Institut für Regelungs- und Automatisierungstechnik
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Sliding Mode Control: Basic Theory, Advances and Applications

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Lecture 5

Applications of 2nd Order Sliding Mode Control to Mechanical Systems

- Simulation of constrained Lagrangian systems
- Overhead crane control
- Contact force regulation in High-Speed train pantographs

L5 – Simulation of DAE systems

Differential Algebraic Equations are often used to model constrained Lagrange-Hamiltonian systems

$$\begin{cases} \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) + \mathbf{k}(\mathbf{q}) = \boldsymbol{\tau} + \left[\mathbf{J}_{\mathbf{q}}^{\boldsymbol{\Phi}}(\mathbf{q})\right]^{\mathrm{T}} \boldsymbol{\lambda} \\ \boldsymbol{\Phi}(\mathbf{q}) = \mathbf{0} \end{cases}$$

$\mathbf{q} \in \mathrm{R}^n$ is the vector of generalized Lagrange position coordinates

$\boldsymbol{\tau} \in \mathrm{R}^n$ is the vector of the generalized input forces

$\boldsymbol{\lambda} \in \mathrm{R}^m$ is the vector of the Lagrange multipliers (constraint reaction forces)

$[\mathbf{J}_{\mathbf{q}}^{\Phi}]^{\mathrm{T}}\boldsymbol{\lambda}$ is the vector of the generalized reaction forces

$\mathbf{M}$ is inertia matrix

$\mathbf{C}$ term takes into account for the Coriolis and inertial forces

$\mathbf{g}$ represents the gravitational (potential) forces

$\mathbf{k}$ represents the elastic forces

$\boldsymbol{\Phi}$: $\mathbf{R}^n \rightarrow \mathbf{R}^m$ is a smooth function representing the constraint

L5 – Simulation of DAE systems

Differentiating twice the constraint function the Lagrange multipliers appears (the relative degree between the constraint and the Lagrange multipliers is 2

$$\ddot{\mathbf{\Phi}}(\mathbf{q}) = \mathbf{f}(\mathbf{q}, \dot{\mathbf{q}}, \boldsymbol{\tau}) + \mathbf{J}_{\mathbf{q}}^{\mathbf{\Phi}}(\mathbf{q}) \cdot \mathbf{M}^{-1}(\mathbf{q}) \left[\mathbf{J}_{\mathbf{q}}^{\mathbf{\Phi}}(\mathbf{q})\right]^{\mathrm{T}} \cdot \boldsymbol{\lambda}$$

The matrix left multiplying the Lagrange multipliers is definite positive by construction and by the properties of Lagrange-Hamiltonian systems

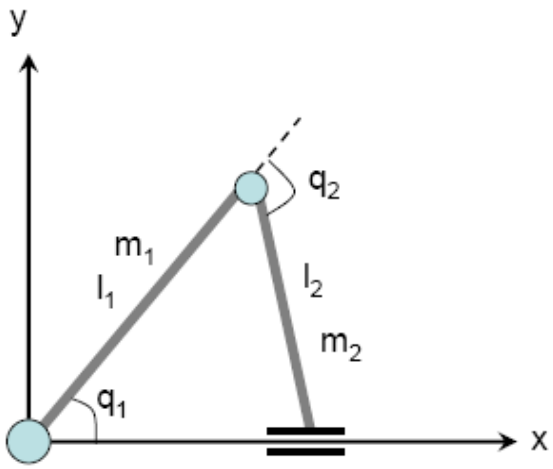
The constraint function can be considered to defining a sliding variable to be constrained to zero by means of the "control" λ

$\mathbf{s} = \dot{\mathbf{\Phi}}(\mathbf{q}) + c\mathbf{\Phi}(\mathbf{q})$ Classic 1-SMC+LP filter or Super-Twisting 2-SMC

$\mathbf{s} = \mathbf{\Phi}(\mathbf{q})$ 2-SMC+LP filter

L5 – Simulation of DAE systems

Example: Constrained planar manipulator



$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} 0.83 + 0.5cos(q_2) & 0.16 + 0.25cos(q_2) \\ 0.16 + 0.25cos(q_2) & 0.16 \end{bmatrix},$$

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} -1.01\dot{q}_2 sin(q_2) & 1.01(\dot{q}_1 + \dot{q}_2)sin(q_2) \\ 1.01\dot{q}_1 sin(q_2) & 0 \end{bmatrix},$$

$$\mathbf{g}(\mathbf{q}) = \begin{bmatrix} 9.8cos(q_1) + 2.45cos(q_1 + q_2) \\ 2.45cos(q_1 + q_2) \end{bmatrix}, \qquad \mathbf{k}(\mathbf{q}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

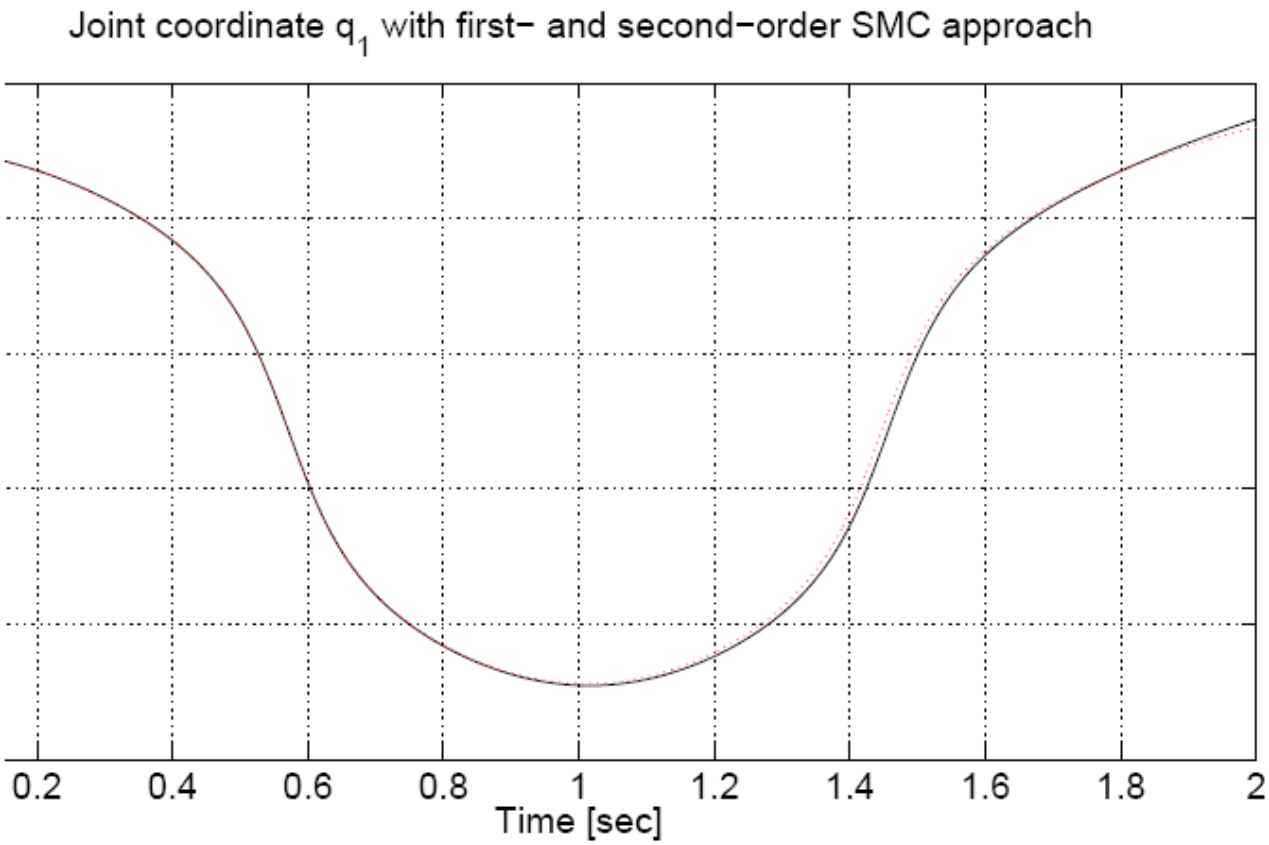
$$\Phi(\mathbf{q}) = l_1 \sin(q_1) + l_2 \sin(q_1 + q_2) = 0$$

The manipulator is forced by the unique torque applied to the first arm

$$\boldsymbol{\tau} = \begin{bmatrix} \sin(t) \\ 0 \end{bmatrix}$$

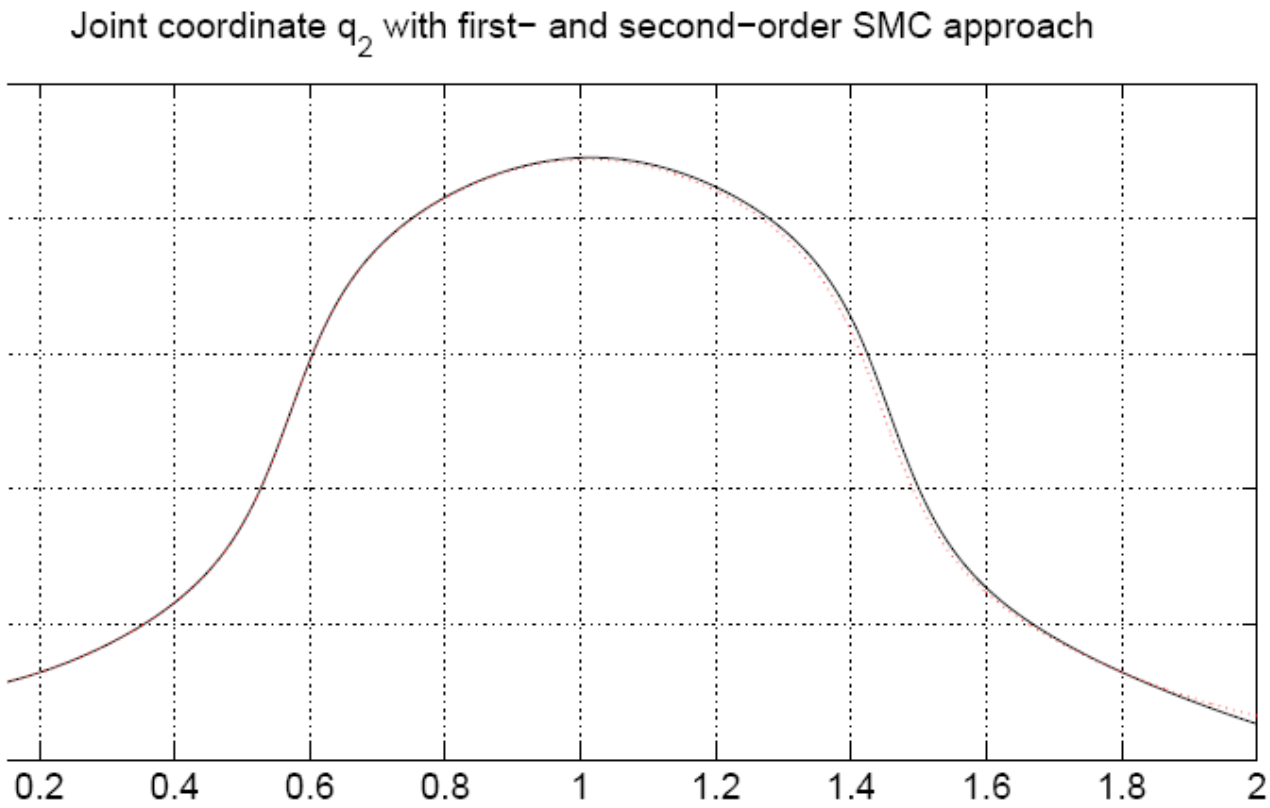
L5 – Simulation of DAE systems

Example: Constrained planar manipulator ($T_s=10^{-5}$ s)



