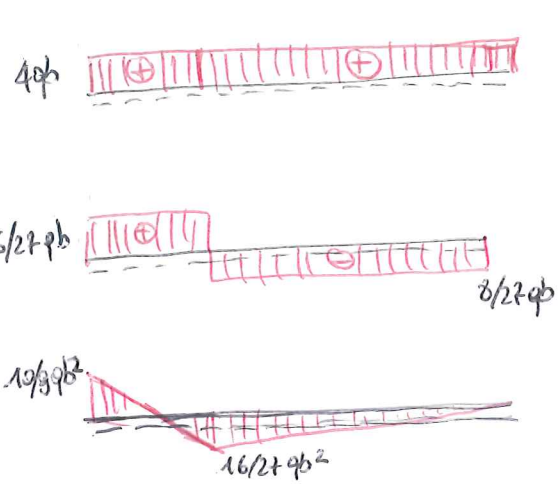
PROVA SCRITTA IN AULA, 17/04/2020, PARTE II, TEMA 5 — NOME - COGNOME - MATRICOLA - CDS

ESERCIZIO N°1

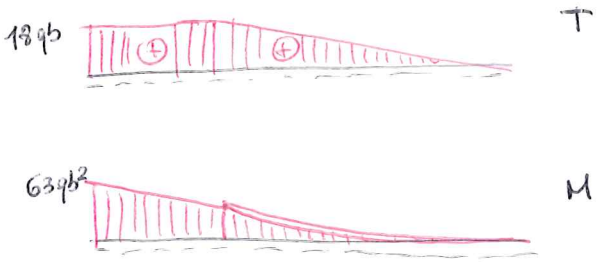


$H_A = 4qb$ $\quad V_A = 46/27\,qb$ $\quad M_A = 10/9\,qb^2$ $\quad V_C = 8/27\,qb$

$N_{AB} = 4qb$ $\quad T_{AB} = 46/27\,qb$ $\quad M_{AB} = -10/9\,qb^2 + 46/27\,qb\,(x_1)$

$N_{CB} = 4qb$ $\quad T_{CB} = -8/27\,qb$ $\quad M_{CB} = 8/27\,qb\,(x_2)$

ESERCIZIO N°2



$H_A = 0$ $\quad V_A = 18qb$ $\quad M_A = 63qb^2$

$N_{AB} = 0$ $\quad T_{AB} = 18qb$ $\quad M_{AB} = -63qb^2 + 18qb(x_1)$

$N_{BC} = 0$ $\quad T_{BC} = 18qb - 6qb x_2$ $\quad M_{BC} = -27qb^2 + 18qb x_2 + 3q x_2^2$

cc. A = $v(z_1=0) = 0$; $v'(z_1=0) = 0$

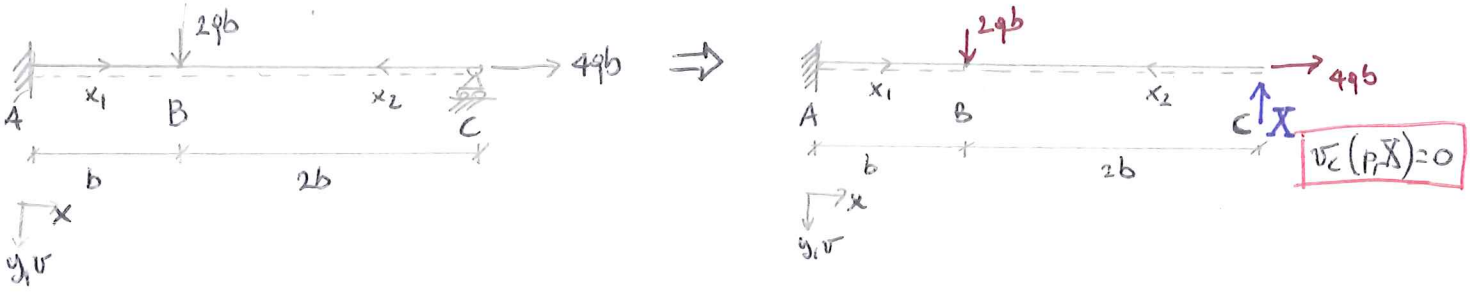
cc B = $v(z_1=2b) = v(z_2=0)$; $v'(z_1=2b) = v'(z_2=0)$

