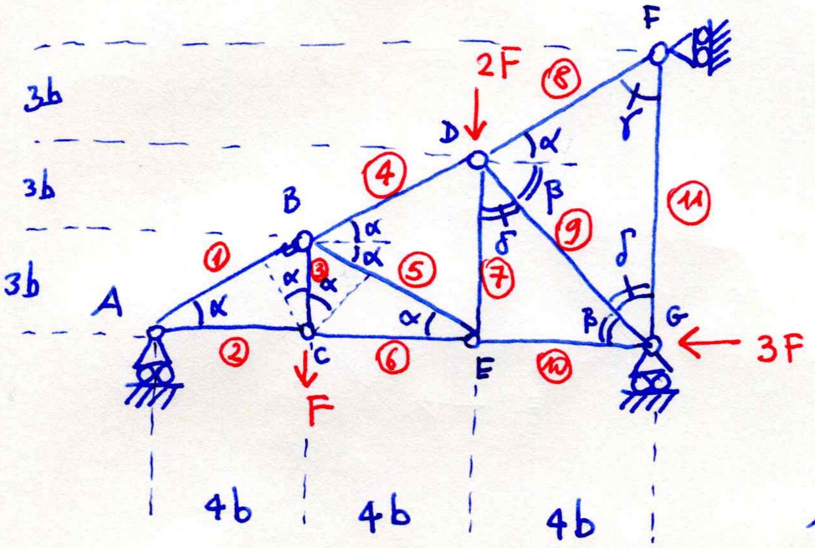
ESERCIZIO: STRUTTURA RETICOLARE



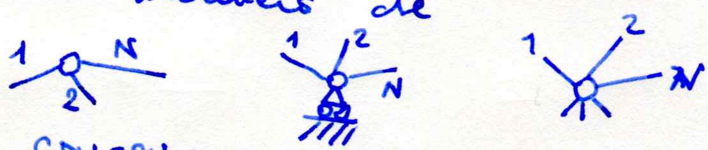
Osserviamo preventivamente il numero di gradi di libertà e di vincoli.

METODO TRADIZIONALE

$$GDL = 3 \times 11 = 33$$

$$GDV: 3(A) + 6(B) + 4(C) + 6(D) + 6(E) + 3(F) + 5(G) = 33$$

dove ricordiamo che



GDV = 2N−2   GDV = 2N−1   GDV = 2N

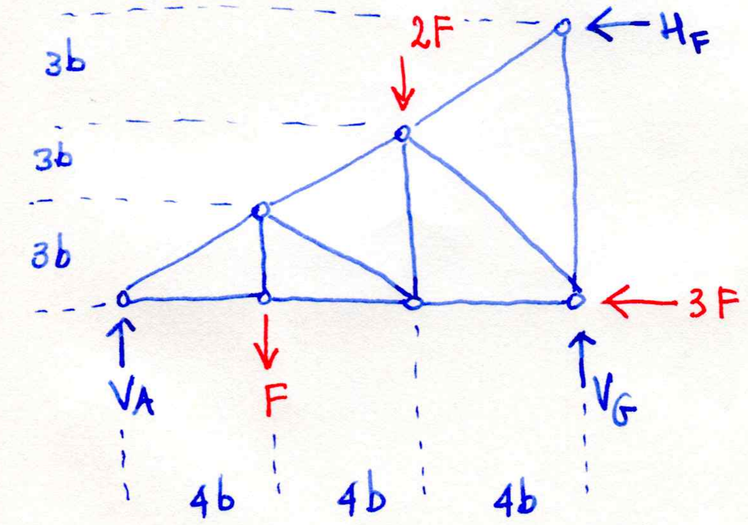
Poiché stiamo studiando una struttura reticolare, possiamo usare anche il metodo speciale per strutture reticolari (a ogni trave/asta corrispondono due gradi di vincolo, inoltre vanno considerati anche i vincoli esterni e infine ogni nodo aggiunge due gradi di libertà)

METODO SPECIALE PER STRUTTURE RETICOLARI

$$GDL = 2 \times 7 = 14 \qquad GDV = 1 \times 11 + 1(A) + 1(F) + 1(G) = 14$$

Con entrambi i metodi otteniamo che GDL = GDV e, quindi, la struttura è isostatica.

~~Per ricavare~~ Il diagramma di corpo libero è dato da



Per ricavare le reazioni vincolari (V_A, V_G, H_F) basta risolvere il sistema delle equazioni cardinali

$$\begin{cases} \rightarrow R_x = 0 \\ \uparrow R_y = 0 \\ \circlearrowleft M_z = 0 \end{cases}$$

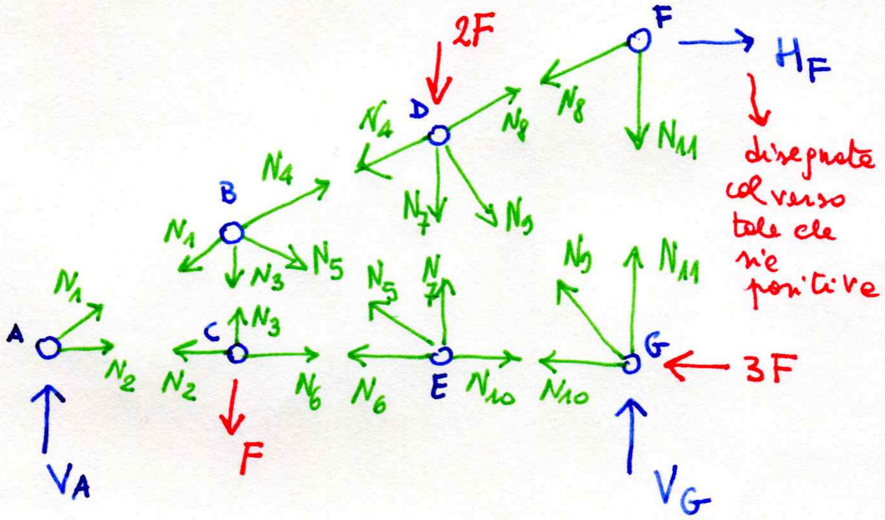
ANALISI STATICA

$$\begin{cases} \rightarrow R_x = 0 \\ \uparrow R_y = 0 \\ \circlearrowleft M_{z(A)} = 0 \end{cases} ; \quad \begin{cases} -H_F - 3F = 0 \\ V_A + V_G - F - 2F = 0 \\ -F4b - 2F8b + V_G 12b + H_F 9b = 0 \end{cases} ; \quad \begin{cases} H_F = -3F \\ V_A + V_G = 3F \\ 9H_F + 12V_G = 20F \end{cases}$$

da cui si ricava $H_F = 3F\ (\rightarrow)$, $V_G = \frac{47}{12}F\ (\uparrow)$, $V_A = -\frac{11}{12}F\ (\uparrow)$

METODO DELL'EQUILIBRIO DEI NODI

Eliminiamo tutte le travi/aste, sostituendole con le azioni assiali che ciascuna trasmette ai nodi d'estremità



