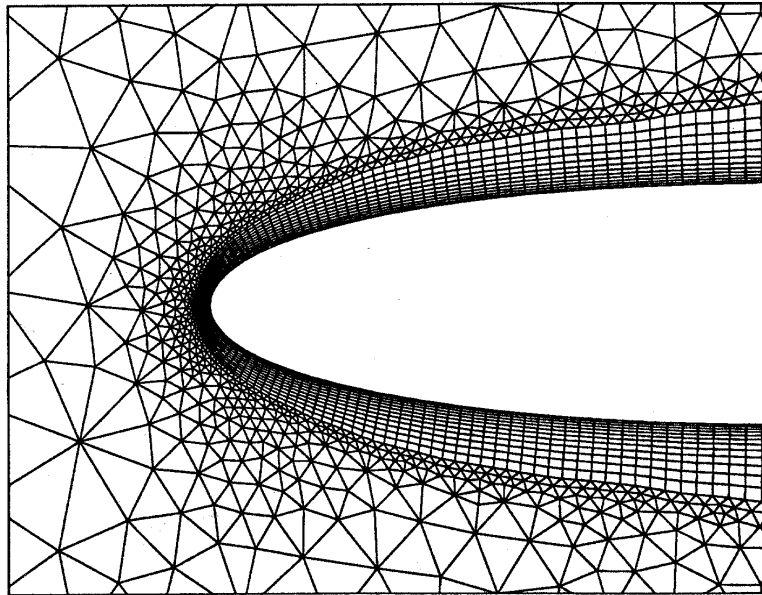
Aircraft landing gear

**octree mesh (3D)**

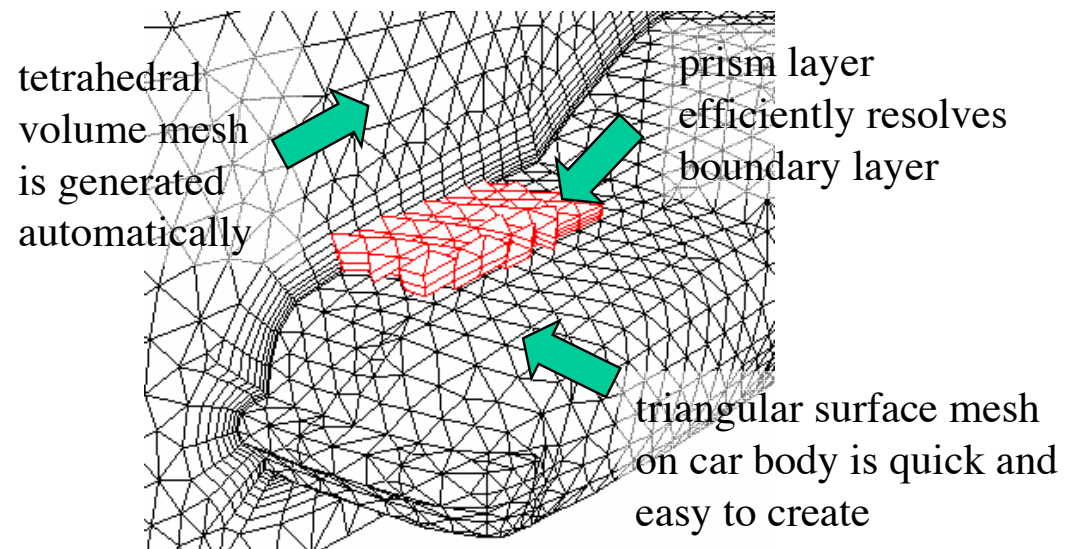
# Tipi di griglia

## Griglie ibride

- Combinano tipi di celle diverse a seconda della necessità (le più appropriate)
  - triangoli e quadrilateri in 2D
  - tetraedri, prismi e piramidi in 3D