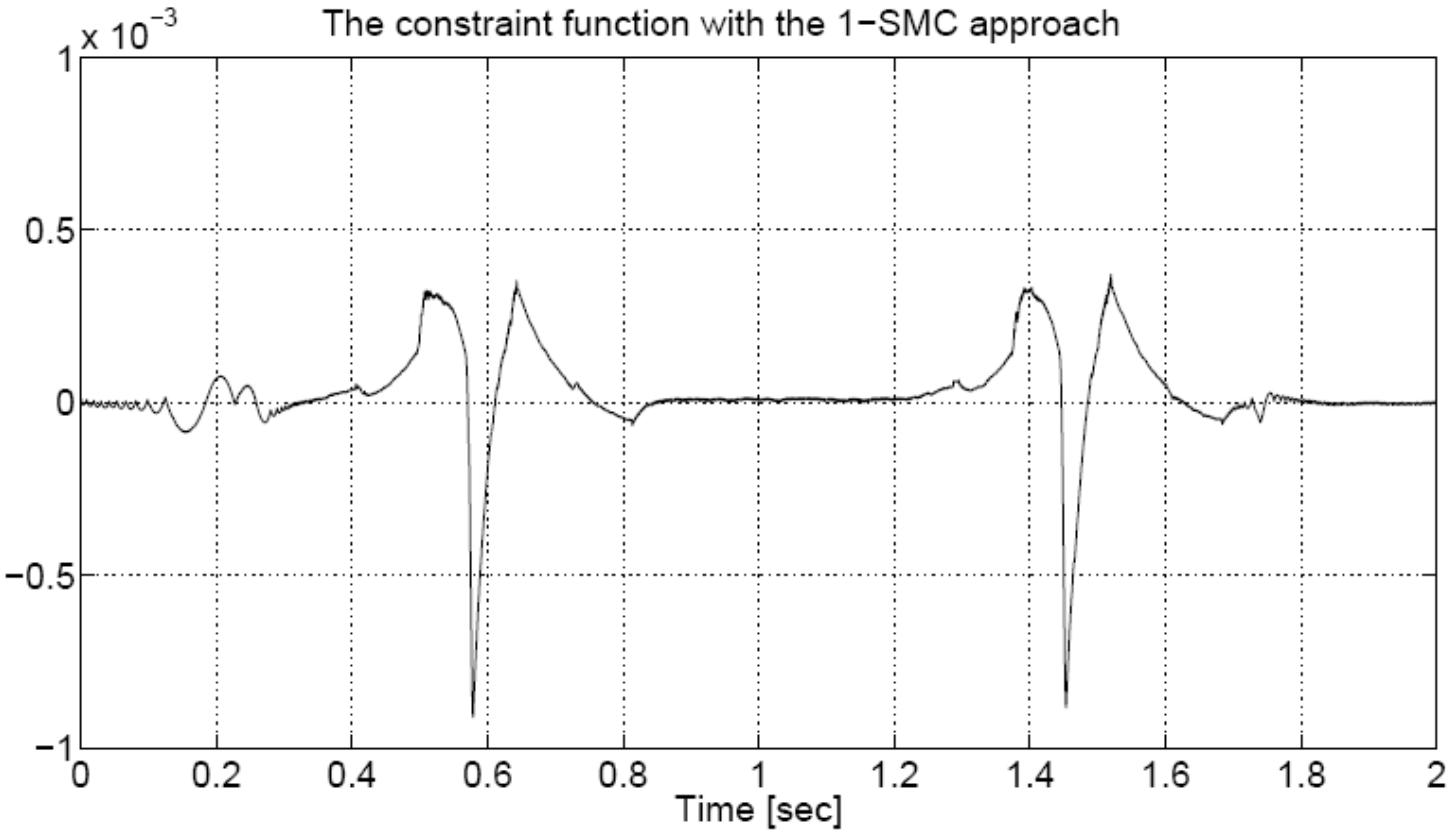
L5 – Simulation of DAE systems

Example: Constrained planar manipulator ($T_s=10^{-5}$ s)



L5 – Simulation of DAE systems

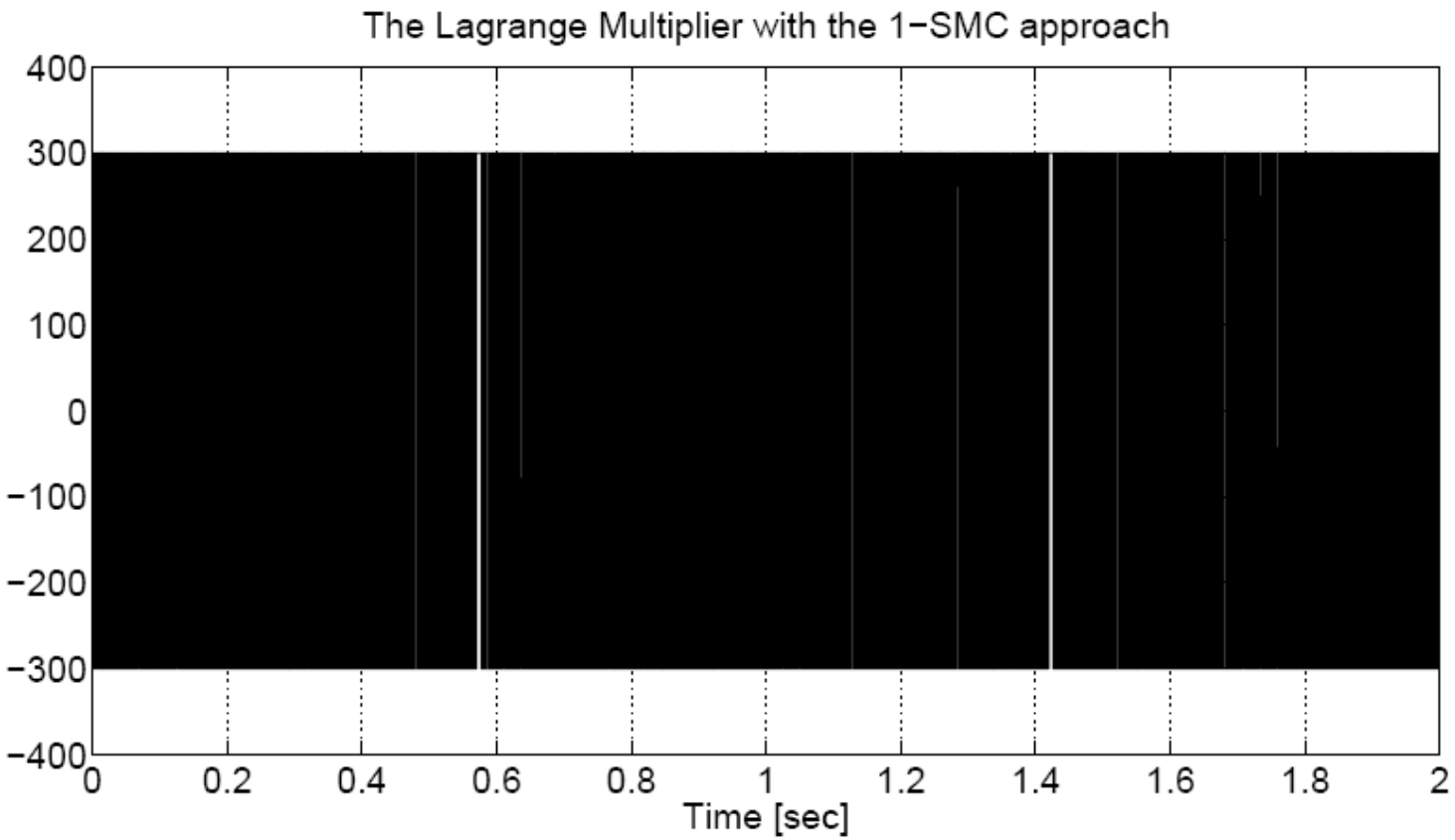
Example: Constrained planar manipulator ($T_s=10^{-5}$ s)