NOTA

Per il seguito è utile ricordare che

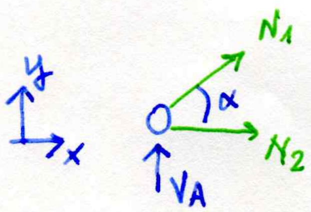
$\sin\alpha = \frac{3}{5}$ $\quad \cos\alpha = \frac{4}{5}$

$\sin\beta = \frac{6}{\sqrt{52}}$ $\quad \cos\alpha = \frac{4}{\sqrt{52}}$

$\sin\gamma = \cos\alpha$ $\quad \cos\gamma = \sin\alpha$

$\sin\delta = \cos\beta$ $\quad \cos\delta = \sin\beta$

NODO (A)

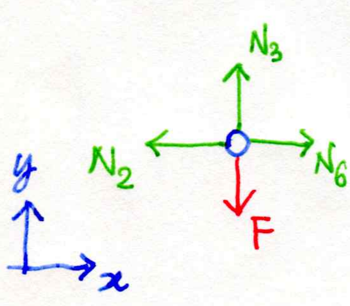


$$\begin{cases} \rightarrow R_x = 0 \\ \uparrow R_y = 0 \end{cases} ; \quad \begin{cases} N_1 \cos\alpha + N_2 = 0 \\ V_A + N_1 \sin\alpha = 0 \end{cases} ; \quad \begin{cases} N_2 = -N_1 \cos\alpha \\ N_1 = -\frac{V_A}{\sin\alpha} \end{cases}$$

$$N_1 = -\frac{5}{3} \cdot \left(-\frac{11}{12}F\right) = \frac{55}{36}F \quad \text{(tirante)}$$

$$N_2 = -\frac{4}{5} \cdot \frac{55}{36}F = -\frac{11}{9}F \quad \text{(puntone)}$$

NODO (C)

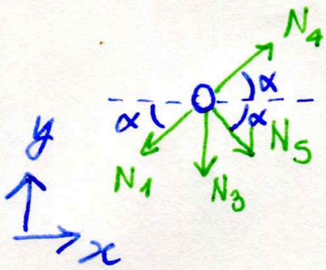


$$\begin{cases} \rightarrow R_x = 0 \\ \uparrow R_y = 0 \end{cases} ; \quad \begin{cases} -N_2 + N_6 = 0 \\ N_3 - F = 0 \end{cases} ; \quad \begin{cases} N_6 = N_2 \\ N_3 = F \end{cases}$$

$$N_6 = -\frac{11}{9}F \quad \text{(puntone)}$$

$$N_3 = F \quad \text{(tirante)}$$

NODO (B)



$$\begin{cases} \rightarrow R_x = 0 \\ \uparrow R_y = 0 \end{cases} ; \quad \begin{cases} -N_1 \cos\alpha + N_5 \cos\alpha + N_4 \cos\alpha = 0 \\ -N_3 - N_1 \sin\alpha - N_5 \sin\alpha + N_4 \sin\alpha = 0 \end{cases}$$

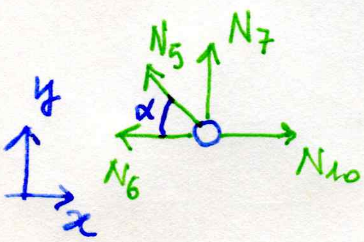
$$\begin{cases} N_4 + N_5 = N_1 \quad \Rightarrow N_5 = N_1 - N_4 \\ (N_4 - N_5)\sin\alpha = N_3 + N_1 \sin\alpha \end{cases}$$

$$\begin{cases} N_5 = N_1 - N_4 \\ (2N_4 - N_1)\sin\alpha = N_3 + N_1 \sin\alpha \end{cases} ; \quad \begin{cases} N_5 = N_1 - N_4 \\ N_4 = \dfrac{N_3}{2\sin\alpha} + N_1 \end{cases}$$

$$N_4 = \frac{5}{6} F + \frac{55}{36} F = \frac{85}{36} F \quad \text{(tirante)}$$

$$N_5 = \frac{55}{36} F - \frac{85}{36} F = -\frac{5}{6} F \quad \text{(puntone)}$$

NODO (E)



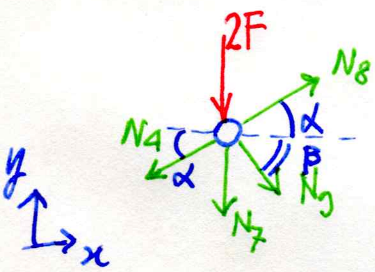
$$\begin{cases} \rightarrow R_x = 0 \\ \uparrow R_y = 0 \end{cases} ; \quad \begin{cases} -N_6 + N_{10} - N_5 \cos\alpha = 0 \\ N_7 + N_5 \sin\alpha = 0 \end{cases}$$

$$\begin{cases} N_{10} = N_6 + N_5 \cos\alpha = -\dfrac{11}{9} F - \dfrac{5}{6} F \cdot \dfrac{4}{5} = -\dfrac{11}{9} F - \dfrac{2}{3} F \\ N_7 = -N_5 \sin\alpha = \dfrac{5}{6} F \cdot \dfrac{3}{5} = \dfrac{F}{2} \end{cases}$$

$$N_{10} = -\frac{17}{9} F \quad \text{(puntone)}$$

$$N_7 = \frac{F}{2} \quad \text{(tirante)}$$

NODO (D)



$$\begin{cases} \rightarrow R_x = 0 \\ \uparrow R_y = 0 \end{cases} ; \quad \begin{cases} N_8 \cos\alpha + N_9 \sin\beta - N_4 \cos\alpha = 0 \\ -2F - N_7 - N_4 \sin\alpha - N_9 \sin\beta + N_8 \sin\alpha = 0 \end{cases}$$

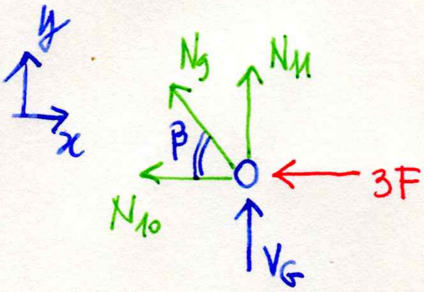
$$\begin{cases} \dfrac{4}{5} N_8 + \dfrac{4}{\sqrt{52}} N_9 - \dfrac{4}{5} N_4 = 0 \\ -N_7 - 2F - \dfrac{6}{\sqrt{52}} N_9 - \dfrac{3}{5} N_4 + \dfrac{3}{5} N_8 = 0 \end{cases}$$

$$\begin{cases} \dfrac{N_8}{5} + \dfrac{N_9}{\sqrt{52}} = \dfrac{1}{5} \cdot \dfrac{85}{36} F \\ \dfrac{3}{5} N_8 - \dfrac{6}{\sqrt{52}} N_9 = \dfrac{F}{2} + 2F + \dfrac{3}{5} \cdot \dfrac{85}{36} F \end{cases}$$

da cui si deduce

$$\begin{cases} N_8 = \dfrac{15}{4} F \quad \text{(tirante)} \\ N_9 = -\dfrac{5\sqrt{13}}{9} F \quad \text{(puntone)} \end{cases}$$