**quads + tri (2D)**

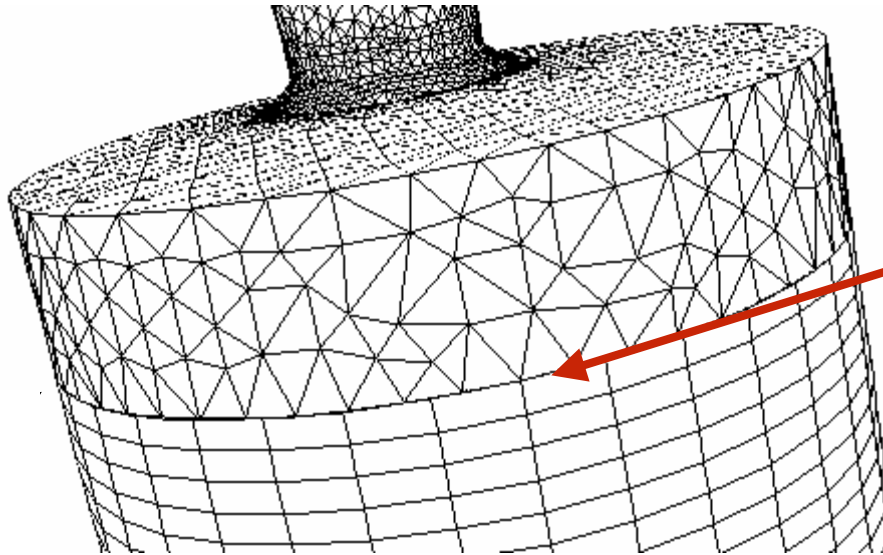


**prisms + tets (3D)**

# Tipi di griglia

## Griglie ibride

- Combinano tipi di celle diverse a seconda della necessita' (le più appropriate)
  - triangoli e quadrilateri in 2D
  - tetraedri, prismi e piramidi in 3D



**non-conformal  
interface**

**prisms + tets (3D)**

# Riepilogo

## Topologia

- Griglia strutturata (structured mesh): la griglia e' formata da celle che possono essere identificate da una terna (i,j,k)
- Griglia non-strutturata (unstructured mesh): non c'e' regolarità
- Multi-blocco (multiblock): la griglia e' formata da più di un blocco, ognuno dei quali può essere strutturato o non-strutturato.

## Tipo di celle

- Triangolari (tri-mesh)
- Quadrilateri (quad-mesh)
- Esaedri (hex-mesh)
- Tetraedri (tet-mesh)
- Prismi, piramidi (prisms, pyramids)
- Griglia ibrida
  - tri e quad in 2D
  - tets, prisms and hex in 3D
  - boundary layer mesh (prisms in BL, tets outside)
- Griglia poliedrica (polyhedral mesh)
- Griglia non-conforme (non-conformal mesh): i nodi della griglia non combaciano su un interfaccia

# Qualità di una griglia

- A parità di numero di celle, griglie esaedriche forniscono in generale soluzioni più accurate, specialmente se le linee della griglia sono allineate con il flusso
- La densità della mesh deve essere abbastanza elevata da catturare le caratteristiche importanti del flusso
- La griglia in vicinanza delle pareti deve essere abbastanza fine da risolvere lo strato limite (se necessario). Nello strato limite, quadrilateri (quads), esaedri (hex) e prismi (prisms) sono preferibili rispetto a triangoli (tris), tetraedri (tets) e piramidi (pyramids)
- Tre misure della qualità di una griglia
  - SKEWNESS
  - SMOOTHNESS
  - ASPECT RATIO

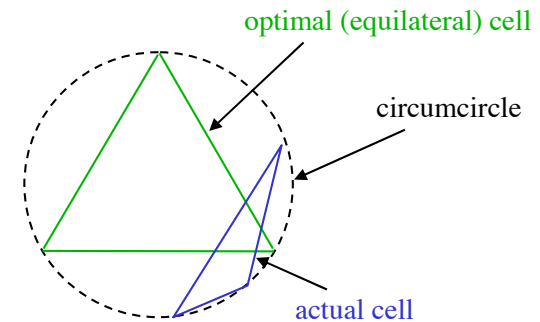
# Qualità di una griglia

## SKEWNESS

Dà una misura della deviazione di una cella da una cella “ideale”