The magnitude of the switching control is set to U=300

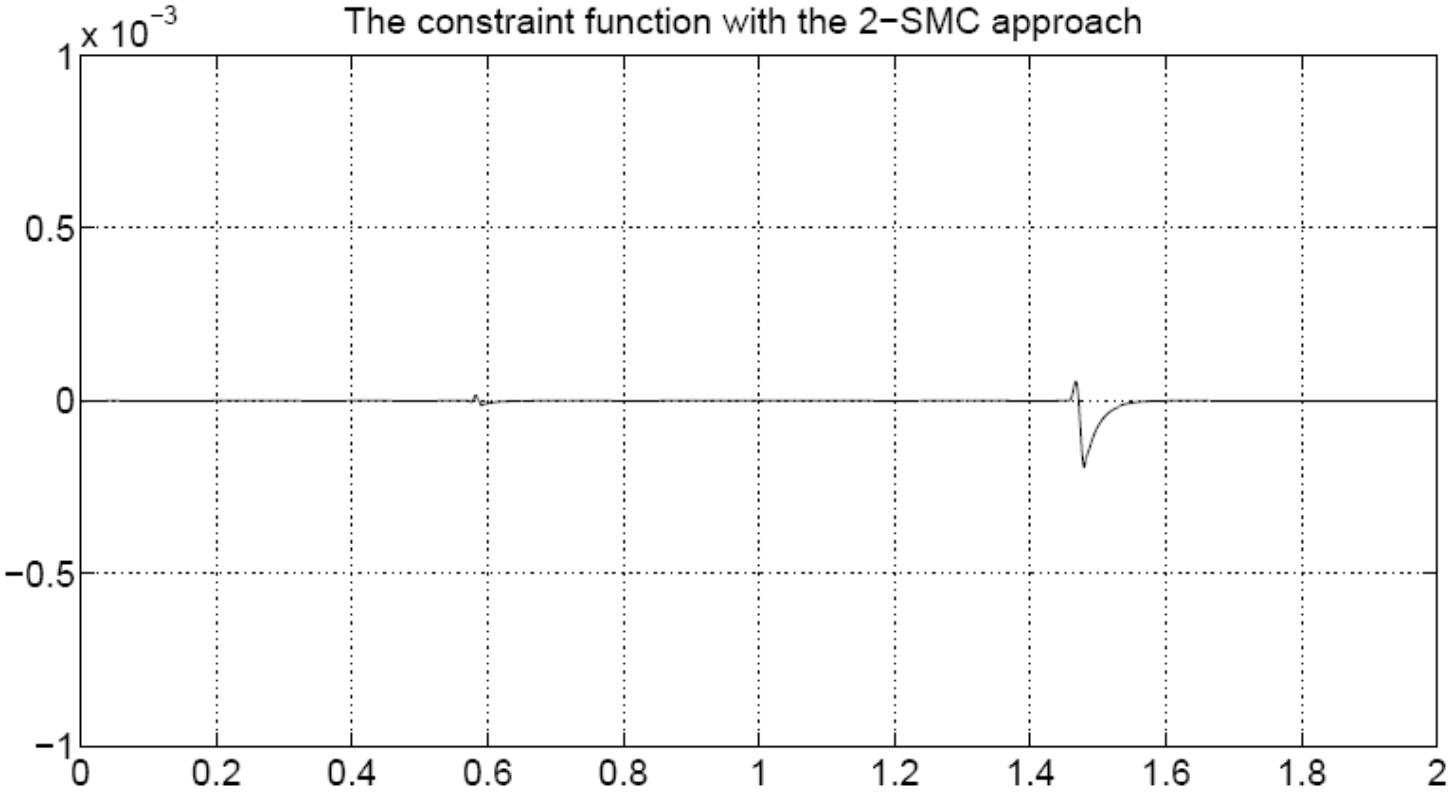
L5 – Simulation of DAE systems

Example: Constrained planar manipulator (Ts=10^{-5} s)



L5 – Simulation of DAE systems

Example: Constrained planar manipulator (Ts=10^{-5} s)

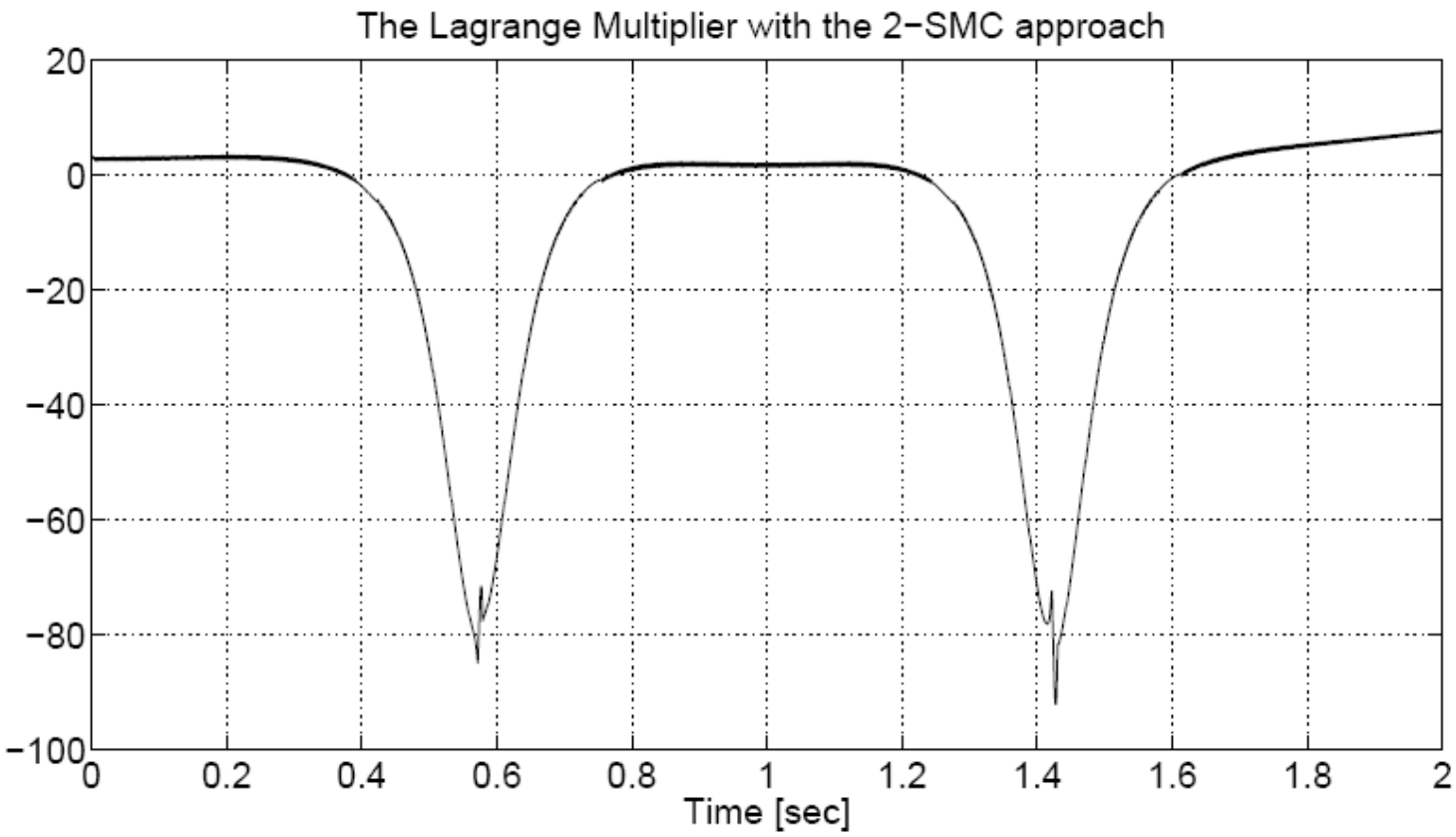


The magnitude of the continuous term is λ=10

The magnitude of the switching term is α=100

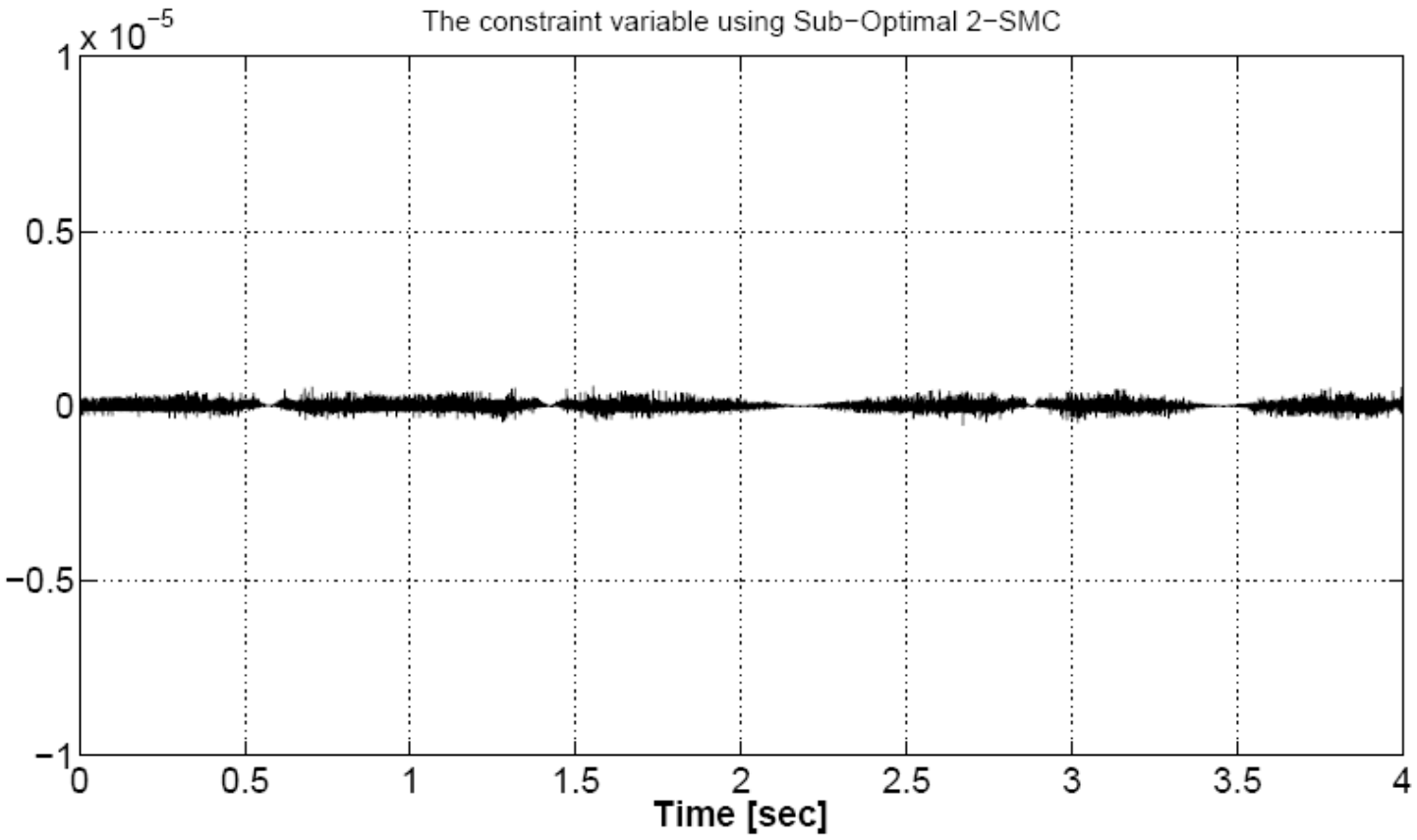
L5 – Simulation of DAE systems

Example: Constrained planar manipulator (Ts=10^{-5} s)



L5 – Simulation of DAE systems

Example: Constrained planar manipulator (Ts=10^{-5} s)