NODO (G)



$$\begin{cases} \rightarrow R_x = 0 \\ \uparrow R_y = 0 \end{cases} ; \quad \begin{cases} -3F - N_{10} - N_9 \cos\beta = 0 \\ N_{11} + V_G + N_9 \sin\beta = 0 \end{cases}$$

Ci basta considerare solo la seconda equazione

$$N_{11} = -V_G - N_9 \sin\beta$$

$$= -\frac{47}{12}F + \frac{5\sqrt{13}}{9}F \cdot \frac{6}{\sqrt{52}}$$

$$N_{11} = -\frac{47}{12}F + \frac{5}{3}F = -\frac{9}{4}F \quad (\text{puntone})$$

I risultati ottenuti finora possono essere riassunti nella seguente tabella e anche graficamente come segue

$N_1 = N_{AB} = \frac{55}{36}F$ [T]

$N_2 = N_{AC} = -\frac{11}{9}F$ [P]

$N_3 = N_{BC} = F$ [T]

$N_4 = N_{BD} = \frac{85}{36}F$ [T]

$N_5 = N_{BE} = -\frac{5}{6}F$ [P]

$N_6 = N_{CE} = -\frac{11}{9}F$ [P]

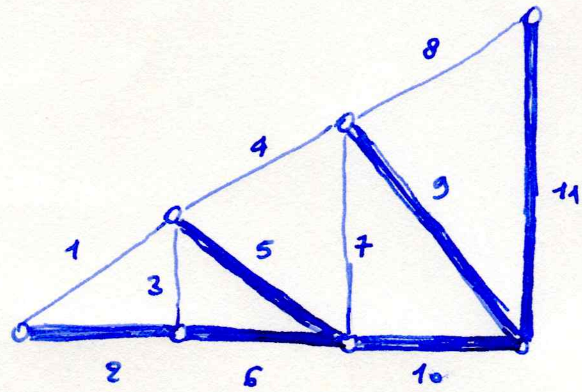
$N_7 = N_{DE} = \frac{F}{2}$ [T]

$N_8 = N_{DF} = \frac{15}{4}F$ [T]

$N_9 = N_{DG} = -\frac{5\sqrt{13}}{9}F$ [P]

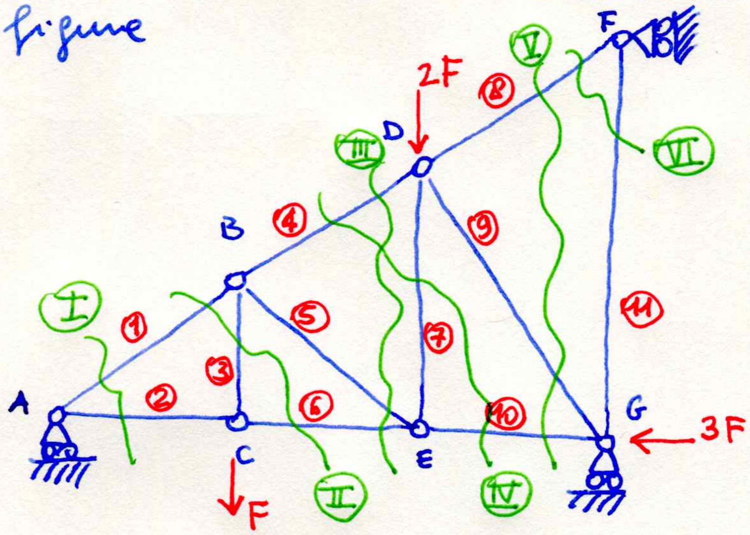
$N_{10} = N_{EG} = -\frac{17}{9}F$ [P]

$N_{11} = N_{FG} = -\frac{9}{4}F$ [P]



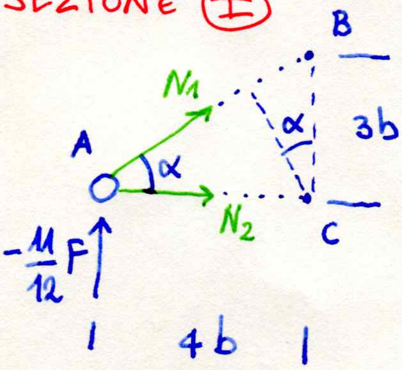
METODO DELLE SEZIONI DI RITTER

Si provvede a sezionare la struttura con le sei sezioni indicate in figura



Poi procediamo a scrivere opportune equazioni di equilibrio per una delle due parti in cui la struttura risulta suddivisa.

SEZIONE (I)



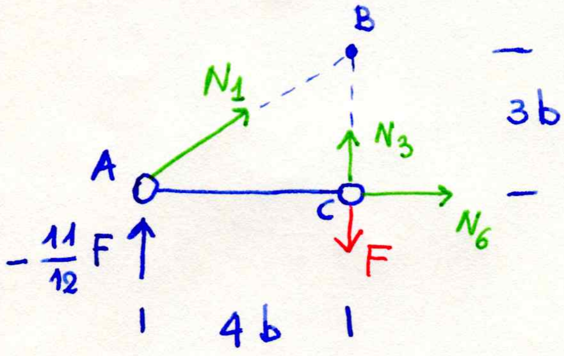
È immediato osservare che, scrivendo l'equazione d'equilibrio del momento scegliendo come polo B, l'azione assiale N_1 non contribuirà, consentendo di calcolare direttamente N_2. Il contrario accade scegliendo come polo C.

$$\circlearrowleft M^{(I)}_{z(B)} = 0 \quad : \quad -\left(-\frac{11}{12}F\right)4b + N_2 \cdot 3b = 0 \quad \text{da cui} \quad \boxed{N_2 = -\frac{11}{9}F}$$

$$\circlearrowleft M^{(I)}_{z(C)} = 0 \quad : \quad -\left(-\frac{11}{12}F\right)4b - N_1 \cdot 3b \cos\alpha = 0$$

$$\frac{11}{3}F\,b - N_1 \cdot \frac{12}{5}b = 0 \quad \text{da cui} \quad \boxed{N_1 = \frac{55}{36}F}$$

SEZIONE (II)