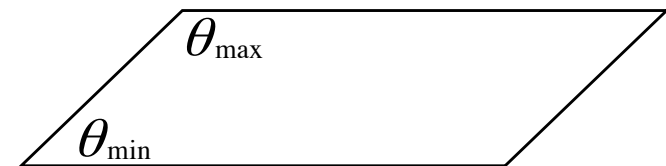
1) per tri/tets

$$S = \frac{\text{optimal cell size} - \text{cell size}}{\text{optimal cell size}}$$



2) per quads/hexs

$$s = \max \left[ \frac{\theta_{\max} - 90}{90}, \frac{90 - \theta_{\min}}{90} \right]$$



# Qualità di una griglia

## SKEWNESS

Dà una misura della deviazione di una cella da una cella “ideale”

### 3) metodo generale

$$S = \max \left[ \frac{\theta_{\max} - \theta_e}{180 - \theta_e}, \frac{\theta_e - \theta_{\min}}{\theta_e} \right]$$

$\theta_{\max}$  = angolo massimo in una faccia o cella

$\theta_{\min}$  = angolo minimo in una faccia o cella

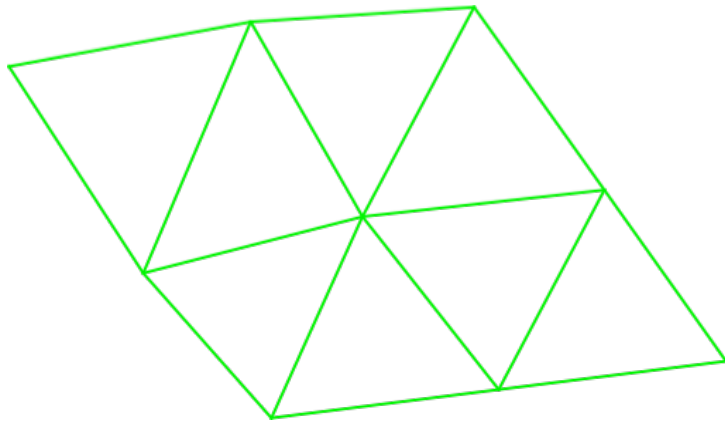
$\theta_e$  = angolo ottimale in una faccia o cella  
(60 deg per triangolo, 90 deg per quadrilatero)



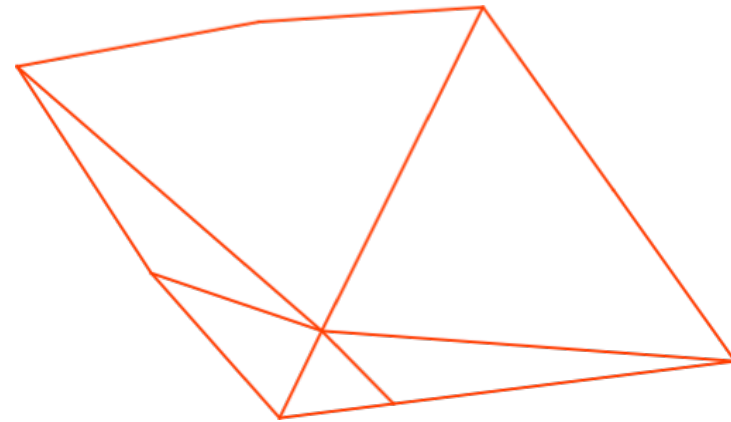
# Qualità di una griglia

## SMOOTHNESS

Dà una misura della variazione di dimensione tra celle adiacenti (deve essere graduale)



variazione graduale  
di dimensione

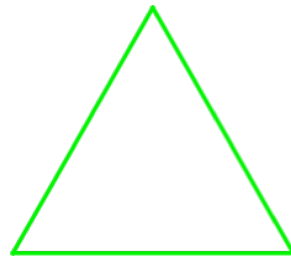
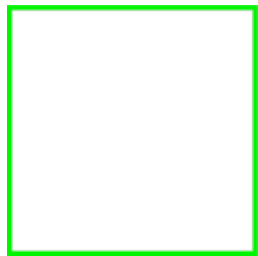


variazione rapida  
di dimensione

# Qualità di una griglia

## ASPECT RATIO

Rapporto tra la lunghezza spigolo (edge) più lungo e quella dello spigolo più corto