The magnitude of the control is U=300

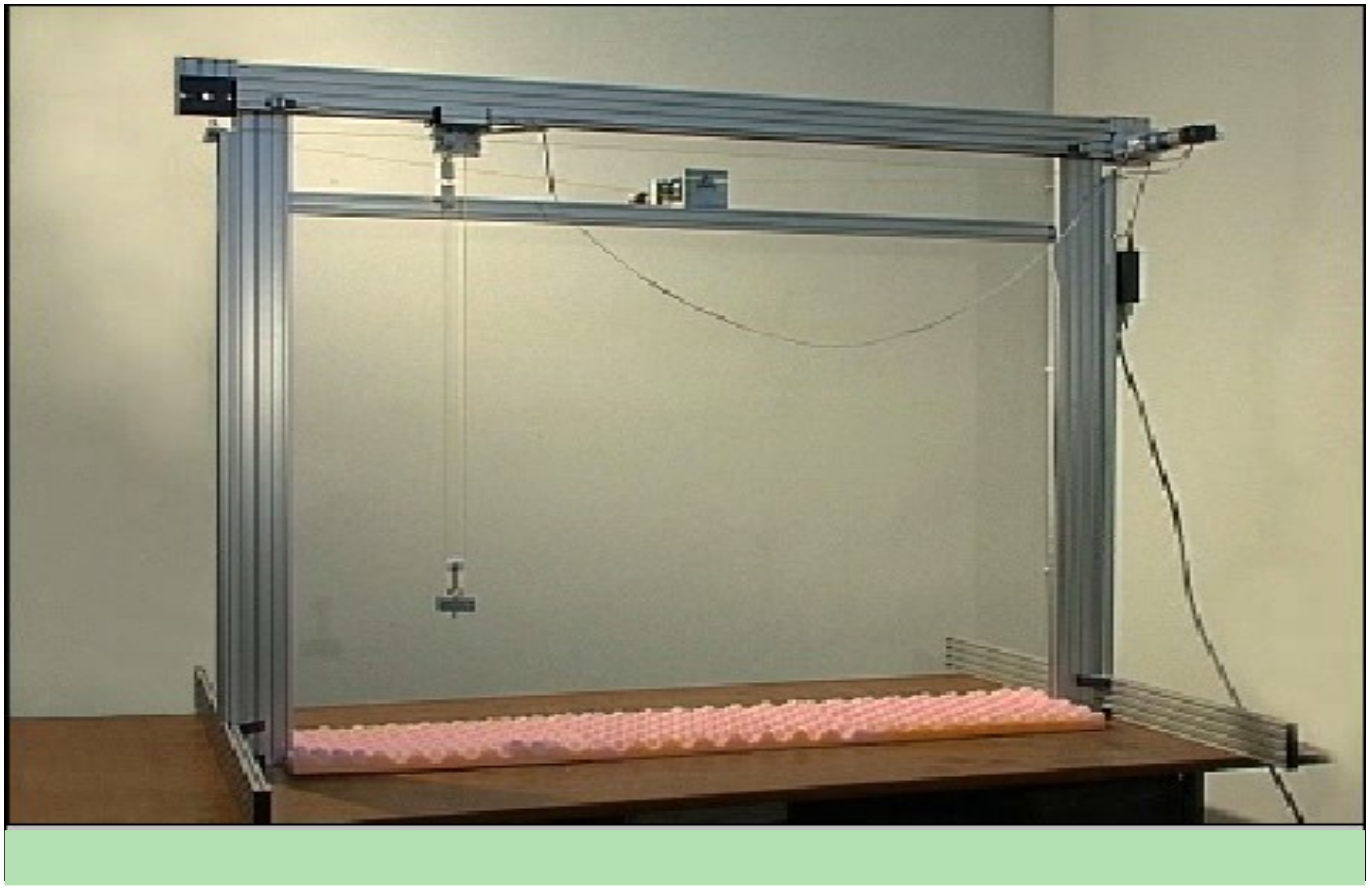
The anticipation parameter is β=0.8

Since the accuracy is very high the plot of the Lagrange multiplier clearly shows two singular (∞) points

L5 – Overhead crane control

Overhead cranes are used to move heavy loads, and for shipping procedures of containers

Laboratory-sized prototype



L5 – Overhead crane control

Overhead Crane Model

Mechanical subsystem

$$A_1\ddot{x}_t + B_1\left(\ddot{l}\sin\varphi + l\ddot{\varphi}\cos\varphi + 2\dot{l}\dot{\varphi}\cos\varphi - l\dot{\varphi}^{\,2}\sin\varphi\right) = T_1$$

$$B_2\ddot{x}_t\sin\varphi + A_2\ddot{l} - B_2 l\left(\dot{\varphi}^{\,2} + g\cos\varphi\right) = T_2$$

$$\ddot{x}_t\cos\varphi + l\ddot{\varphi} + 2\dot{l}\dot{\varphi} + g\sin\varphi = 0$$

x_t: *trolley position*

l: *rope length*

φ: *load oscillation angle*

T_1, T_2: trolley and hoisting torques

Electrical torque actuators: DC Motors)

$$L\dot{\mathbf{T}} + R\mathbf{T} - K\begin{vmatrix}\dot{x}_t\\ \dot{l}\end{vmatrix} = \begin{vmatrix}V_1\\ V_2\end{vmatrix}$$

V_1, V_2: motor voltages

L5 – Overhead crane control

Sliding manifold design

$$\sigma_1 = \dot{x}_t - \dot{x}_t^* + c_x\left(x_t - x_t^*\right) - k\,\varphi$$
$$\sigma_2 = \dot{l} - \dot{l}^* + c_y\left(l - l^*\right)$$

The zero-dynamics

$$\dot{x}_t = \dot{x}_t^* - c_x\left(x_t - x_t^*\right) + k\varphi$$
$$\dot{l} = \dot{l}^* - c_y\left(l - l^*\right)$$
$$\ddot{\varphi} = -\frac{g}{l}\varphi - \frac{1}{l}\left[\ddot{x}_t^* - c_x\left(x_t - x_t^*\right)\right] - \frac{k + 2\left(\dot{l}^* - c_y\left(l - l^*\right)\right)}{l}\dot{\varphi}$$

The sliding dynamics

$$\begin{bmatrix}\ddot{\sigma}_1\\ \ddot{\sigma}_2\end{bmatrix} = \begin{bmatrix}f_1(\cdot)\\ f_2(\cdot)\end{bmatrix} + \mathbf{G}(\cdot)\begin{bmatrix}V_1\\ V_2\end{bmatrix}$$

f_i are uncertain but bounded

G is almost diagonal

V_i are the motor supply voltages

L5 – Overhead crane control

Zero-dynamics stability

1. Traveling phase

$$\dot{x}^* = V_0 = const.$$
$$c_x = 0$$

STABILITY CONDITION

$$k_m \le k \le k_M$$

The crane parameters do not affect the above stability condition, which depends only on the desired trajectory

2. Arrival phase

(neighbor of the destination point (x_f , y_f))

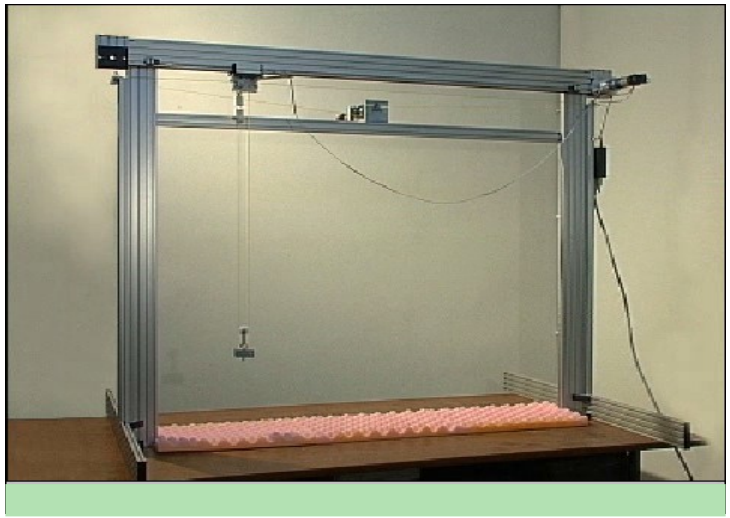
$$x^* = x_f$$
$$y^* = y_f$$
$$\dot{x}^* = \dot{l}^* = 0$$

STABILITY CONDITION

$$0 < k < \frac{g}{c_x}$$

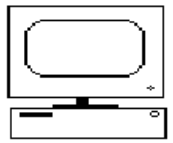
L5 – Overhead crane control

Experimental set-up



x_t , l , φ
(trolley and hoist motor positions and swing angle)

v_t , v_l
(trolley and hoist motor voltages)



PC UNIT

2-SM differentiators are implemented for estimating the trolley and rope velocities

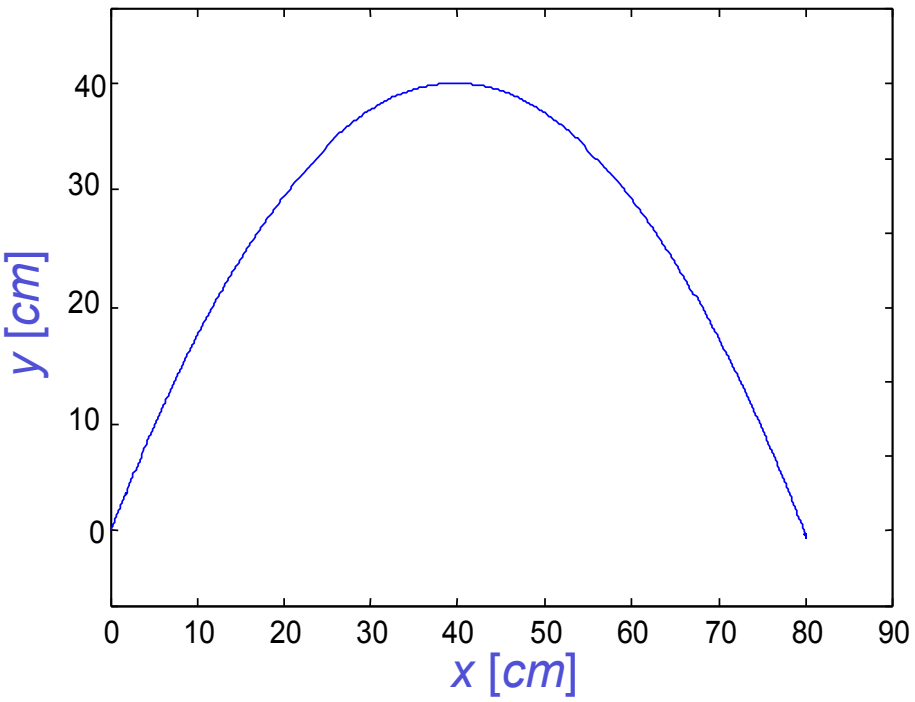
2 instances of the sub-optimal 2-SMC are implemented

