Università degli Studi di Cagliari

DICAAR - Facoltà di Ingegneria-Architettura

CORSO DI STATICA E SCIENZA DELLE COSTRUZIONI

A.A. 2019-2020

Prova scritta in aula del 17.04.2020

Parte II - Testo 1

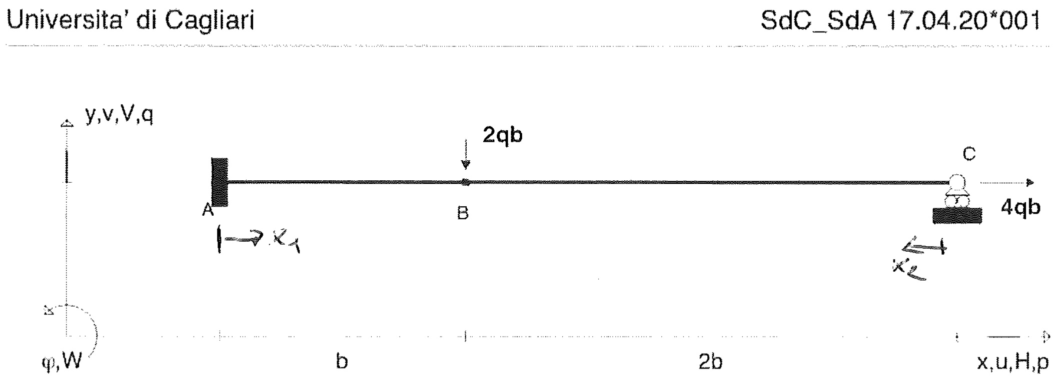
CdS **Edilizia** □ CdS **AdC** □ CdS **SdA** □

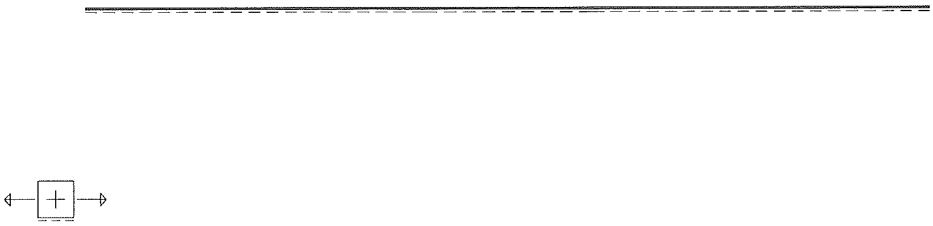
Nota: I risultati numerici vanno riportati a penna su questo stesso foglio, nei riquadri predisposti; i calcoli (in forma ordinata) vanno allegati sui soli fogli a quadretti che sono stati forniti.

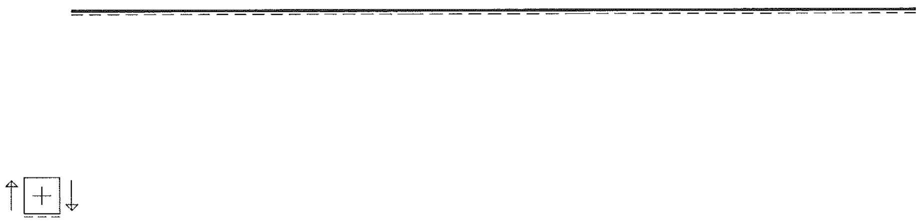
Allievo:.................................... e-mail:....................... Matricola:.......

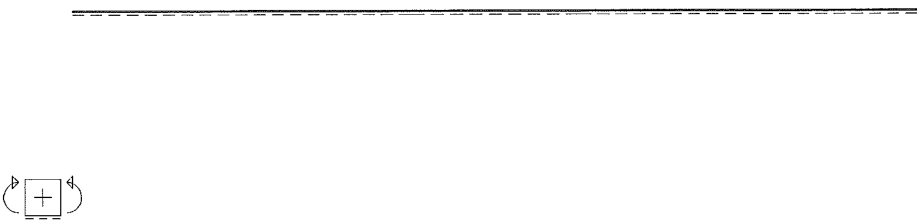
Esercizio n. 1 (19 punti)

Risolvere mediante il Principio dei Lavori Virtuali (PLV) la struttura iperstatica riportata in Figura, assumendo, come incognita iperstatica, l'appoggio in C, V_C.
Dopo avere determinato l'iperstatica *tenendo conto solo della deformabilità flessionale*, calcolare le reazioni vincolari, le equazioni delle azioni interne e tracciare nello spazio predisposto nella pagina a fronte i corrispondenti grafici.