ASPECT RATIO 1



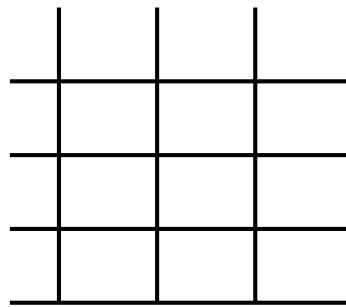
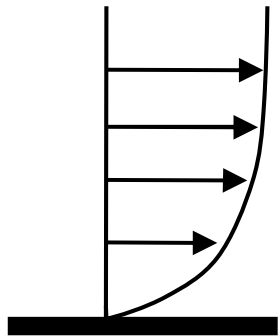
ASPECT RATIO  $\gg 1$

# Qualità di una griglia

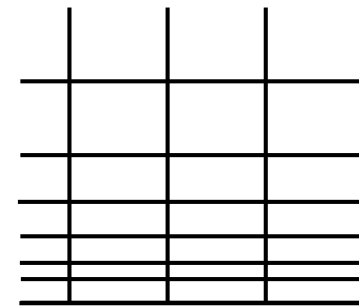
## LINEE GUIDA

- skewness  $< 0.9$
- rapporto dimensione celle adiacenti  $< 1.2$
- aspect ratio vicino a uno in situazioni di fluido multi-dimensionale (nessuna direzione preferenziale), aspect ratio sensibilmente maggiori possono essere utilizzati in problemi essenzialmente monodimensionali

esempio: strato limite (boundary layer)



**NO**



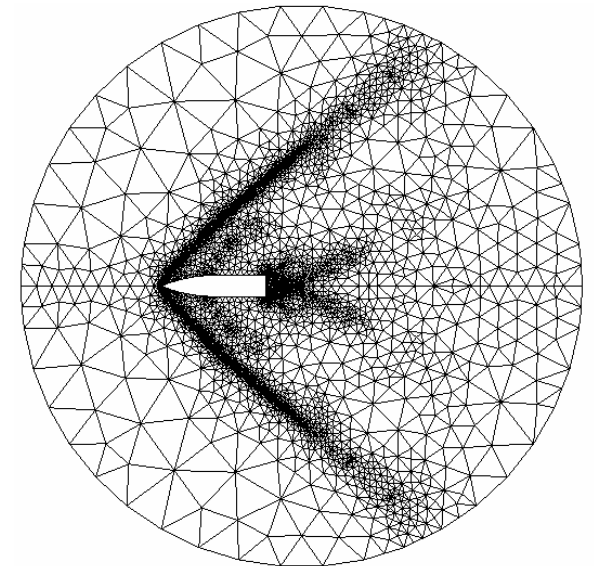
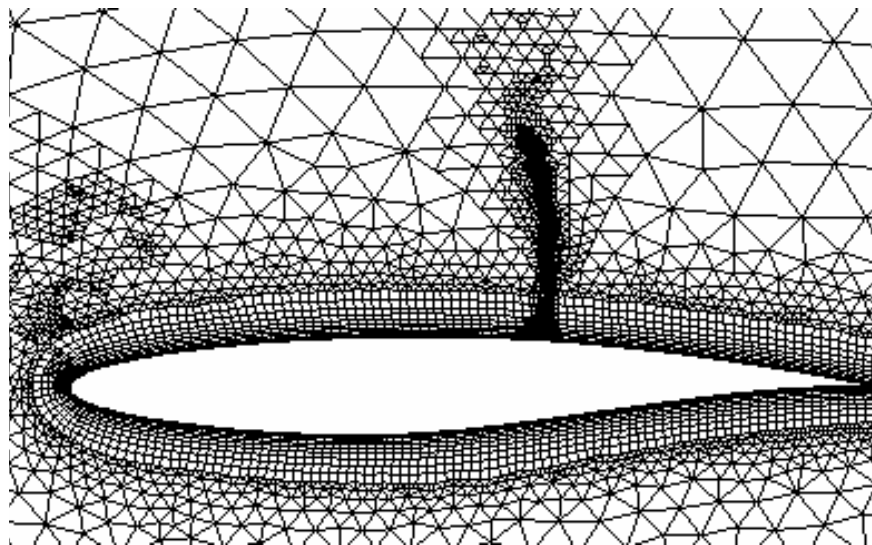
**SI**