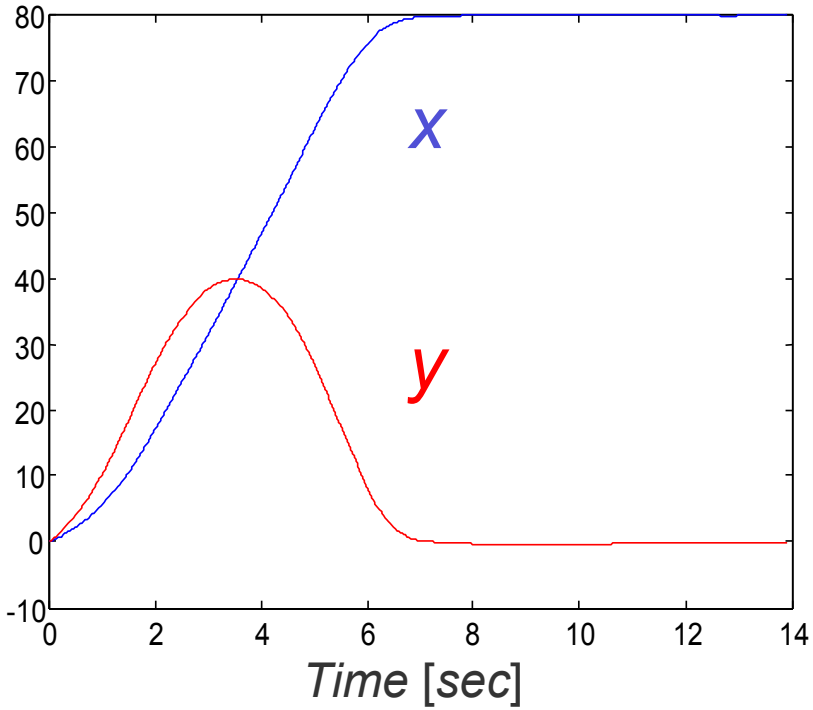
Direct discretization of the differentiators and controllers with T_s=2ms is implemented

L5 – Overhead crane control

Experiment 1 – simple traveling and hoisting

The actual and reference load coordinates



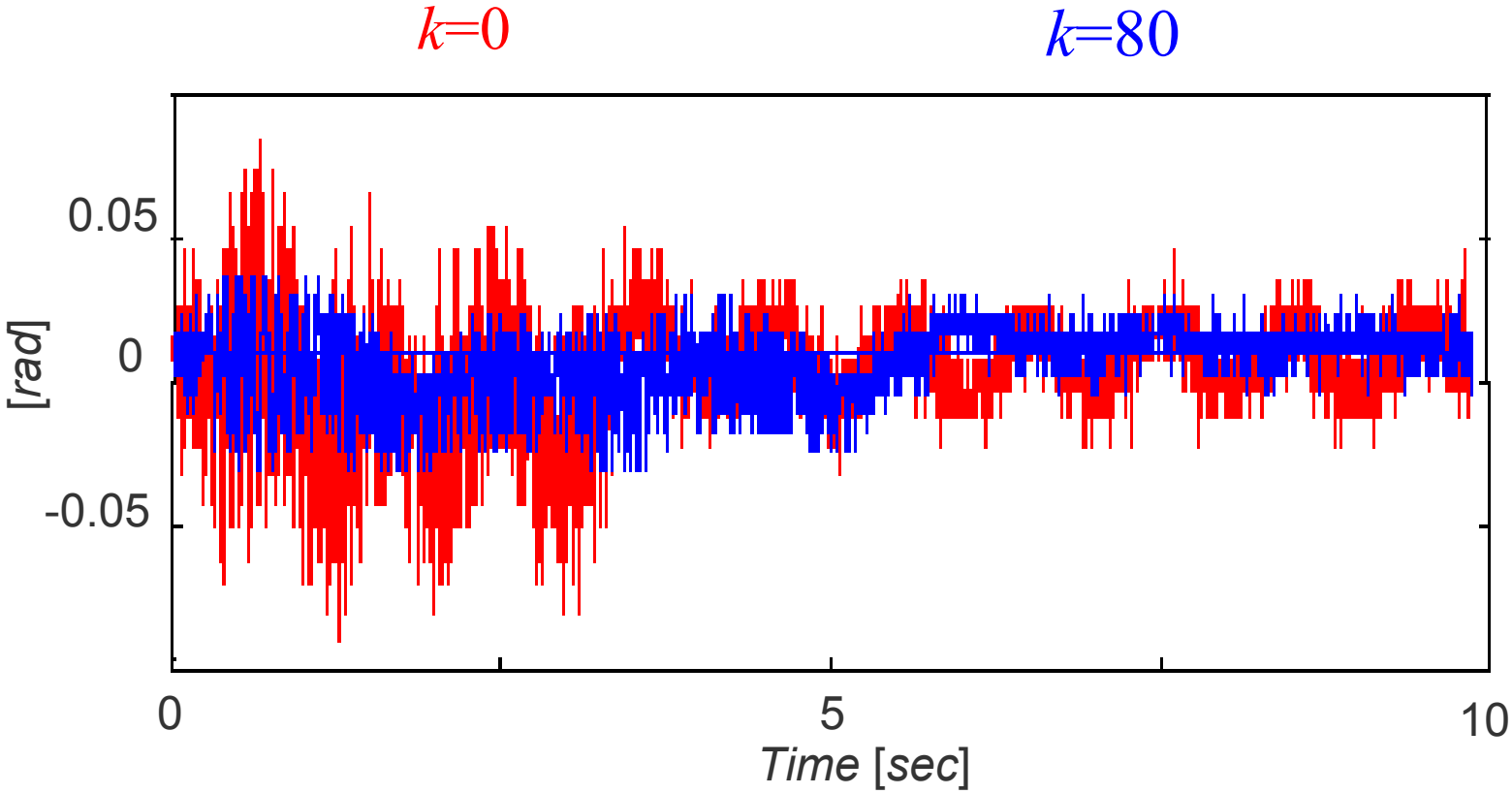


Traveling velocity of the trolley is 15 m/s

L5 – Overhead crane control

Experiment 1 – simple traveling and hoisting

The load swing

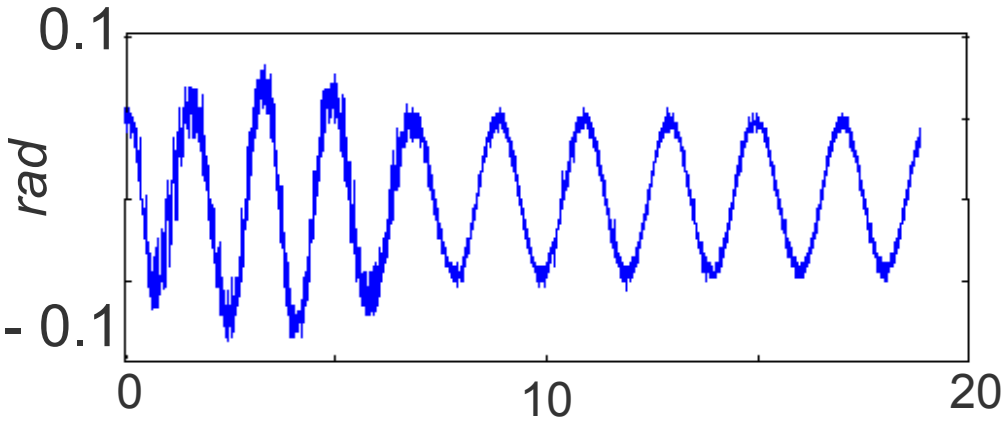


Traveling velocity of the trolley is 15 m/s

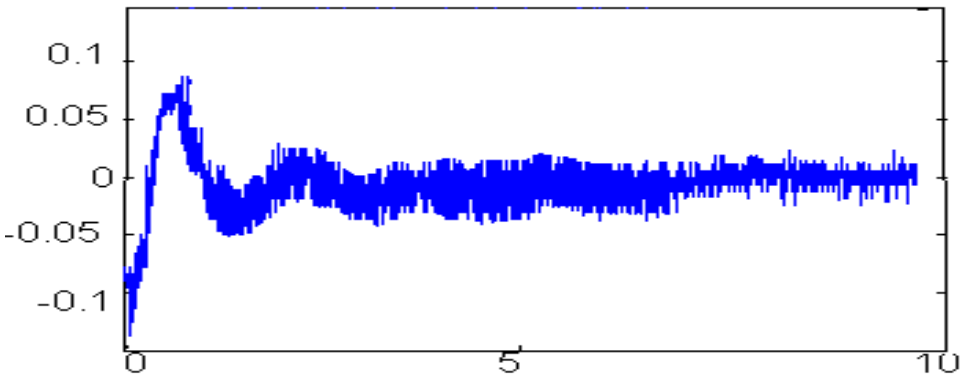
L5 – Overhead crane control

Experiment 2 – traveling and hoisting with initial oscillation

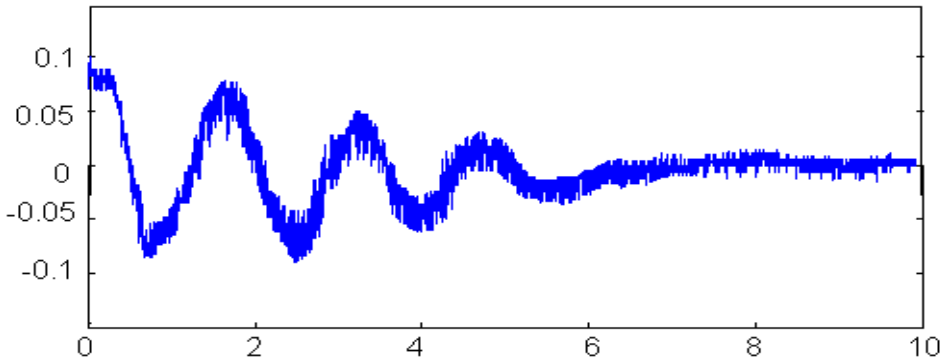
k=0 (no swing control)



