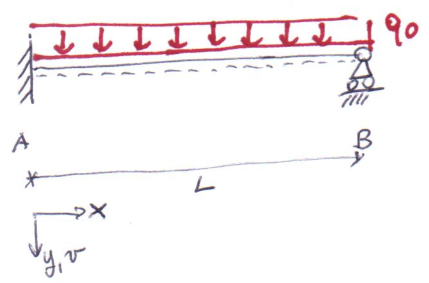
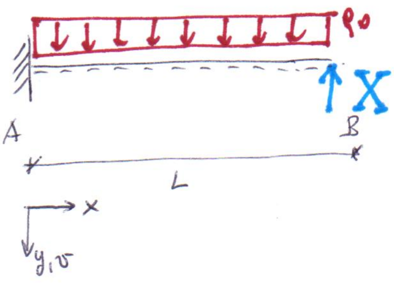
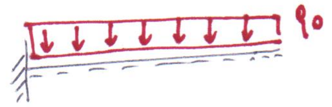
RISOLUZIONE DI STRUTTURA IPERSTATICA E CALCOLO DI COMPONENTE DI SPOSTAMENTO

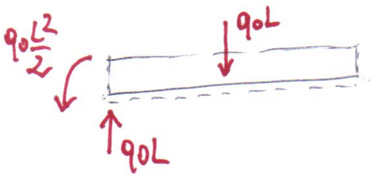




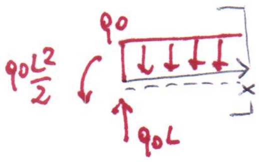
$$\boxed{v_{(B)}(q_0, X) = 0}$$

S0





$H_{0A} = 0$; $V_{0A} = q_0 L$; $M_{0A} = q_0 \frac{L^2}{2}$

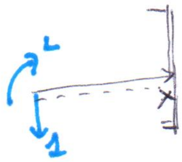


$N_0(x) = 0$; $T_0(x) = q_0 L - q_0 x = q_0(L-x)$

$M_0(x) = -q_0\frac{L^2}{2} + q_0 L x - q_0\frac{x^2}{2} = -\frac{q_0}{2}(L-x)^2$

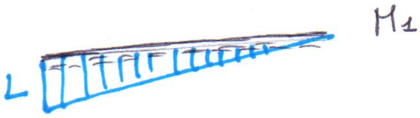
S1

$H_{1A} = 0$; $V_{1A} = -1$; $M_{1A} = -L$



$N_1(x) = 0$; $T_1(x) = -1$;

$M_1(x) = L - x$



$$\varepsilon_x = \frac{1}{EA}\left(N_0(x) + X N_1(x)\right) = 0$$

$$\gamma_x = \frac{1}{GA_T}\left(T_0(x) + X T_1(x)\right) = \frac{q_0(L-x) + X(-1)}{GA_T}$$

$$\chi_x = \frac{1}{EJ_x}\left(M_0(x) + X M_1(x)\right) = \frac{-\frac{q_0}{2}(L-x)^2 + X(L-x)}{EJ_x}$$

$\delta L_e = \delta L_i \qquad \delta L_e = 1 \cdot v_B = 1 \cdot 0 = 0 \qquad \delta L_i = 0$

$$\delta L_i = \cancel{\int_0^L N_1(x)\,\varepsilon_x\,dx} + \int_0^L T_1(x)\,\gamma_x\,dx + \int_0^L M_1(x)\,\chi_x\,dx =$$

$$= \int_0^L T_1(x)\,\frac{T_0(x) + X T_1(x)}{GA_T}\,dx + \int_0^L M_1(x)\,\frac{M_0(x) + X M_1(x)}{EJ}\,dx =$$

$$= \int_0^L -1\left(\frac{q_0(L-x) + X(-1)}{GA_T}\right)dx + \int_0^L (L-x)\left(\frac{-\frac{q_0}{2}(L-x)^2 + X(L-x)}{EJ_x}\right)dx =$$

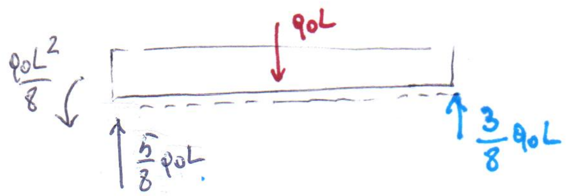
$$= \int_0^L \left(-\frac{q_0(L-x)}{GA_T} + \frac{X}{GA_T} \right) dx + \int_0^L \left(\frac{-q_0/2\,(L-x)^3}{EJ_x} + \frac{X(L-x)^2}{EJ_x} \right) dx =$$

$$= \int_0^L \frac{-q_0(L-x)}{GA_T} dx + \int_0^L \frac{-q_0(L-x)^3}{2EJ_x} dx + X \left[\int_0^L \frac{1}{GA_T} dx + \int_0^L \frac{(L-x)^2}{EJ_x} dx \right] = 0$$

$$\frac{1}{GA_T} \int_0^L q_0(L-x)\, dx + \frac{1}{2EJ_x} \int_0^L q_0(L-x)^3 dx = X \left[\frac{1}{GA_T} \int_0^L dx + \frac{1}{EJ_x} \int_0^L (L-x)^2 dx \right]$$

$$\frac{q_0 L^2}{2GA_T} + \frac{q_0 L^4}{8EJ_x} = X \left(\frac{L}{GA_T} + \frac{L^3}{3EJ_x} \right) \rightarrow X = \frac{q_0 \left(\frac{L^2}{2GA_T} + \frac{L^4}{8EJ_x} \right)}{\frac{L}{GA_T} + \frac{L^3}{3EJ_x}}$$