# Qualità di una griglia

## LINEE GUIDA

- Un numero maggiore di celle aumenta l'accuratezza della soluzione, ma allo stesso tempo aumentano i tempi di calcolo.
- Le celle non devono essere “sprecate” in zone del fluido poco importanti, ma addensate nelle zone di interesse (strato limite, zone di rapida variazione delle grandezze fluidodinamiche)
- La griglia (e quindi la soluzione) può essere rifinita aumentando la densità di celle nelle zone di interesse (grid adaption)

GRID  
ADAPTION



# Conclusioni

Una griglia di qualità rappresenta il primo (essenziale) passo per una simulazione numerica (fluidodinamica) soddisfacente

# Extra

Le prossime pagine sono estratte dal manuale di Fluent. Contengono definizioni e linee guida importanti per la creazione di una griglia soddisfacente.



# Che cos'è la qualità di una griglia per Ansys?

## 6.2.2 Mesh Quality

The quality of the mesh plays a significant role in the accuracy and stability of the numerical computation. The attributes associated with mesh quality are node point distribution, smoothness, and skewness.

Regardless of the type of mesh used in your domain, checking the quality of your grid is essential. Depending on the cell types in the mesh (tetrahedral, hexahedral, polyhedral, etc.), different quality criteria are evaluated:

- Cell squish on all meshes (Section [30.4](#)).
- Cell equivolume skew on tri/tet elements (Section [30.4](#)).
- Face squish on polyhedral meshes (Section [30.4](#)).
- "Aspect ratio" on all meshes.

The "aspect ratio" is a measure of the stretching of a cell, and is defined as the ratio of the maximum distance between the cell centroid and face centroids to the minimum distance between the nodes of the cell (see Figure [6.2.2](#)). If the quality of your grid is questionable, then a warning will appear in the console noting the problems **FLUENT** has detected with your mesh. The warnings that you see use rules of thumb and although it is a warning, you may still be able to run the case successfully.

**Cell Squish Index** (in the **Grid...** category) is a measure of the quality of a mesh, and is calculated from the dot products of each vector pointing from the centroid of a cell toward the center of each of its faces, and the corresponding face area vector as

$$\max_i \left[ 1 - \frac{\vec{A}_i \cdot \vec{r}_{c0/xf_i}}{|\vec{A}_i| |\vec{r}_{c0/xf_i}|} \right] \quad (30.4-6)$$

Therefore, the worst cells will have a **Cell Squish Index** close to 1.

**Cell Equiangle Skew** (in the **Grid...** category) is a nondimensional parameter calculated using the normalized angle deviation method, and is defined as

$$\max \left[ \frac{q_{\max} - q_e}{180 - q_e}, \frac{q_e - q_{\min}}{q_e} \right] \quad (30.4-3)$$