cc C = //

$$v(z_1) = +\frac{63}{2}\frac{qb^2x^2}{EJ} - \frac{3qbx^3}{EJ} \qquad v'(z_1) = +\frac{63qb^2x}{EJ} - \frac{9qbx^2}{EJ}$$

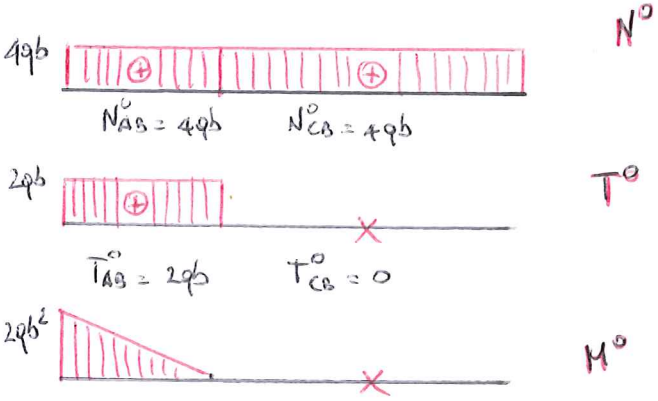
$$v(z_2) = +\frac{102qb^4}{EJ} + \frac{90qb^3x}{EJ} + \frac{27qb^2x^2}{2EJ} - \frac{3qbx^3}{EJ} + \frac{9x^4}{4EJ} \qquad v'(z_1) = +\frac{27qb^2x}{EJ} - \frac{9qbx^2}{2EJ} + \frac{9x^3}{EJ}$$

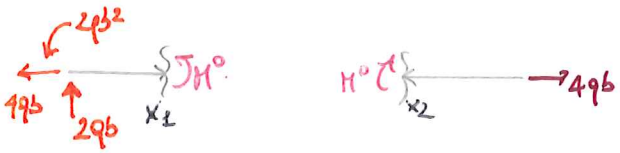
$v(B) = -102qb^4/EJ$ (1) $\qquad \varphi(B) = -90qb^3/EJ$ (2)



SP0

$$\begin{cases} \rightarrow R_x = 0 & H_A^\circ + 4qb = 0 & H_A^\circ = -4qb \\ \uparrow R_y = 0 & V_A^\circ - 2qb = 0 & V_A^\circ = 2qb \\ \sum M_{z(A)} = 0 & M_A^\circ - 2qb^2 = 0 & M_A^\circ = 2qb^2 \end{cases}$$

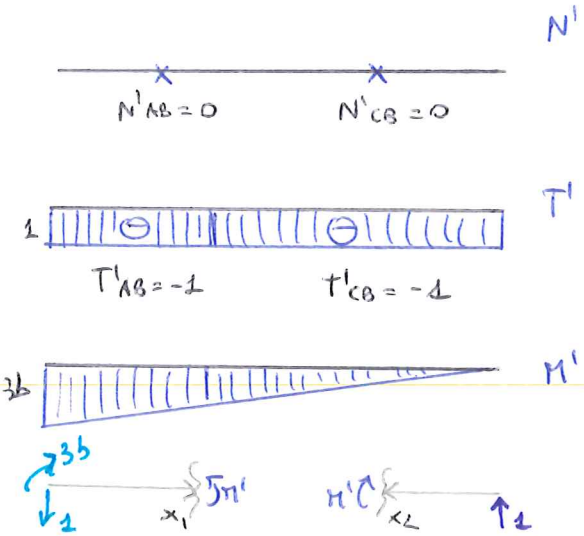




$$M_{AB}^\circ = -2qb^2 + 2qb\,x_1 \qquad M_{CB}^\circ = 0$$

SA 1

$$\begin{cases} \rightarrow R_x = 0 & H_A' = 0 & H_A' = 0 \\ \uparrow R_y = 0 & V_A' + 1 = 0 & V_A' = -1 \\ \sum M_{z(A)} = 0 & M_A' + 1 \cdot 3b = 0 & M_A' = -3b \end{cases}$$