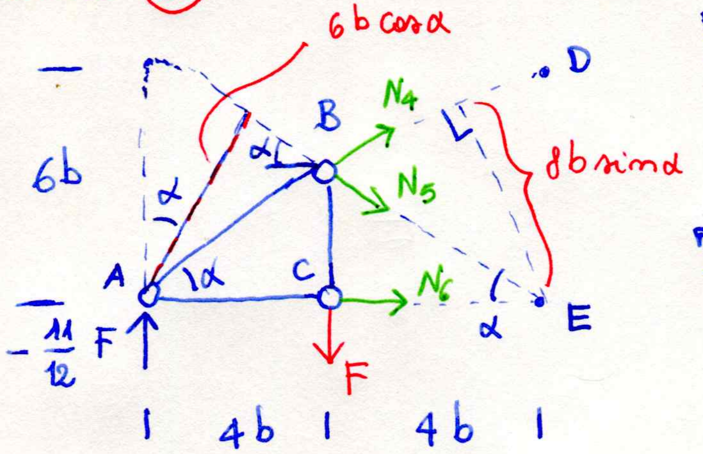
$$\circlearrowleft M^{(II)}_{z(A)} = 0 \; : \; N_3\, 4b - F\, 4b = 0$$

da cui $\boxed{N_3 = F}$

$$\circlearrowleft M^{(II)}_{z(B)} = 0 \; : \; -\frac{11}{12}F \cdot 4b + N_6 \cdot 3b = 0$$

da cui $\boxed{N_6 = -\frac{11}{9}F}$

SEZIONE III



$$\circlearrowleft M_{z(A)}^{III} = 0 : \quad -F\,4b - N_5 \cdot 6b\cos\alpha = 0$$

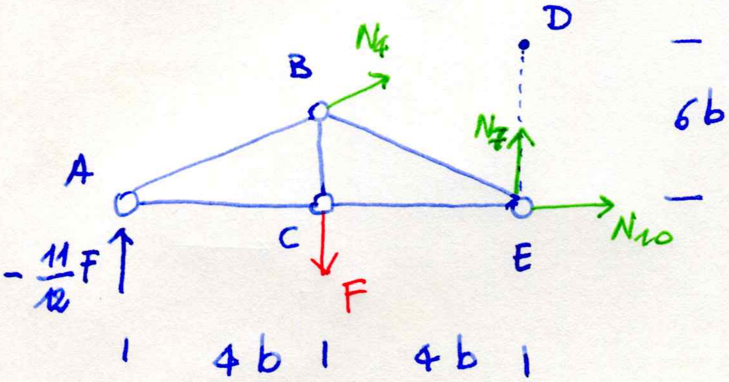
da cui $\boxed{N_5 = -\frac{5}{6}F}$

$$\circlearrowleft M_{z(E)}^{III} = 0 :$$

$$-\left(-\frac{11}{12}F\right)8b + F\,4b - N_4\,8b\sin\alpha = 0$$

da cui $\boxed{N_4 = \frac{85}{36}F}$

SEZIONE IV



$$\circlearrowleft M_{z(A)}^{IV} = 0 :$$

$$-F\,4b + N_7\,8b = 0$$

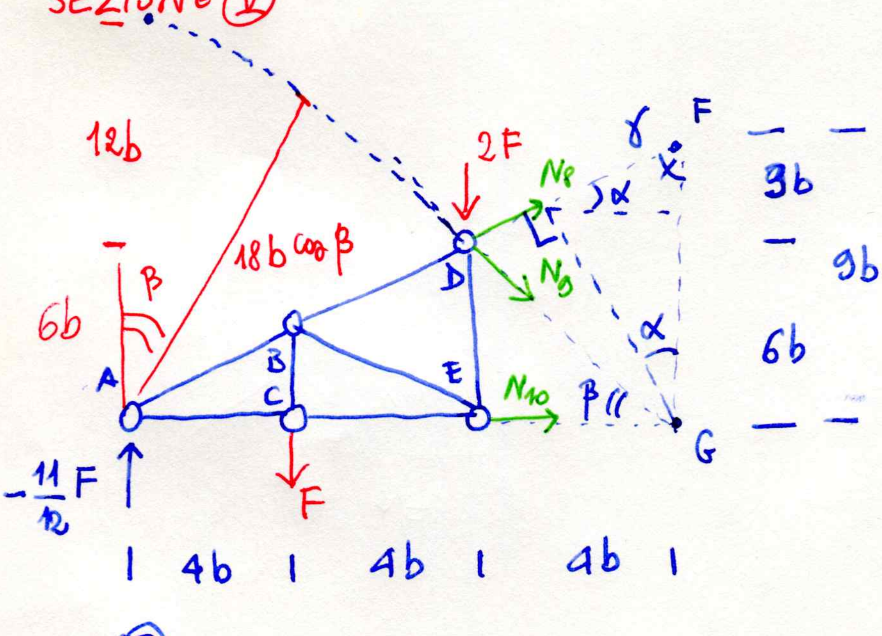
da cui $\boxed{N_7 = \frac{F}{2}}$

$$\circlearrowleft M_{z(D)}^{IV} = 0 :$$

$$-\left(-\frac{11}{12}F\right)\cdot 8b + F\cdot 4b + N_{10}\,6b = 0$$

da cui $\boxed{N_{10} = -\frac{17}{9}F}$

SEZIONE V



$$\circlearrowleft M_{z(A)}^{V} = 0$$

$$-F\,4b - 2F\,8b - N_9 \cdot 18b\cos\beta = 0$$

$$-20F - N_9 \cdot 18\,\frac{4}{2\sqrt{13}} = 0$$

da cui

$$N_9 = -\frac{5}{9}\sqrt{13}\,F$$

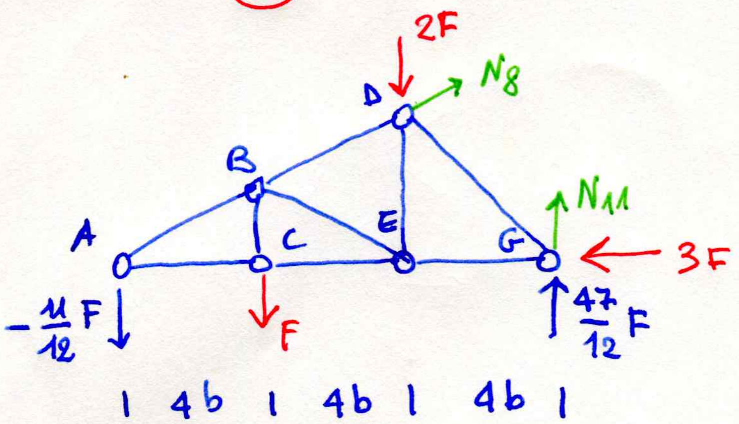
$$\circlearrowleft M_{z(G)}^{V} = 0 : \quad -\left(-\frac{11}{12}F\right)\cdot 12b + F\,8b + 2F\cdot 4b - N_8\,9b\sin\gamma = 0$$

$$\sin\gamma = \cos\alpha = \frac{4}{5}$$

$$11Fb + 8Fb + 8Fb - \frac{36}{5}N_8 = 0$$

da cui $\boxed{N_8 = \frac{15}{4}F}$

SEZIONE (VI)