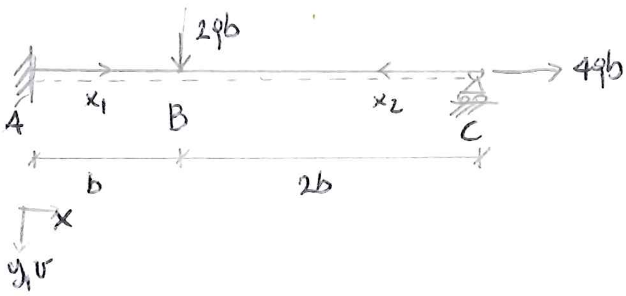




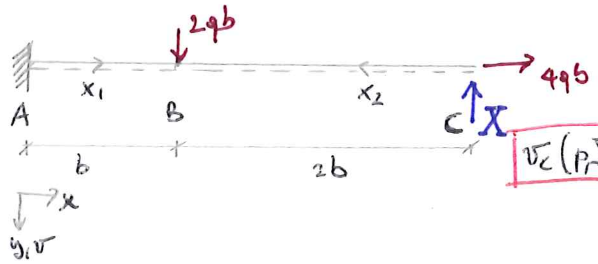
H_A (⇨) = ...; V_A (⇧) = ...; M_A (↻) = ...; V_C (⇧) = ...

N_{AB} = ...; T_{AB} = ...; M_{AB} = ...;

N_{CB} = ...; T_{CB} = ...; M_{CB} = ...;



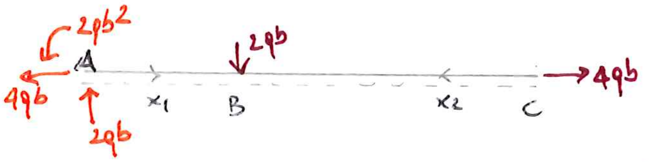
$\Rightarrow$

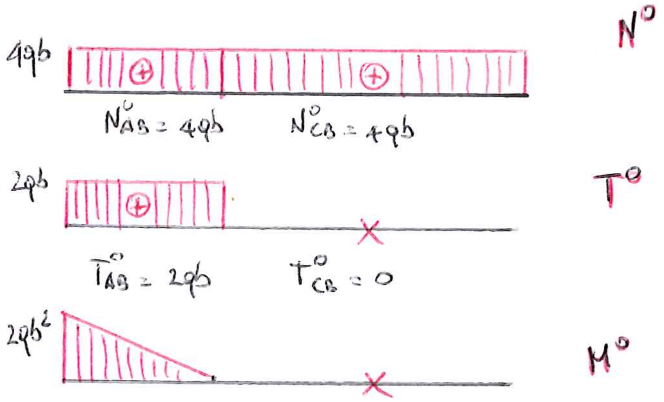


$\boxed{v_C(P,X)=0}$

SP₀

$$\begin{cases} \rightarrow R_x = 0 & H_A^0 + 4qb = 0 & \underline{H_A^0 = -4qb} \\ \uparrow R_y = 0 & V_A^0 - 2qb = 0 & \underline{V_A^0 = 2qb} \\ \circlearrowleft M_{z(A)} = 0 & M_A^0 - 2qb^2 = 0 & \underline{M_A^0 = 2qb^2} \end{cases}$$