$$M_{AB}' = 3b - 1 \cdot x_1 \qquad M_{CB}' = 1 \cdot x_2$$

$\delta\mathcal{L}_e = \delta\mathcal{L}_i$

$$\delta\mathcal{L}_e = 1 \cdot v_C = 1 \cdot 0 = 0 \qquad \delta\mathcal{L}_e = 0 \rightarrow \delta\mathcal{L}_i = 0$$

$$\delta\mathcal{L}_i = \int_0^L N'(x)\,\varepsilon_x\,dx + \int_0^L T'(x)\,\gamma_x\,dx + \int_0^L M'(x)\,\chi_x\,dx = 0$$

$$\chi_x = \frac{M^\circ(x) + X M'(x)}{EJ} \qquad \delta\mathcal{L}_i = \int_0^L M'(x)\,\frac{M^\circ(x) + X M'(x)}{EJ}\,dx = \int_0^L \frac{M'(x) M^\circ(x)}{EJ}\,dx + X \int_0^L \frac{M'(x)^2}{EJ}\,dx = 0$$

ΤΜΗΜΑ	L	M^0	M^1	$M^0 M^1$	M^{1^2}
AB	b	$-2qb^2+2qbx$	$3b-x$	$-6qb^3+2qb^2x+6qb^2x-2qbx^2$	$9b^2-6bx+x^2$
BC	2b	//	x	//	x^2

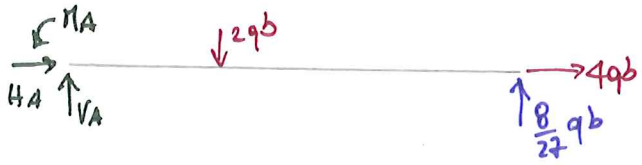
$$\delta f_i = \int_0^b \frac{1}{EJ}\left(-6qb^3+8qb^2x-2qbx^2\right)dx + X\int_0^b \frac{1}{EJ}\left(9b^2-6bx+x^2\right)dx + X\int_0^b \frac{1}{EJ}\left(x^2\right)dx = 0$$

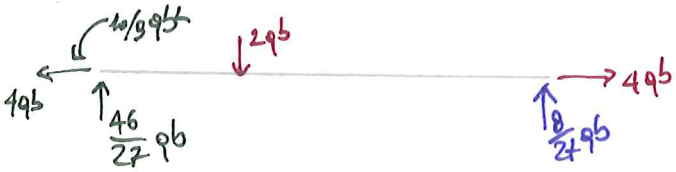
$$\delta f_i = \frac{1}{EJ}\left[-6qb^3x+8qb^2\frac{x^2}{2}-2qb\frac{x^3}{3}\right]_0^b + \frac{1}{EJ}\left[X\left[9b^2x-6b\frac{x^2}{2}+\frac{x^3}{3}\right]_0^b + X\left[\frac{x^3}{3}\right]_0^{2b}\right] = 0$$

$$\delta f_i = \frac{1}{EJ}\left(-6qb^4+4qb^4-\frac{2}{3}qb^4\right) + \frac{1}{EJ}\left[X\left(9b^3-3b^3+\frac{b^3}{3}+\frac{8b^3}{3}\right)\right] = 0$$