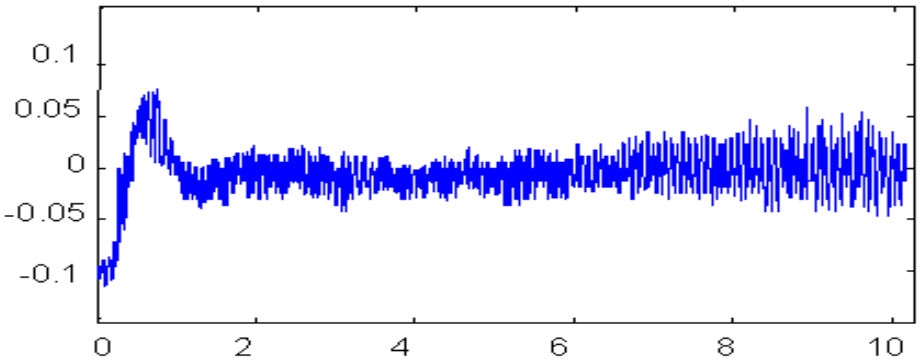
k=360



k=80

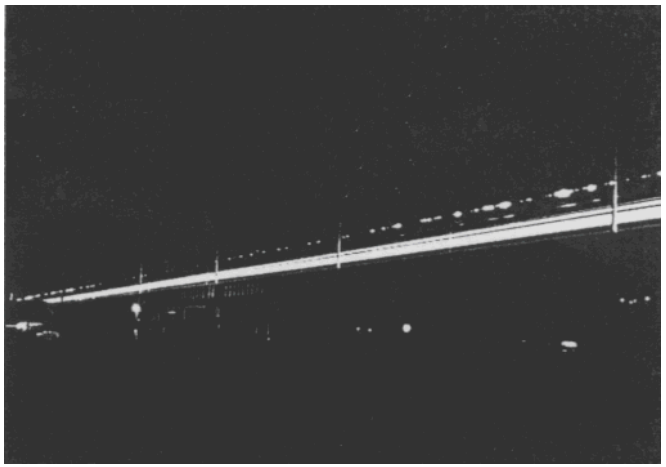


k=640 (unstable oscillations)



L5 – Overhead crane control - Video

L5 – Contact Force Regulation



Pantograph-catenary interaction causes oscillations due to the moving force on the contact wire.

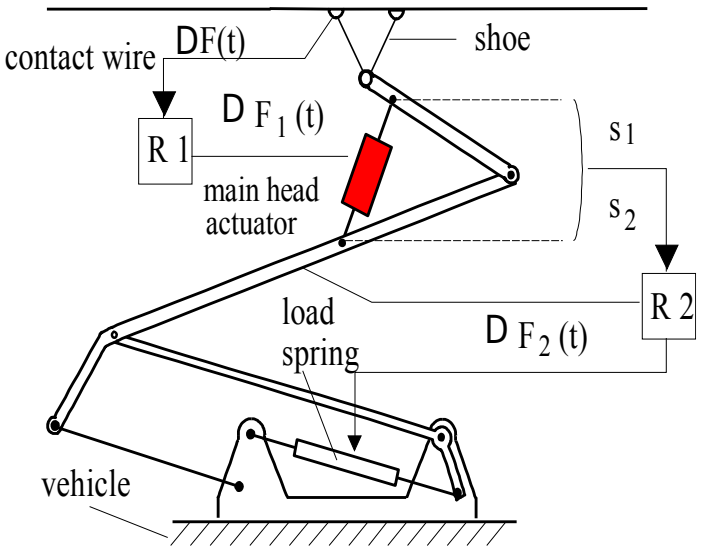
Contact force variations cause:

- Loss of contact and electric arcs
- Wire and shoe wearing

Possible solutions:

- Increase the tension of the contact wire
- More rigid catenary systems
- Reduce the pantograph weight
- **Use of active pantographs**

L5 – Contact Force Regulation



Two options for actuator location

Head (upper frame) actuated

minor control effort

difficulties in positioning

Lower frame actuated

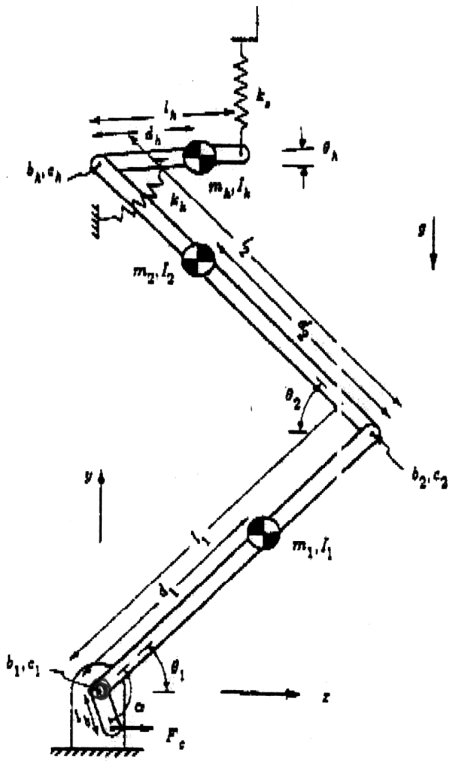
major control effort

simple positioning

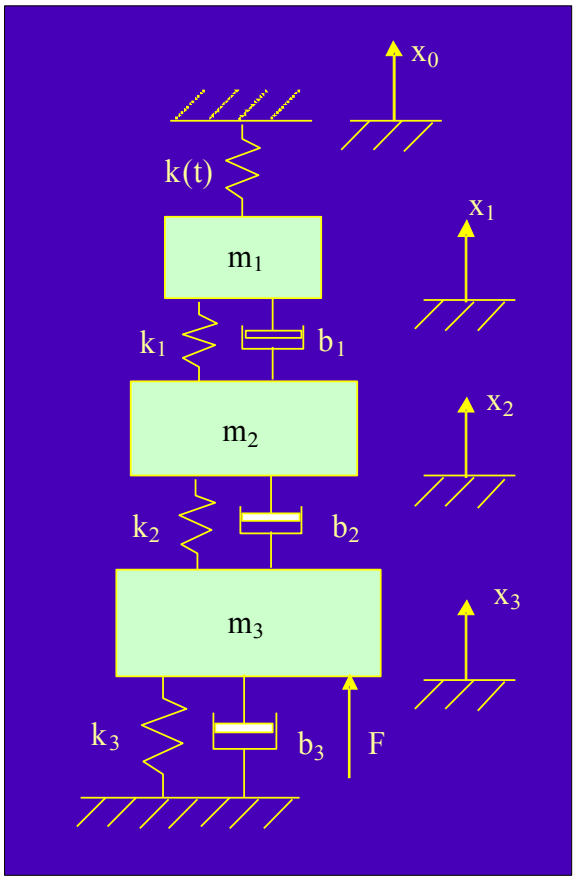
Low-cost changes to passive pantographs: wire actuators

L5 – Contact Force Regulation

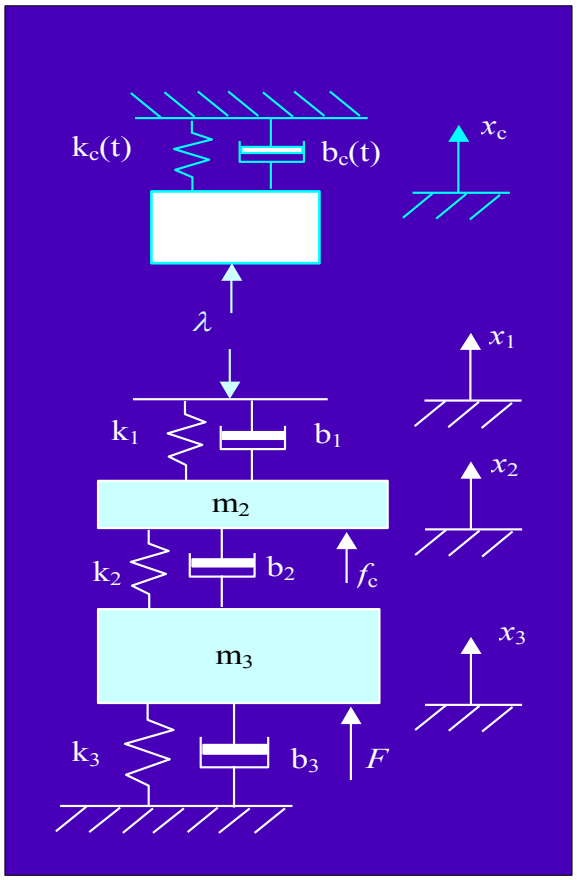
Pantograph models



nonlinear
3 d.o.f.



linear
3 d.o.f.



Linear 3DOF
including loss
of contact

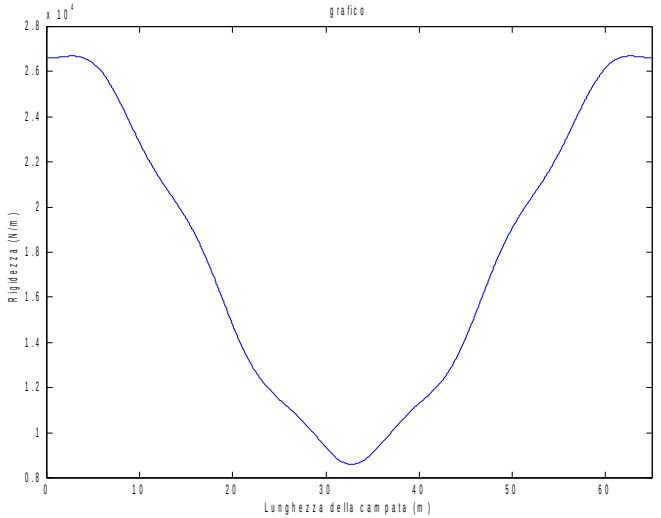
L5 – Contact Force Regulation

Pantograph models

$$m_c(x) = m_{c_0} + \sum_{i=1}^{p} m_{s_i} \cos\left(\frac{2\pi}{L_s} ix\right) + \sum_{i=1}^{q} m_{p_i} \sin\left(\frac{2\pi}{L_p} ix\right),$$

$$b_c(x) = b_{c_0} + \sum_{i=1}^{p} b_{s_i} \cos\left(\frac{2\pi}{L_s} ix\right) + \sum_{i=1}^{q} b_{p_i} \sin\left(\frac{2\pi}{L_p} ix\right),$$

$$k_c(x) = k_{c_0} + \sum_{i=1}^{p} k_{s_i} \cos\left(\frac{2\pi}{L_s} ix\right) + \sum_{i=1}^{q} k_{p_i} \sin\left(\frac{2\pi}{L_p} ix\right),$$