where

$q_{\max}$  = largest angle in the face or cell

$q_{\min}$  = smallest angle in the face or cell

$q_e$  = angle for an equiangular face or cell

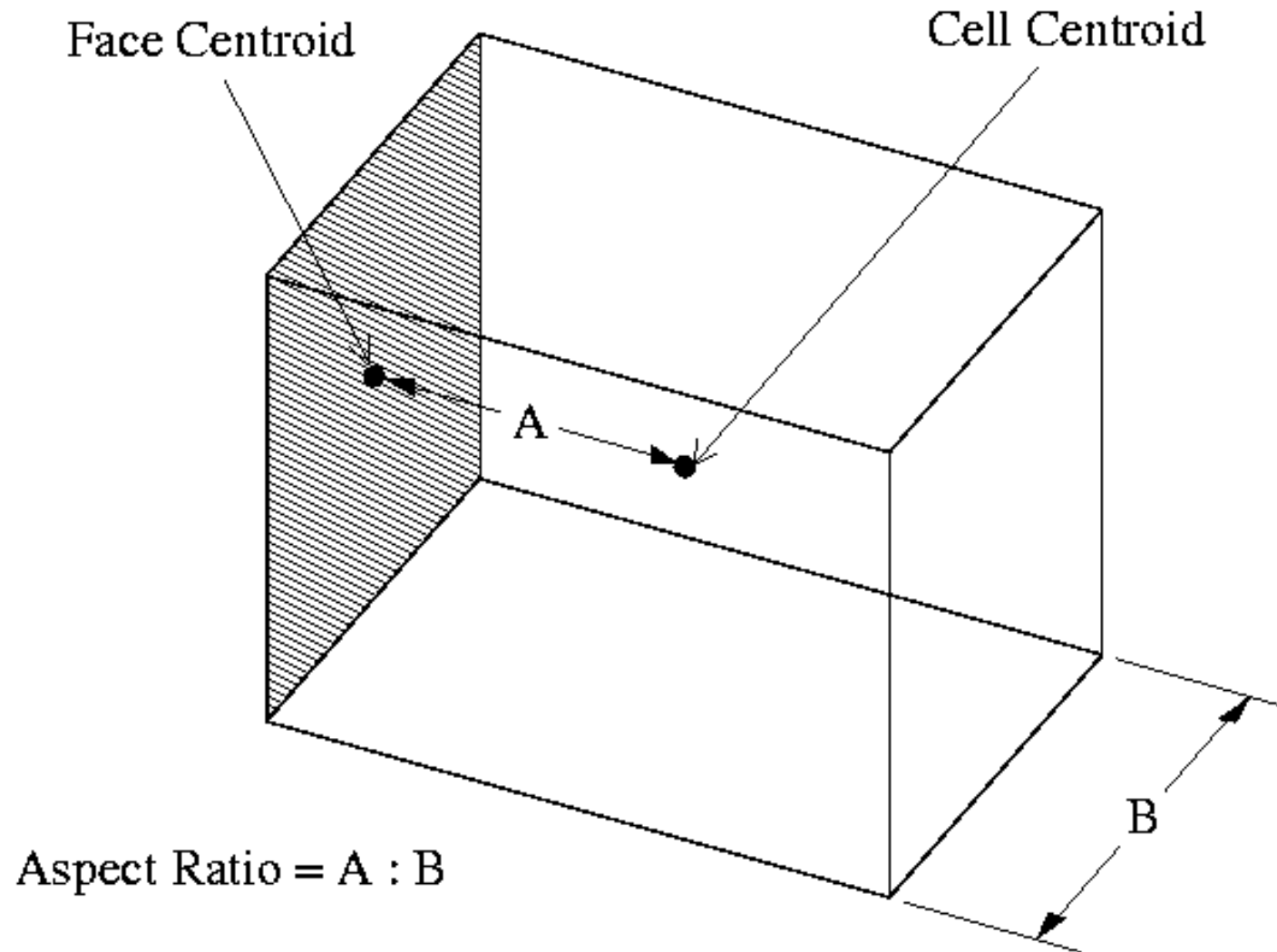
(e.g., 60 for a triangle and 90 for a square)

A value of 0 indicates a best case equiangular cell, and a value of 1 indicates a completely degenerate cell. Degenerate cells (slivers) are characterized by nodes that are nearly coplanar (collinear in 2D). **Cell Equiangle Skew** applies to all elements.