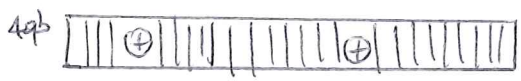
$$\delta f_i = -\frac{8qb^4}{3EJ} + \frac{9b^3X}{EJ} = 0 \qquad X\frac{9b^3}{EJ} = \frac{8qb^4}{3EJ} \qquad X = \frac{8qb^4}{3EJ}\cdot\frac{EJ}{9b^3}$$

$$\boxed{X = \frac{8}{27}qb}$$

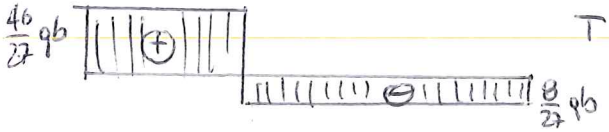




$$\begin{cases} H_A = H_A^0 + X H_A^1 & H_A = 4qb \\ V_A = V_A^0 + X V_A^1 = 2qb + \left(\frac{8}{27}qb\right)(-1) = \frac{46}{27}qb = V_A & \\ M_A = M_A^0 + X M_A^1 = 2qb^2 + \left(\frac{8}{27}qb\right)(-3b) = \frac{10}{9}qb^2 = M_A & \end{cases}$$

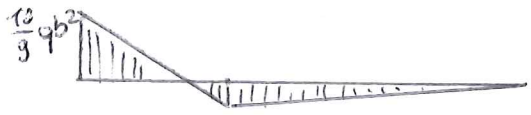


$$N = N^0 + X N^1 \qquad N_{AB} = 4qb \qquad N_{CB} = 4qb$$



$$T = T^0 + X T^1 \qquad T_{AB} = 2qb + \frac{8}{27}qb(-1) = \frac{46}{27}qb$$

$$T_{CB} = 0 + \frac{8}{27}qb(-1) = -\frac{8}{27}qb$$

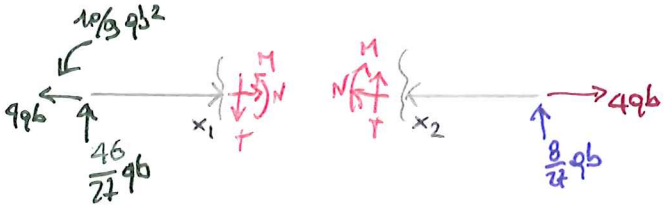


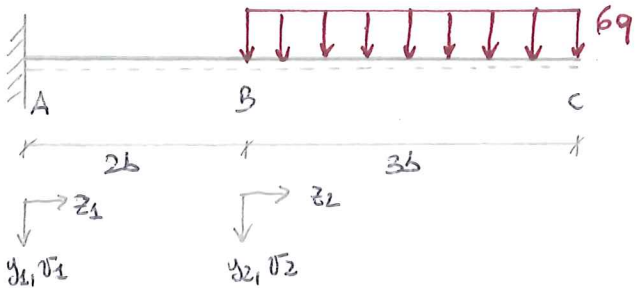
$$M = M^0 + X M^1 \qquad M_{AB} = -2qb^2 + 2qbx_1 + \frac{8}{27}qb(3b-x_1) =$$

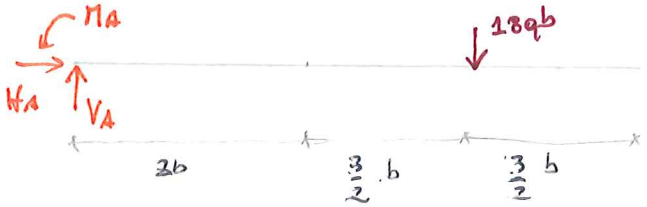
$$= -2qb^2 + 2qbx_1 + \frac{8}{9}qb^2 - \frac{8}{27}qbx_1 =$$

$$= \frac{10}{9}qb^2 - \frac{46}{27}qbx_1$$

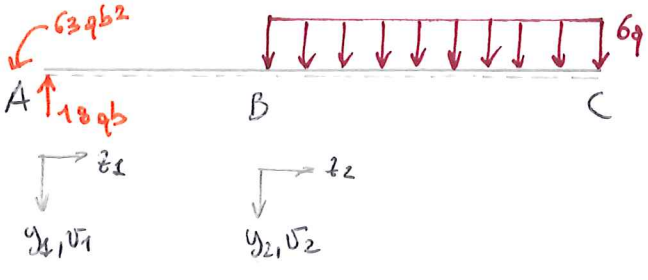
$$M_{CB} = 0 + \frac{8}{27}qb(x_2) = \frac{8}{27}qbx_2$$