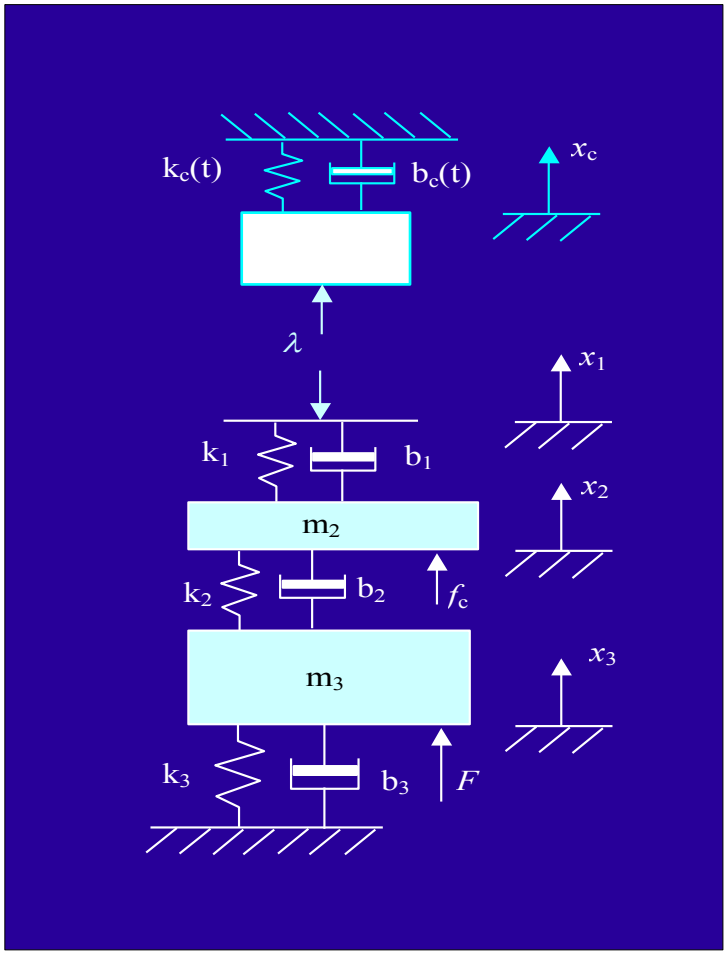


$$k_c(t) = k_{c_0}\left(1 + \alpha \cos\frac{2\pi v}{L_s} t + \beta \sin\frac{2\pi v}{L_p} t\right)$$

The approximate model of the catenary is reduced to a time varying lumped parameter model depending on the train velocity

L5 – Contact Force Regulation

Pantograph models without loss of contact



$$\dot{\mathbf{z}}(t) = \mathbf{A}(t)\mathbf{z}(t) + \mathbf{B}\mathbf{f}(t)$$
$$y(t) = \mathbf{C}\mathbf{z}(t)$$

$$\mathbf{A}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{k_c(t)+k_1}{m_c(t)} & -\frac{b_c(t)+b_1}{m_c(t)} & \frac{k_1}{m_c(t)} & \frac{b_1}{m_c(t)} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{k_1}{m_2} & \frac{b_1}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{b_1+b_2}{m_2} & \frac{k_2}{m_2} & \frac{b_2}{m_2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{k_2}{m_3} & \frac{b_2}{m_3} & -\frac{k_2+k_3}{m_3} & -\frac{b_2+b_3}{m_3} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m_2} & 0 \\ 0 & 0 \\ 0 & \frac{1}{m_3} \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} -k_1 & -b_1 & k_1 & b_1 & 0 & 0 \end{bmatrix}$$

L5 – Contact Force Regulation

Pantograph dynamics

Dynamics of the wire-type actuators

$$\ddot{f}_c + 2\zeta_f \omega_{nf} \dot{f}_c + \omega_{nf}^2 f_c = \omega_{nf}^2 K_{a_f} u_f$$
$$\ddot{F} + 2\zeta_F \omega_{nF} \dot{F} + \omega_{nF}^2 F = \omega_{nF}^2 K_{a_F} u_F$$

The sliding variable

$$\sigma = y - y_d \qquad y_d = 100N$$

Normal-form dynamics considering the upper frame actuation

$$\dddot{\sigma}(t) = \varphi(\mathbf{y}(t), \mathbf{w}(t), t) + \gamma \cdot u_f(t),$$
$$\dot{\mathbf{w}}(t) = \boldsymbol{\psi}(\mathbf{y}(t), \mathbf{w}(t), t),$$

Relative degree is three because of the 2nd order dynamics of the actuator

The internal dynamics are BIBS stable

L5 – Contact Force Regulation

Controller design

$$u = -U\,\mathrm{sgn}\left(\hat{\sigma} - \beta\hat{\sigma}_{ex}\right)$$

The controller gain is tuned taking into account the nominal dynamics of the system with an additional integrator for anti-chattering purpose, disregarding the actuator dynamics

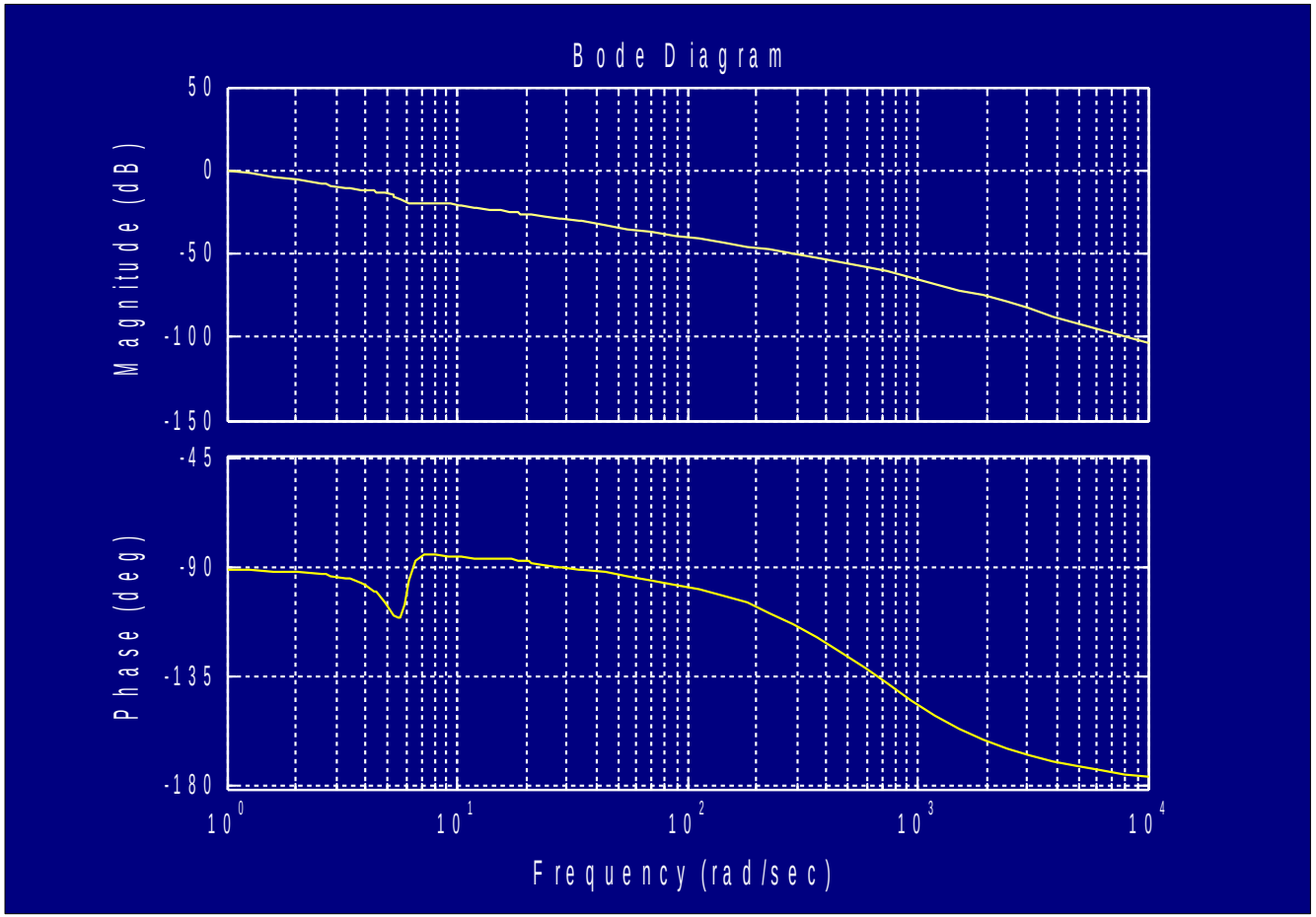
The anticipation parameter is tuned by means of a DF analysis techniques which suggest also the **introduction in-the-loop of a properly-tuned linear low-pass filter to enhance the system performance.**

L5 – Contact Force Regulation

Controller design

The integrator is introduced for anti-chattering purpose and the resulting relative degree is two