**Face Squish Index** (in the **Grid...** category) is a measure of the quality of a mesh, and is calculated from the dot products of each face area vector, and the vector that connects the centroids of the two adjacent cells as

$$1 - \frac{\vec{A}_i \cdot \vec{r}_{c0/c1}}{|\vec{A}_i| |\vec{r}_{c0/c1}|} \quad (30.4-17)$$

Therefore, the worst cells will have a **Face Squish Index** close to 1.



## Node Density and Clustering

---

Since you are discretely defining a continuous domain, the degree to which the salient features of the flow (such as shear layers, separated regions, shock waves, boundary layers, and mixing zones) are resolved, depends on the density and distribution of nodes in the mesh. In many cases, poor resolution in critical regions can dramatically alter the flow characteristics. For example, the prediction of separation due to an adverse pressure gradient depends heavily on the resolution of the boundary layer upstream of the point of separation.

Resolution of the boundary layer (i.e., mesh spacing near walls) also plays a significant role in the accuracy of the computed wall shear stress and heat transfer coefficient. This is particularly true in laminar flows where the grid adjacent to the wall should obey

$$y_p \sqrt{\frac{u_\infty}{\nu x}} \leq 1 \quad (6.2-1)$$

where  $y_p$  = distance to the wall from the adjacent cell centroid

$u_\infty$  = free-stream velocity

$\nu$  = kinematic viscosity of the fluid

$x$  = distance along the wall from the starting point of the boundary layer

Equation [6.2-1](#) is based upon the Blasius solution for laminar flow over a flat plate at zero incidence [[322](#)].

Proper resolution of the mesh for turbulent flows is also very important. Due to the strong interaction of the mean flow and turbulence, the numerical results for turbulent flows tend to be more susceptible to grid dependency than those for laminar flows. In the near-wall region, different mesh resolutions are required depending on the near-wall model being used. See Section [12.11](#) for guidelines.

In general, no flow passage should be represented by fewer than 5 cells. Most cases will require many more cells to adequately resolve the passage. In regions of large gradients, as in shear layers or mixing zones, the grid should be fine enough to minimize the change in the flow variables from cell to cell. Unfortunately, it is very difficult to determine the locations of important flow features in advance. Moreover, the grid resolution in most complicated 3D flow fields will be constrained by CPU time and computer resource limitations (i.e., memory and disk space). Although accuracy increases with larger grids, the CPU and memory requirements to compute the solution and postprocess the results also increase. Solution-adaptive grid refinement can be used to increase and/or decrease grid density based on the evolving flow field, and thus provides the potential for more economical use of grid points (and hence reduced time and resource requirements). See Chapter [26](#) for information on solution adaption.

## Smoothness

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Truncation error is the difference between the partial derivatives in the governing equations and their discrete approximations. Rapid changes in cell volume between adjacent cells translate into larger truncation errors. **FLUENT** provides the capability to improve the smoothness by refining the mesh based on the change in cell volume or the gradient of cell volume. For information on refining the grid based on change in cell volume. (See Sections [26.4](#) and [26.8](#)).

## Cell Shape

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The shape of the cell (including its skewness and aspect ratio) also has a significant impact on the accuracy of the numerical solution.

- *Skewness* is defined as the difference between the shape of the cell and the shape of an equilateral cell of equivalent volume. Highly skewed cells can decrease accuracy and destabilize the solution. For example, optimal quadrilateral meshes will have vertex angles close to 90 degrees, while triangular meshes should preferably have angles of close to 60 degrees and have all angles less than 90 degrees.
- *Aspect ratio* is a measure of the stretching of the cell. As discussed in Section [6.1.3](#), for highly anisotropic flows, extreme aspect ratios may yield accurate results with fewer cells. However, a general rule of thumb is to avoid aspect ratios in excess of 5:1.

## Flow-Field Dependency

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The effect of resolution, smoothness, and cell shape on the accuracy and stability of the solution process is dependent on the flow field being simulated. For example, very skewed cells can be tolerated in benign flow regions, but can be very damaging in regions with strong flow gradients.

Since the locations of strong flow gradients generally cannot be determined a priori, you should strive to achieve a high-quality mesh over the entire flow domain.