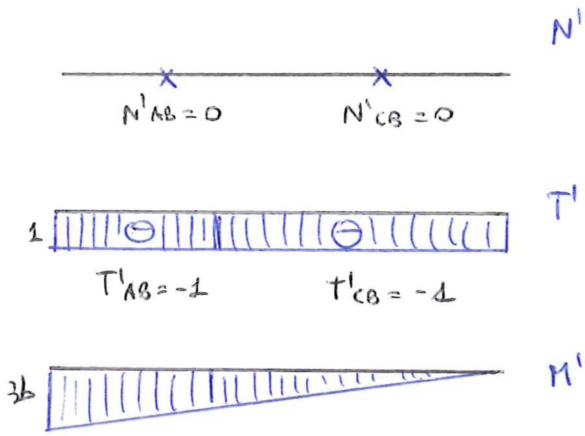
$N^0_{AB} = 4qb$ $\quad N^0_{CB} = 4qb$

$T^0_{AB} = 2qb$ $\quad T^0_{CB} = 0$

$M^0_{AB} = -2qb^2 + 2qb\, x_1$ $\qquad M^0_{CB} = 0$

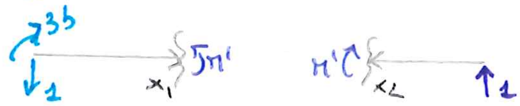
SA 1

$$\begin{cases} \rightarrow R_x = 0 & H_A' = 0 & \underline{H_A' = 0} \\ \uparrow R_y = 0 & V_A' + 1 = 0 & \underline{V_A' = -1} \\ \circlearrowleft M_{z(A)} = 0 & M_A' + 1 \cdot 3b = 0 & \underline{M_A' = -3b} \end{cases}$$



$N'_{AB} = 0$ $\quad N'_{CB} = 0$

$T'_{AB} = -1$ $\quad T'_{CB} = -1$



$M'_{AB} = 3b - 1 \cdot x_1$ $\qquad M'_{CB} = 1 \cdot x_2$

$\boxed{\delta \mathcal{L}_e = \delta \mathcal{L}_i}$

$\delta \mathcal{L}_e = 1 \cdot v_c = 1 \cdot 0 = 0$ $\qquad \delta \mathcal{L}_e = 0 \rightarrow \delta \mathcal{L}_i = 0$

$$\delta \mathcal{L}_i = \int_0^L \cancel{N'(x)\, \varepsilon_x}\, dx + \int_0^L \cancel{T'(x)\, \gamma_x}\, dx + \int_0^L M'(x)\, \chi_x\, dx = 0$$

$$\chi_x = \frac{M^0(x) + X M'(x)}{EJ} \qquad \delta \mathcal{L}_i = \int_0^L M'(x) \frac{M^0(x) + X M'(x)}{EJ} dx = \int_0^L \frac{M'(x) M^0(x)}{EJ} dx + X \int_0^L \frac{M'(x)^2}{EJ} dx = 0$$

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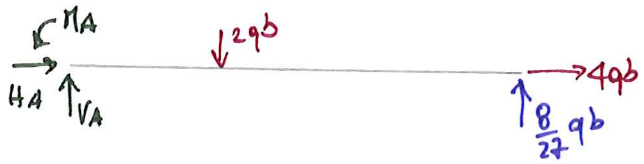
TRATTO	L	M^0	M^1	$M^0 M^1$	$M^{1\,2}$
AB	b	$-2qb^2+2qbx$	$3b-x$	$-6qb^3+2qb^2x+6qb^2x-2qbx^2$	$9b^2-6bx+x^2$
BC	$2b$	//	x	//	x^2

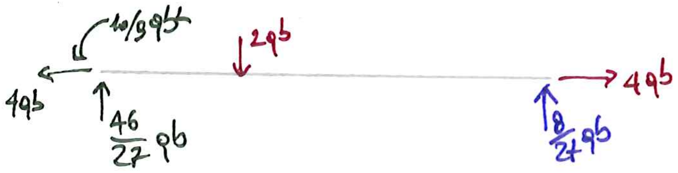
$$\delta \mathcal{L}_i = \int_0^b \frac{1}{EJ}\left(-6qb^3+8qb^2x-2qbx^2\right)dx + X\int_0^b \frac{1}{EJ}\left(9b^2-6bx+x^2\right)dx + X\int_0^{2b}\frac{1}{EJ}\left(x^2\right)dx = 0$$

$$\delta \mathcal{L}_i = \frac{1}{EJ}\left[-6qb^3x+8qb^2\frac{x^2}{2}-2qb\frac{x^3}{3}\right]_0^b + \frac{1}{EJ}\left[X\left[9b^2x-6b\frac{x^2}{2}+\frac{x^3}{3}\right]_0^b + X\left[\frac{x^3}{3}\right]_0^{2b}\right] = 0$$