Qui basta studiare solo una equazione di equilibrio del momento, con polo in A

$$\Sigma M_{z(A)}^{(VI)} = 0$$

$$-F \cdot 4b - 2F \cdot 8b + \frac{47}{12} F \cdot 12b + N_{11} \cdot 12b = 0$$

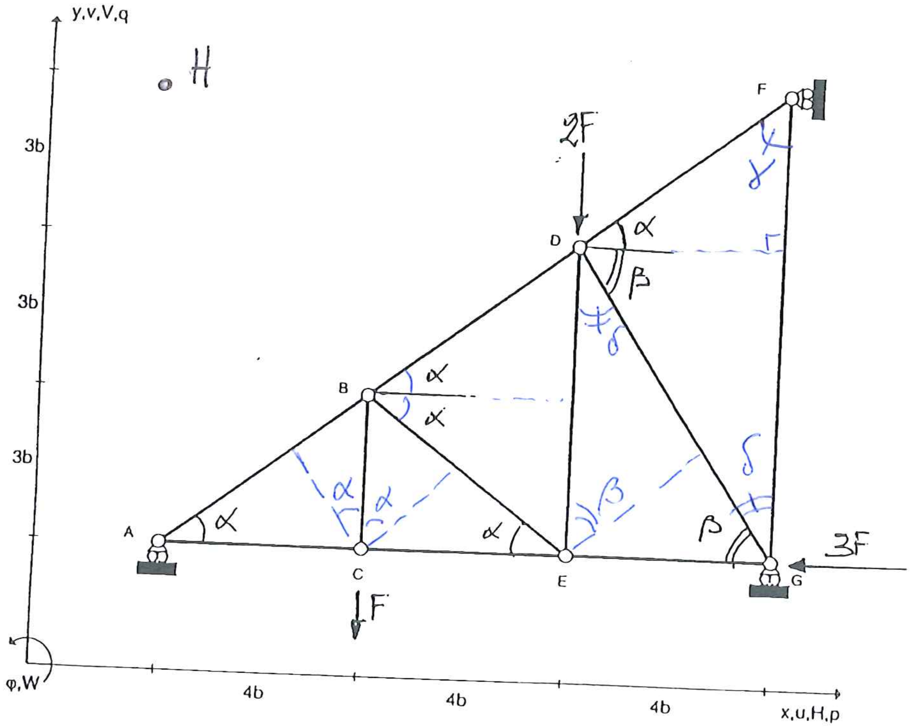
$$-4F - 16F + 47F + 12 N_{11} = 0$$

da cui $\boxed{N_{11} = -\frac{9}{4} F}$

Esercizio n.2 (8 punti)

Risolvere la struttura reticolare indicata in Figura; calcolare i valori delle azioni assiali nelle singole aste e indicare, ingrossandole nel disegno, quali si comportano come puntoni.

Si ricorda che senα=3/5, cosα=4/5, senβ=6 / $\sqrt{52}$ e cosβ= 4/ $\sqrt{52}$. $\cos\gamma = \sin\alpha$; $\sin\gamma = \cos\alpha$

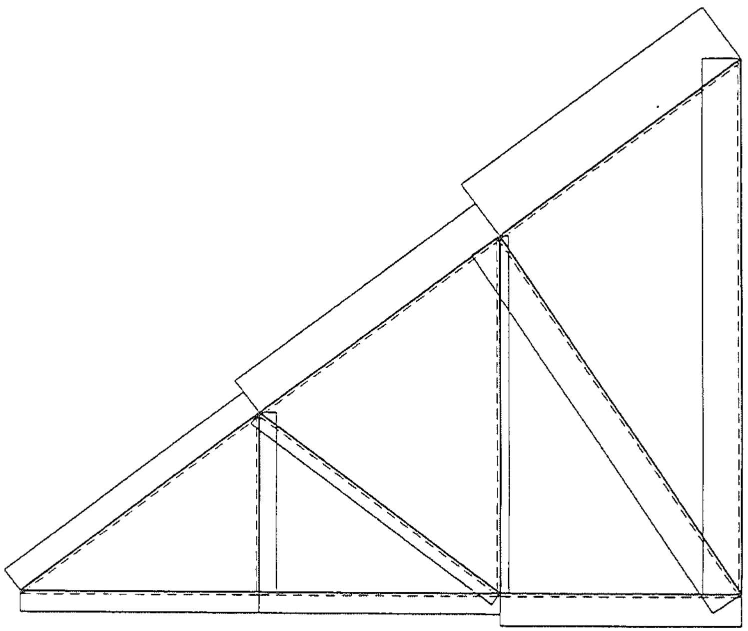


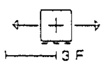
V_A = $-\frac{11}{12}F(\uparrow) = \frac{11}{12}F(\downarrow)$; V_G = $\frac{47}{12}F\ (\uparrow)$; H_F = $3F\ (\rightarrow)$;

N_{AB} = $N_1 = \frac{55}{36}F$; N_{AC} = $N_2 = -\frac{11}{9}F$; N_{BC} = $N_3 = +F$; N_{CE} = $N_6 = -\frac{11}{9}F$;

N_{BE} = $N_5 = -\frac{5}{6}F$; N_{BD} = $N_4 = \frac{85}{36}F$; N_{EG} = $N_{10} = -\frac{17}{9}F$;

N_{DG} = $N_9 = -\frac{5\sqrt{13}}{9}F$; N_{DF} = $N_8 = \frac{15}{4}F$; N_{FG} = $N_{11} = -\frac{9}{4}F$.