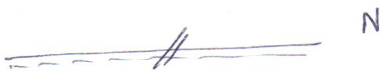
TRASCURANDO LA DEFORMABILITÁ A TAGLIO: $\boxed{X = \frac{3 q_0 L}{8}}$



$H_A = H_{0A} + X H_{1A}$ $\qquad H_A = 0$

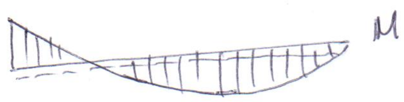
$V_A = V_{0A} + X V_{1A}$ $\qquad V_A = q_0L + \frac{3}{8}q_0L(-1) = \frac{5}{8}q_0L$

$M_A = M_{0A} + X M_{1A}$ $\qquad M_A = q_0\frac{L^2}{2} + \frac{3}{8}q_0L(-L) = \frac{q_0L^2}{8}$



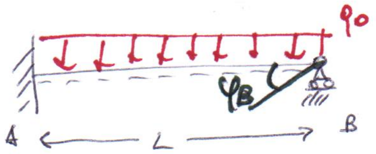
$N(x) = N_0(x) + X N_1(x)$ $\qquad N(x) = 0$

$T(x) = T_0(x) + X T_1(x)$ $\qquad T(x) = q_0(L-x) - \frac{3}{8}q_0L$



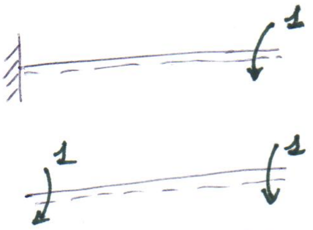
$M(x) = M_0(x) + X M_1(x)$ $\qquad M(x) = -\frac{q_0}{2}(L-x)^2 + \frac{3}{8}q_0L(L-x)$

- CALCOLO DELLA COMPONENTE DI SPOSTAMENTO $\boxed{\varphi_B}$:

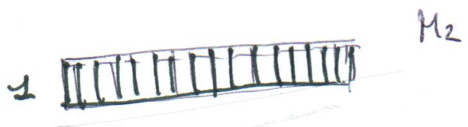


$\boxed{\varphi_B = ?}$

S2)



$H_A = 0; \quad V_A = 0; \quad M_A = -1;$



$M_2(x) = 1$

$$\varepsilon_x = \frac{1}{EA}\left(N_0(x) + \overline{X} N_1(x)\right) = \frac{1}{EA}\left(N_0(x) + \frac{3}{8} q_0 L \, N_1(x)\right)$$

$$\gamma_x = \frac{1}{GA_T}\left(T_0(x) + \overline{X} T_1(x)\right) = \frac{1}{GA_T}\left(T_0(x) + \frac{3}{8} q_0 L \, T_1(x)\right)$$

$$\chi_x = \frac{1}{EJ_x}\left(M_0(x) + \overline{X} M_1(x)\right) = \frac{1}{EJ_x}\left(M_0(x) + \frac{3}{8} q_0 L \, M_1(x)\right)$$

$$\delta\mathcal{L}_e = \delta\mathcal{L}_i \qquad \delta\mathcal{L}_e = 1 \cdot \varphi_B = \varphi_B$$

$$\delta\mathcal{L}_i = \cancel{\int_0^L N_2(x)\,\varepsilon_x\,dx} + \cancel{\int_0^L T_2(x)\,\gamma_x\,dx} + \int_0^L M_2(x)\,\chi_x\,dx =$$

$$= \int_0^L M_2(x)\,\frac{M_0(x) + \overline{X} M_1(x)}{EJ_x}\,dx = \int_0^L M_2(x)\,\frac{M_0(x) + \frac{3}{8} q_0 L \, M_1(x)}{EJ_x}\,dx =$$

$$= \int_0^L \left[\frac{M_2(x) M_0(x)}{EJ_x} + \frac{M_2(x)\,\frac{3}{8} q_0 L \, M_1(x)}{EJ_x}\right] dx =$$

$$= \frac{1}{EJ_x}\int_0^L 1\left[-\frac{q_0}{2}(L-x)^2\right] dx + \frac{1}{EJ_x}\int_0^L 1 \cdot \frac{3}{8} q_0 L\,(L-x)\,dx =$$

$$= \frac{-q_0}{2EJ_x}\left[\frac{x^3}{3}\right]_0^L + \frac{3q_0L}{8EJ_x}\left[Lx - \frac{x^2}{2}\right]_0^L =$$

$$= -\frac{q_0L^3}{6EJ_x} + \frac{3\,q_0L^3}{8EJ_x} - \frac{3\,q_0L^3}{16EJ_x} = \frac{q_0L^3}{48EJ_x}$$

$$\boxed{\varphi_B = \frac{q_0L^3}{48\,EJ_x}}$$