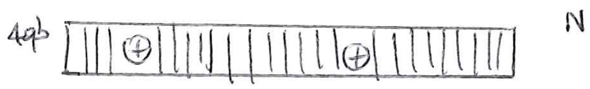
$$\delta \mathcal{L}_i = \frac{1}{EJ}\left(-6qb^4+4qb^4-\frac{2}{3}qb^4\right) + \frac{1}{EJ}\left[X\left(9b^3-3b^3+\frac{b^3}{3}+\frac{8b^3}{3}\right)\right] = 0$$

$$\delta \mathcal{L}_i = -\frac{8\,qb^4}{3EJ} + \frac{9b^3 X}{EJ} = 0 \qquad X\frac{9b^3}{EJ} = \frac{8\,qb^4}{3EJ} \qquad X = \frac{8\,qb^4}{3\cancel{EJ}}\cdot\frac{\cancel{EJ}}{9b^3} \qquad \boxed{X = \frac{8}{27}qb}$$

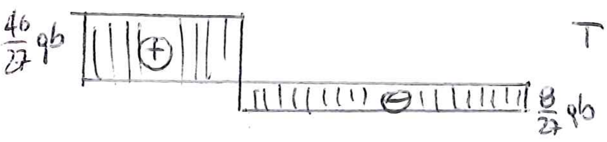




$$\begin{cases} H_A = H_A^0 + X\cancel{H_A^1} & H_A = 4qb \\ V_A = V_A^0 + X V_A^1 = 2qb + \left(\frac{8}{27}qb\right)(-1) = \frac{46}{27}qb = V_A \\ M_A = M_A^0 + X M_A^1 = 2qb^2 + \left(\frac{8}{27}qb\right)(-3b) = \frac{10}{9}qb^2 = M_A \end{cases}$$

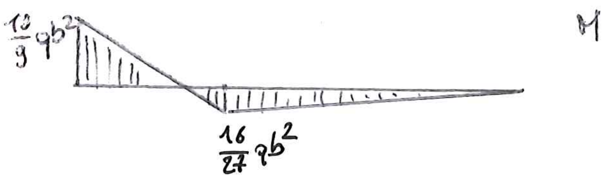


$N = N^0 + X\cancel{N^1}$ $\quad N_{AB} = 4qb \quad N_{CB} = 4qb$



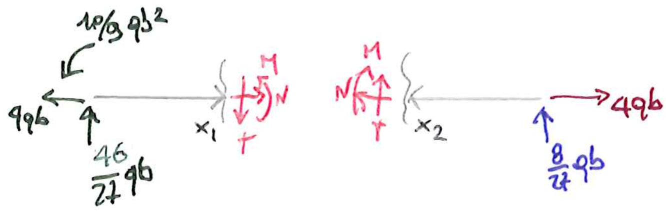
$T = T^0 + X T^1$ $\quad T_{AB} = 2qb + \frac{8}{27}qb(-1) = \frac{46}{27}qb$

$T_{CB} = 0 + \frac{8}{27}qb(-1) = -\frac{8}{27}qb$



$M = M^0 + X M^1$

$$M_{AB} = -2qb^2 + 2qbx_1 + \frac{8}{27}qb(3b-x_1) =$$
$$= -2qb^2 + 2qbx_1 + \frac{8}{9}qb^2 - \frac{8}{27}qbx_1 =$$
$$= -\frac{10}{9}qb^2 + \frac{46}{27}qbx_1$$



$$M_{CB} = 0 + \frac{8}{27}qb(x_2) = \frac{8}{27}qbx_2$$