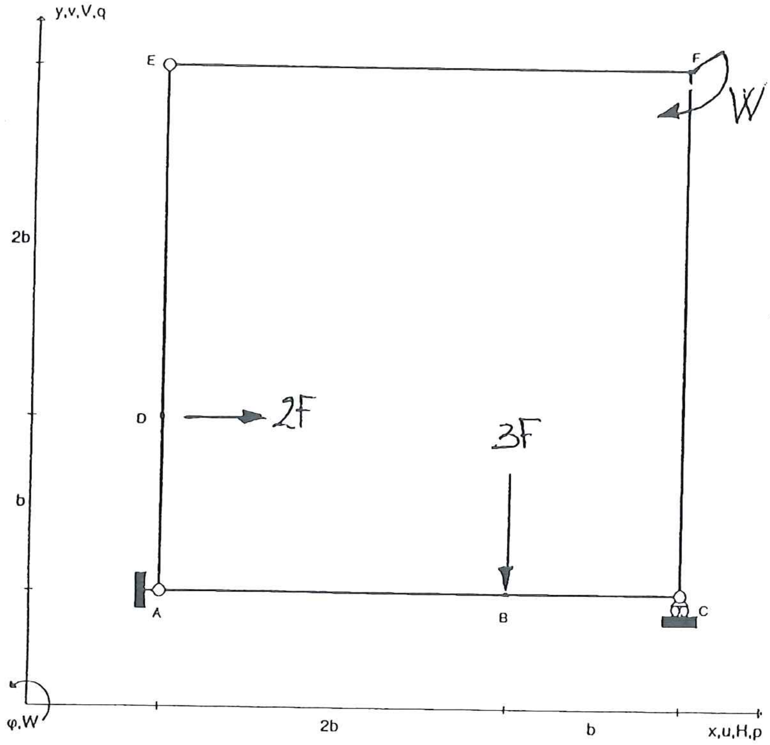


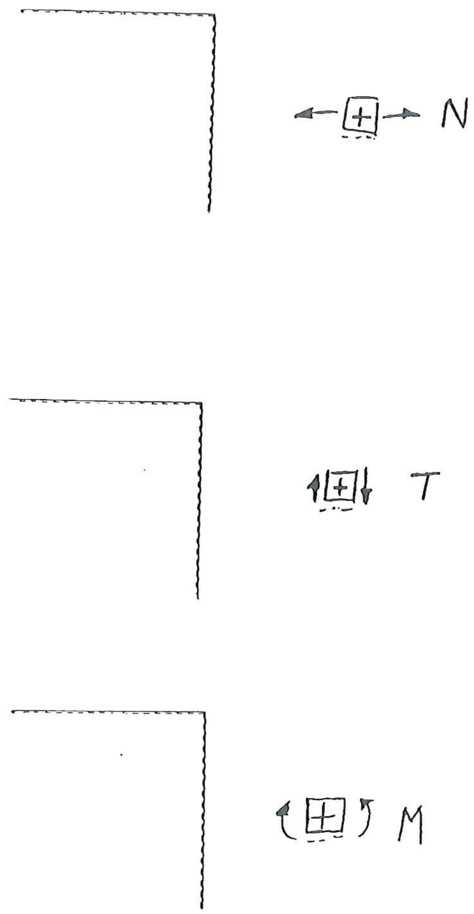
AZIONI INTERNE (coordinate locali)

$N_{AB} = 55/36F$	$N_{AC} = -11/9F$	$N_{BC} = F$	$N_{BE} = -5/6F$
$T_{AB} = 0$	$T_{AC} = 0$	$T_{BC} = 0$	$T_{BE} = 0$
$M_{AB} = 0$	$M_{AC} = 0$	$M_{BC} = 0$	$M_{BE} = 0$
$N_{CE} = -11/9F$	$N_{BD} = 85/36F$	$N_{DE} = 1/2F$	$N_{EG} = -17/9F$
$T_{CE} = 0$	$T_{BD} = 0$	$T_{DE} = 0$	$T_{EG} = 0$
$M_{CE} = 0$	$M_{BD} = 0$	$M_{DE} = 0$	$M_{EG} = 0$
$N_{DG} = -5\sqrt{13}/9F$	$N_{DF} = 15/4F$	$N_{FG} = -9/4F$	
$T_{DG} = 0$	$T_{DF} = 0$	$T_{FG} = 0$	
$M_{DG} = 0$	$M_{DF} = 0$	$M_{FG} = 0$	

Esercizio n.3 (7 punti)

Risolvere la struttura riportata in Figura e tracciare i grafici delle azioni interne sul tratto EFC.





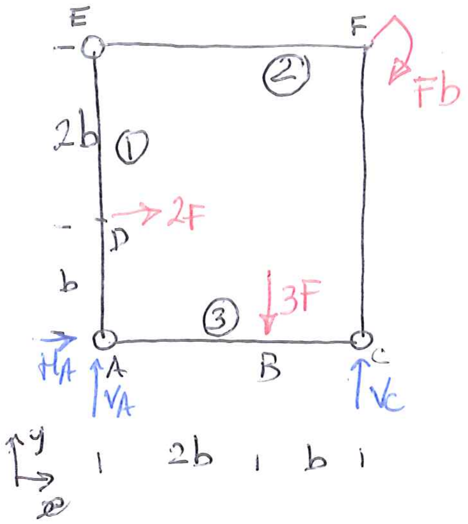
$W = Fb$

$V_A =$ ....; $H_A =$ ....; $V_C =$ ....;

$N_{FC} =$ ....; $N_{EF} =$ ....;

$T_{FC} =$ ....; $T_{EF} =$ ....;

$M_{FC} =$ ....; $M_{EF} =$ ....;



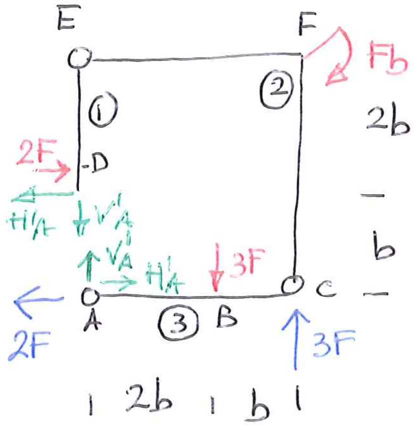
$GDL = 3 \times 3 = 9$

$GDV = 4(A) + 3(C) + 2(E) = 9$

$GDL = GDV \Rightarrow$ STRUTTURA ISOSTATICA

$$\begin{cases} \rightarrow R_x = 0 & H_A + 2F = 0 \\ \uparrow R_y = 0 & V_A - 3F + V_C = 0 \\ \circlearrowleft M_{z(A)} = 0 & -2Fb - 3F \cdot 2b + V_C \cdot 3b - Fb = 0 \end{cases}$$

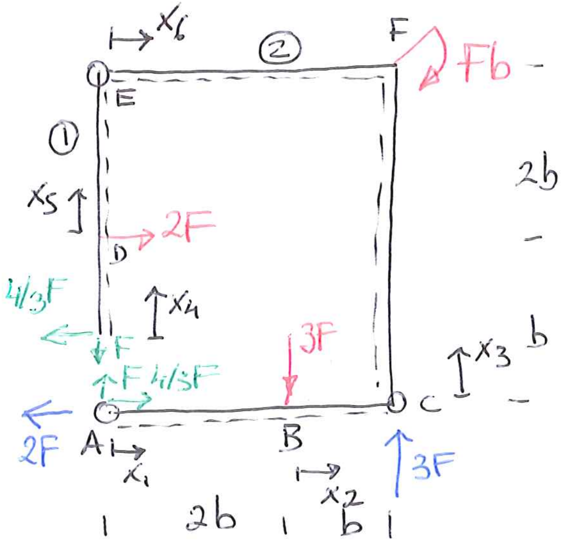
$$\begin{cases} H_A = -2F \\ V_A + V_C = 3F \\ V_C \cdot 3b = 9Fb \end{cases} \Rightarrow \begin{cases} H_A = -2F \\ V_C = 3F \\ V_A = 0 \end{cases}$$



CALCOLO REAZIONI VINCOLO INTERNO A (SULLA TRAVE ①):

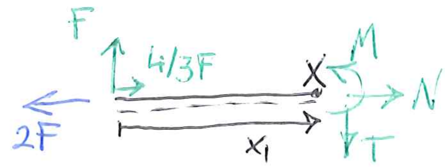
$$\begin{cases} \circlearrowleft M^{①}_{z(E)} = 0 & 2F \cdot 2b - H'_A \cdot 3b = 0 \\ \circlearrowleft M^{③}_{z(C)} = 0 & -V'_A \cdot 3b + 3Fb = 0 \end{cases} \Rightarrow \begin{cases} H'_A = \frac{4}{3}F \\ V'_A = F \end{cases}$$

LA STRUTTURA APERTA IN CONDIZIONI DI EQUILIBRIO È QUINDI:



Ⓘ $A \rightarrow B$

$0 < x_1 < 2b$



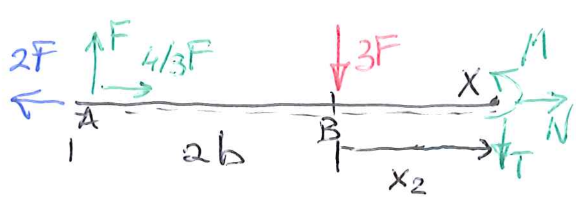
$$\boxed{\begin{aligned} N(x_1) &= +\frac{2}{3}F \\ T(x_1) &= +F \\ M(x_1) &= Fx_1 \end{aligned}}$$

$M(x_1 = 0) = 0$

$M(x_1 = 2b) = 2Fb$

Ⓘ $B \rightarrow C$

$0 < x_2 < b$



$$\boxed{\begin{aligned} N(x_2) &= +\frac{2}{3}F \\ T(x_2) &= -2F \\ M(x_2) &= 2F[b - x_2] \end{aligned}}$$

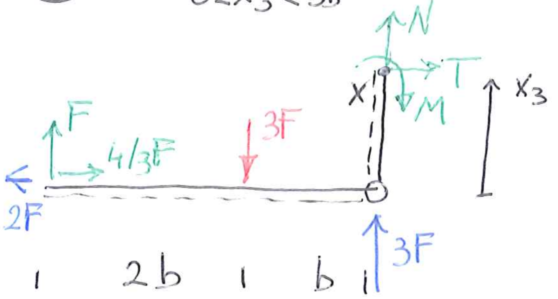
$M(x_2 = 0) = 2Fb$

$M(x_2 = b) = 0$

(III)

C → F

$0 < x_3 < 3b$



$$N(x_3) = -F$$
$$T(x_3) = \frac{2}{3}F$$
$$M(x_3) = -\frac{2}{3}F x_3$$

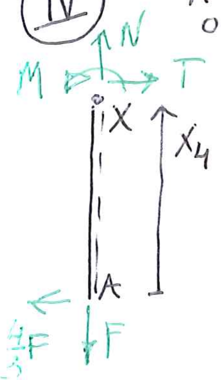
$M(x_3 = 0) = 0$

$M(x_3 = 3b) = -2Fb$

(IV)

A → D

$0 < x_4 < b$



$$N(x_4) = +F$$
$$T(x_4) = \frac{4}{3}F$$
$$M(x_4) = \frac{4}{3}F x_4$$

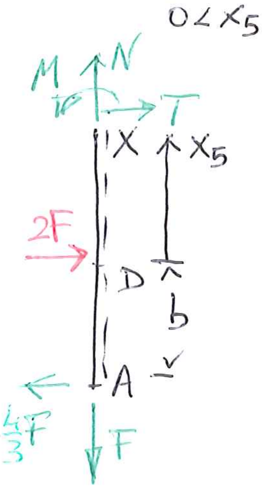
$M(x_4 = 0) = 0$

$M(x_4 = b) = \frac{4}{3}Fb$

(V)

D → E

$0 < x_5 < 2b$



$$N(x_5) = +F$$
$$T(x_5) = -\frac{2}{3}F$$
$$M(x_5) = \frac{2}{3}F[2b - x_5]$$

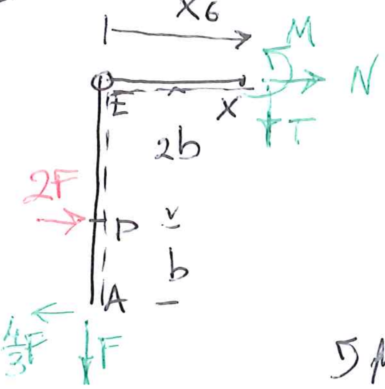
$M(x_5 = 0) = \frac{4}{3}Fb$

$M(x_5 = 2b) = 0$

(VI)

E → F

$0 < x_6 < 3b$

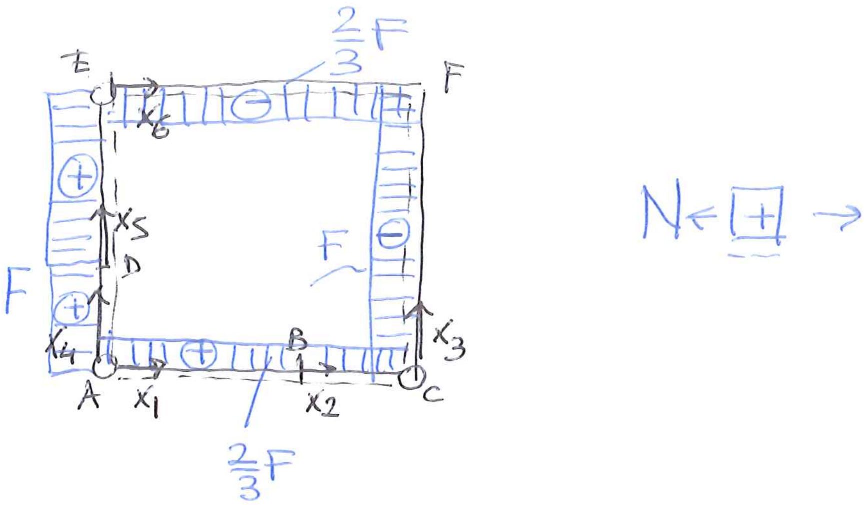


$$N(x_6) = -\frac{2}{3}F$$
$$T(x_6) = -F$$
$$M(x_6) = -F x_6$$

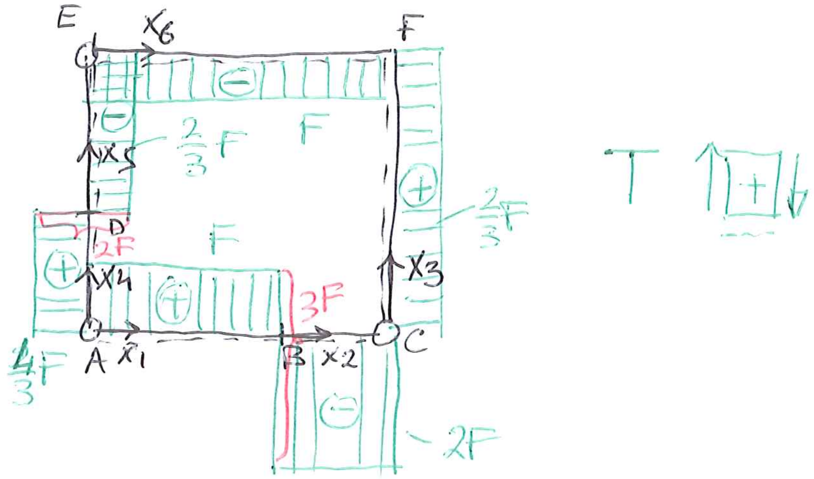
$M(x_6 = 0) = 0$

$M(x_6 = 3b) = -3Fb$.