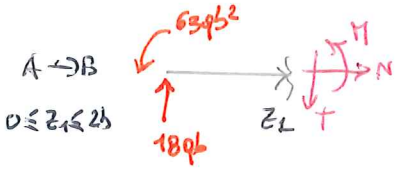


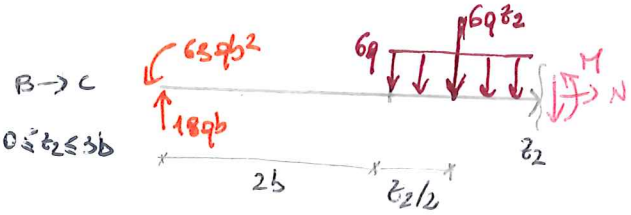




$$\begin{cases} \rightarrow R_x = 0 & H_A = 0 \\ \uparrow R_y = 0 & V_A - 18qb = 0 \quad V_A = 18qb \\ \sum M_{z(A)} = 0 & M_A - 18qb\,\frac{7}{2}b = 0 \quad M_A = 63qb^2 \end{cases}$$



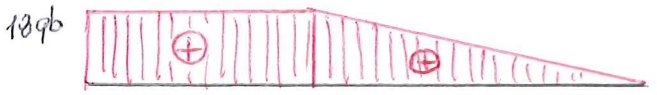




N

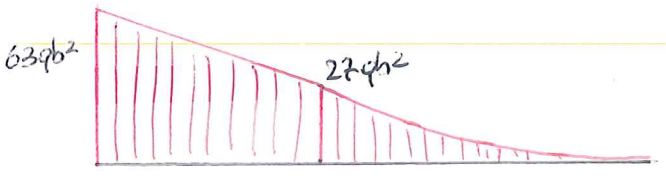
$N_{AB} = 0 \qquad N_{BC} = 0$

T



$T_{AB} = 18qb \qquad T_{BC} = 18qb - 6q z_2$

M



$M_{AB} = -63qb^2 + 18qb\,z_1$

$M_{CB} = -63qb^2 + 36qb^2 + 18qb\,z_2 - 6q z_2 \frac{z_2}{2} =$

$= -27qb^2 + 18qb\,z_2 - 3q z_2^2$

$M_x(z_1) = -63qb^2 + 18qb\,z_1 \qquad 0 \le z_1 \le 2b \qquad A \to B$

$M_x(z_2) = -27qb^2 + 18qb\,z_2 - 3qz_2^2 \qquad 0 \le z_2 \le 3b \qquad B \to C$

$v = v_1 \cup v_2$

$$\left.\begin{aligned} v_1''(z_1) &= +\frac{63qb^2}{EJ} - \frac{18qb}{EJ} z_1 \\ v_1'(z_1) &= +\frac{63qb^2}{EJ} z_1 - \frac{18qb}{EJ}\frac{z_1^2}{2} + A_1 \\ v_1(z_1) &= +\frac{63qb^2}{EJ}\frac{z_1^2}{2} - \frac{18qb}{2EJ}\frac{z_1^3}{3} + A_1 z_1 + A_2 \end{aligned}\right\} \quad 0 \le z_1 \le 2b$$

$$\left.\begin{aligned} v_2''(z_2) &= +\frac{27qb^2}{EJ} - \frac{18qb}{EJ} z_2 + \frac{3q}{EJ} z_2^2 \\ v_2'(z_2) &= +\frac{27qb^2}{EJ} z_2 - \frac{18qb}{EJ}\frac{z_2^2}{2} + \frac{3q}{EJ}\frac{z_2^3}{3} + B_1 \\ v_2(z_2) &= +\frac{27qb^2}{EJ}\frac{z_2^2}{2} - \frac{18qb}{2EJ}\frac{z_2^3}{3} + \frac{3q}{3EJ}\frac{z_2^4}{4} + B_1 z_2 + B_2 \end{aligned}\right\} \quad 0 \le z_2 \le 3b$$

CONDIZIONI A CONTORNO:

IN (A) (VINCOLO ESTERNO = INCASTRO) $\begin{cases} v_1(z_1 = 0) = 0 \\ v_1'(z_1 = 0) = 0 \end{cases}$

IN (B) (VINCOLO INTERNO = CERNIERA) $\begin{cases} v_1(z_1 = 2b) = v_2(z_2 = 0) \\ v_1'(z_1 = 2b) = v_2'(z_2 = 0) \end{cases}$

N.B. IN (C) ESTREMO LIBERO → NON POSSIAMO DIRE NIENTE SULLE C.C.

$v_1(z_1 = 0) = 0 \rightarrow \boxed{A_2 = 0}$

$v_1'(z_1 = 0) = 0 \rightarrow \boxed{A_1 = 0}$

$v_1'(z_1 = 2b) = v_2'(z_2 = 0) \rightarrow B_1 = \frac{63qb^2}{EJ} 2b - \frac{18qb}{2EJ}(2b)^2 + A_1$

$B_1 = \frac{126qb^3}{EJ} - \frac{36qb^3}{EJ} \rightarrow \boxed{B_1 = \frac{90qb^3}{EJ}}$

$v_1(z_1 = 2b) = v_2(z_2 = 0) \rightarrow B_2 = \frac{63qb^2}{2EJ}(2b)^2 - \frac{18qb}{6EJ}(2b)^3 + A_1 2b + A_2$

$B_2 = \frac{126qb^4}{EJ} - \frac{24qb^4}{EJ} \rightarrow \boxed{B_2 = \frac{102qb^4}{EJ}}$

LA LINEA ELASTICA ANALITICA

$$v_1(z_1) = \frac{63qb^2}{2EJ} z_1^2 - \frac{3qb}{EJ} z_1^3 \qquad 0 \le z_1 \le 2b$$

$$v_2(z_2) = \frac{27qb^2}{2EJ} z_2^2 - \frac{3qb}{EJ} z_2^3 + \frac{q}{4EJ} z_2^4 + \frac{90qb^3}{EJ} z_2 + \frac{102qb^4}{EJ} \qquad 0 \le z_2 \le 3b$$

$$v_1'(z_1) = \frac{63qb^2}{EJ} z_1 - \frac{9qb}{EJ} z_1^2 \qquad 0 \le z_1 \le 2b$$

$$v_2'(z_2) = \frac{27qb^2}{EJ} z_2 - \frac{9qb}{EJ} z_2^2 + \frac{q}{EJ} z_2^3 + \frac{90qb^3}{EJ}$$

$$v_B = v_1(z_1 = 2b) = \frac{63qb^2}{2EJ}(2b)^2 - \frac{3qb}{EJ}(2b)^3 = \frac{126qb^4}{EJ} - \frac{24qb^4}{EJ} = \frac{102qb^4}{EJ} \quad (\downarrow)$$

$$\varphi_C = v_2'(z_2 = 3b) = \frac{27qb^2}{EJ}(3b) - \frac{9qb}{EJ}(3b)^2 + \frac{q}{EJ}(3b)^3 + \frac{90qb^3}{EJ} =$$

$$= \frac{81qb^3}{EJ} - \frac{81qb^3}{EJ} + \frac{27qb^3}{EJ} + \frac{90qb^3}{EJ} = \frac{117qb^3}{EJ}$$