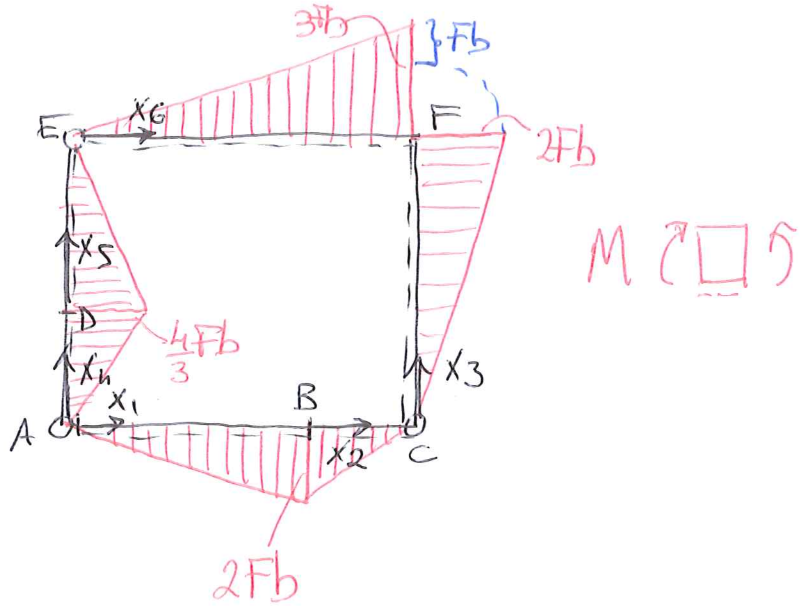
$$\sum M_z(x) = 0 \quad -\frac{4}{3}F \cdot 3b + 2F \cdot 2b + F x_6 + M(x_6) = 0$$



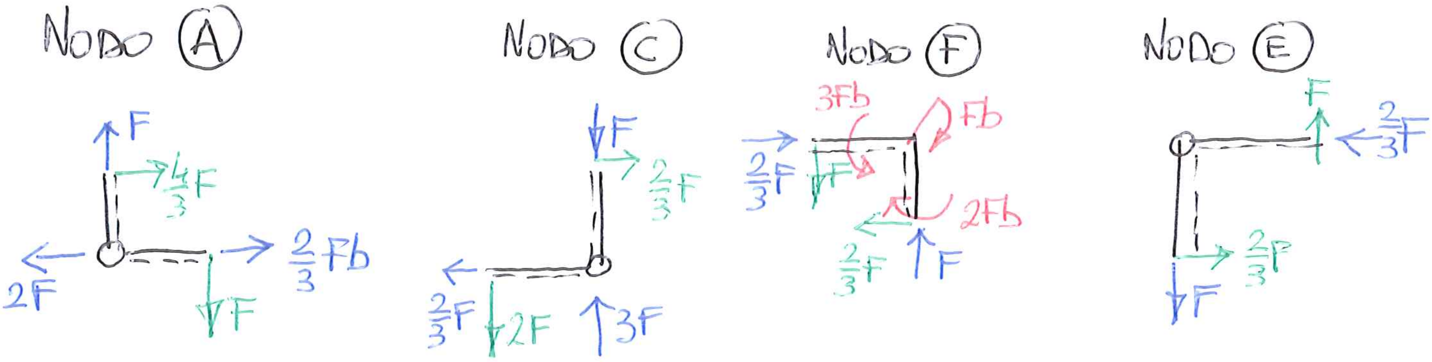
E
F
A
B
C
D
x_1
x_2
x_3
x_4
x_5
x_6
$\frac{2}{3}F$
F
F
$\frac{2}{3}F$
N



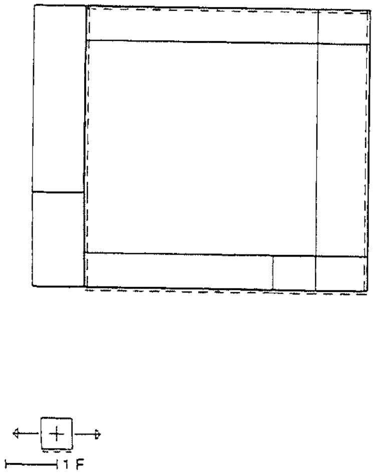
E
F
A
B
C
D
x_1
x_2
x_3
x_4
x_5
x_6
F
$\frac{2}{3}F$
$\frac{2}{3}F$
2F
F
3F
$\frac{4}{3}F$
2F
T

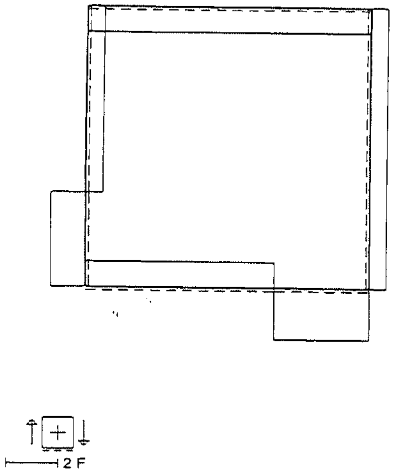


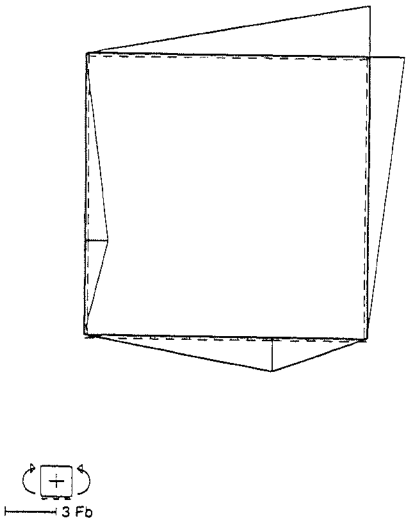
3Fb
Fb
2Fb
E
F
A
B
C
D
x_1
x_2
x_3
x_4
x_5
x_6
$\frac{4}{3}Fb$
2Fb
M



NODO (A)
NODO (C)
NODO (F)
NODO (E)
F
$\frac{4}{3}F$
2F
$\frac{2}{3}Fb$
F
F
$\frac{2}{3}F$
$\frac{2}{3}F$
2F
3F
3Fb
Fb
$\frac{2}{3}F$
F
2Fb
$\frac{2}{3}F$
F
F
$\frac{2}{3}F$
$\frac{2}{3}F$
F







AZIONI INTERNE (coordinate locali)

$N_{AB} = 2/3F$
$T_{AB} = F$
$M_{AB} = Fx$

$N_{BC} = 2/3F$
$T_{BC} = -2F$
$M_{BC} = 2Fb - 2Fx$

$N_{AD} = F$
$T_{AD} = 4/3F$
$M_{AD} = 4/3Fx$

$N_{DE} = F$
$T_{DE} = -2/3F$
$M_{DE} = 4/3Fb - 2/3Fx$

$N_{EF} = -2/3F$
$T_{EF} = -F$
$M_{EF} = -Fx$

$N_{FC} = -F$
$T_{FC} = 2/3F$
$M_{FC} = -2Fb + 2/3Fx$