Harmonic response of the nominal plant with upper frame control and integrator



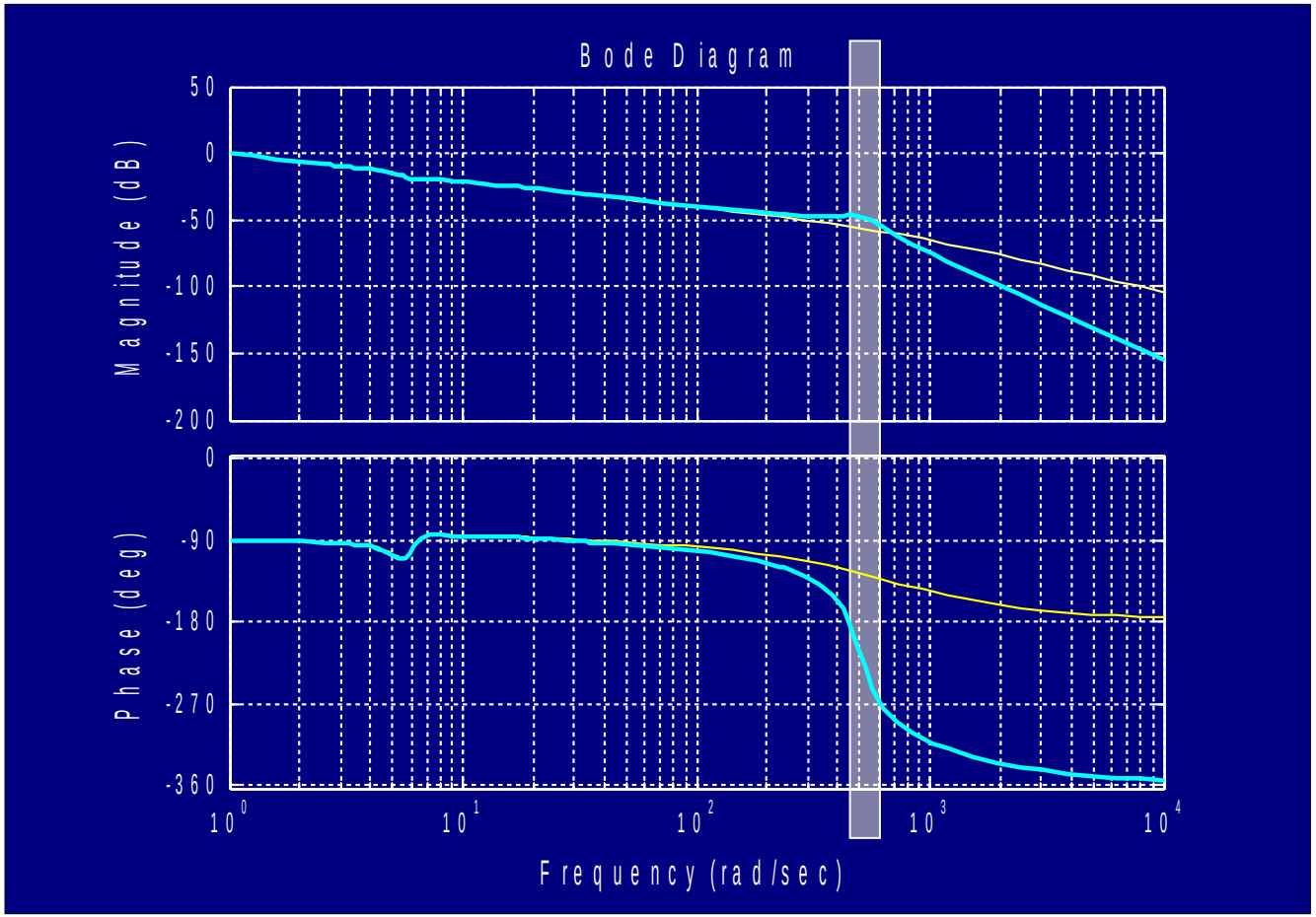
L5 – Contact Force Regulation

Controller design

The introduction of the resonant actuator causes the rise of large oscillations

Harmonic response of the nominal plant with upper frame control and integrator,

plus the wire actuator



L5 – Contact Force Regulation

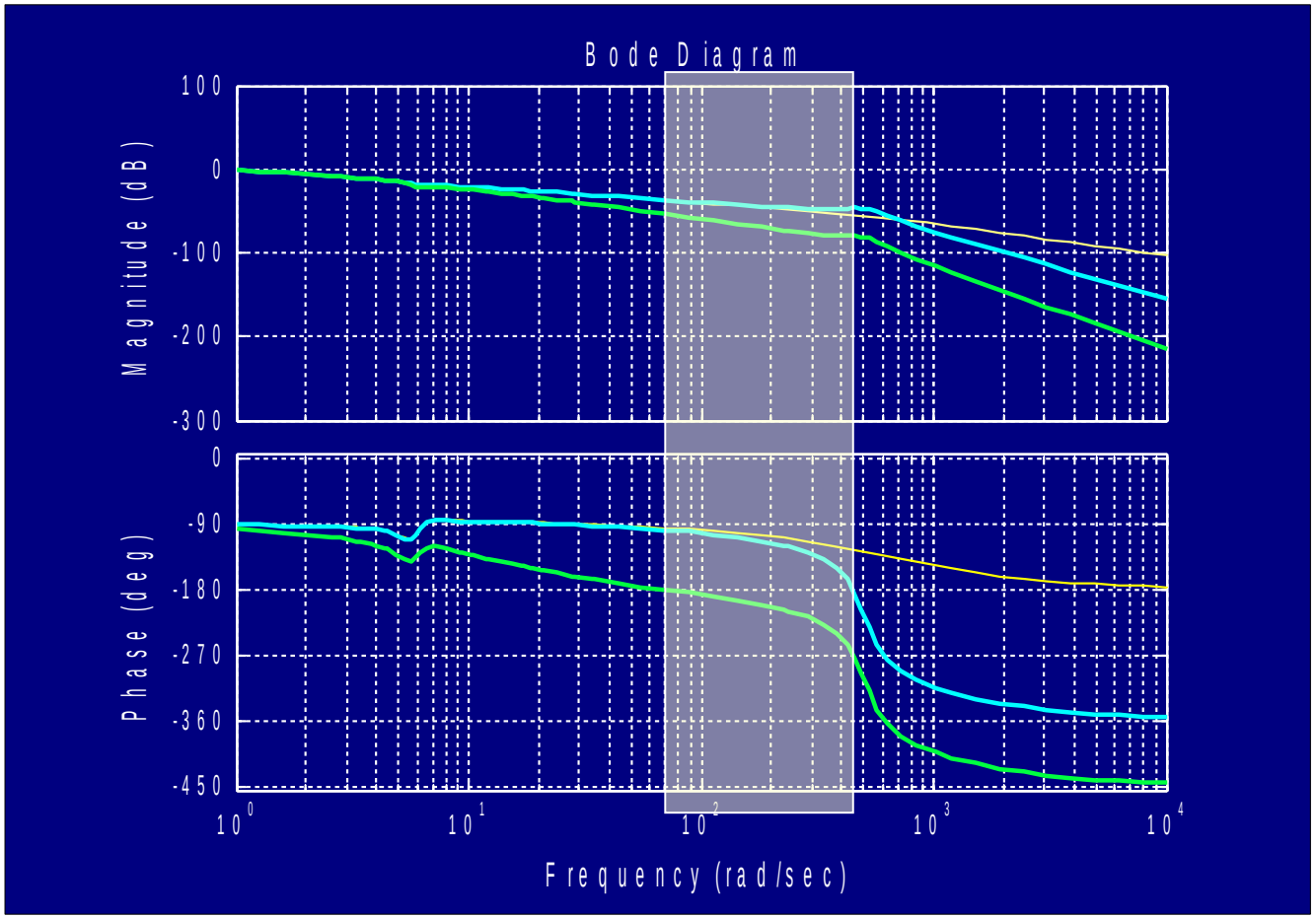
Controller design

The pre-filter increases the relative degree of the input-uotput dynamics, but allows for choosing a suitable value of β such that oscillations are not too large

Harmonic response of the nominal plant with upper frame control and integrator,

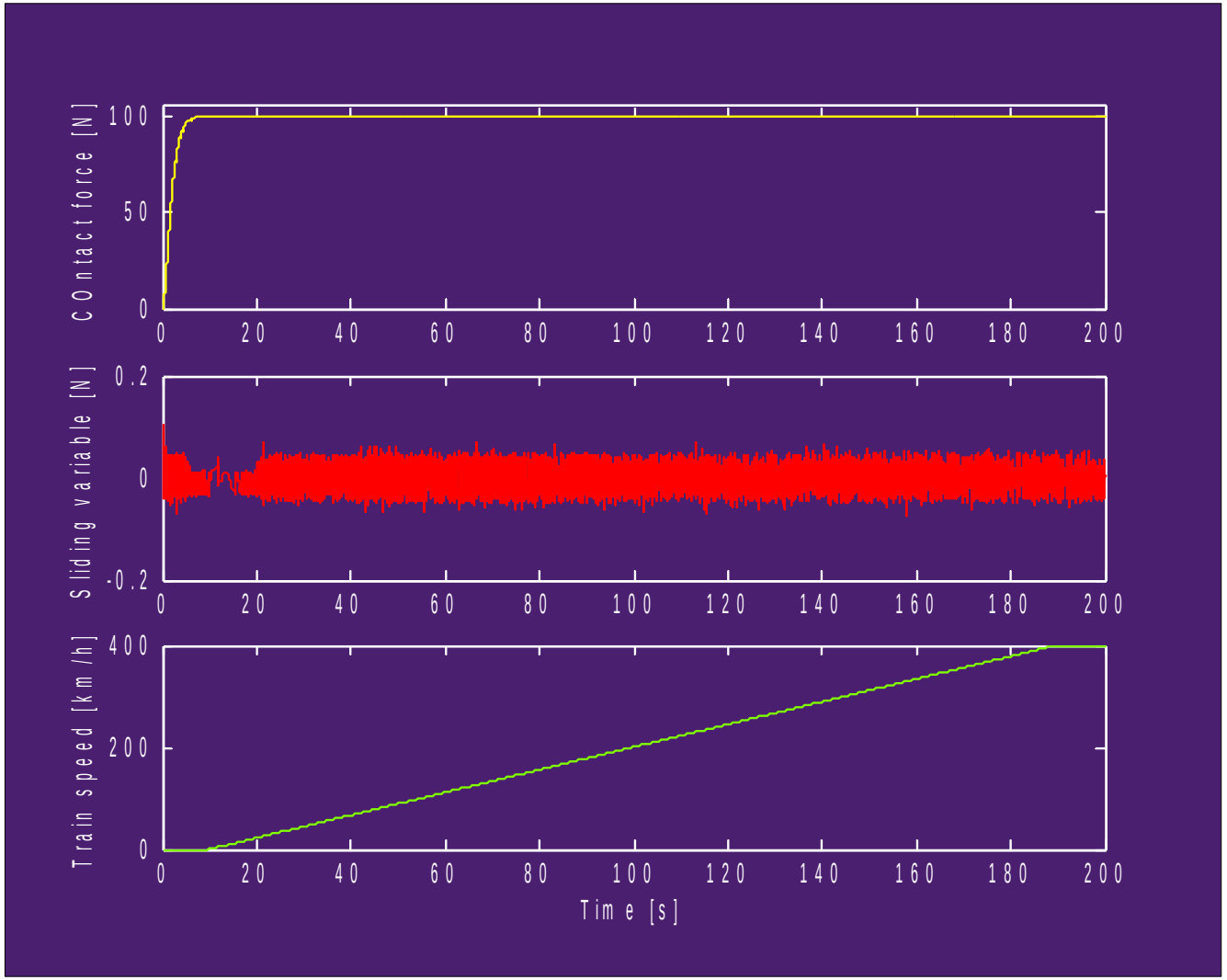
plus the wire actuator

And with the compensating pre-filter with τ=0.1s



L5 – Contact Force Regulation

Simulations



Upper frame control

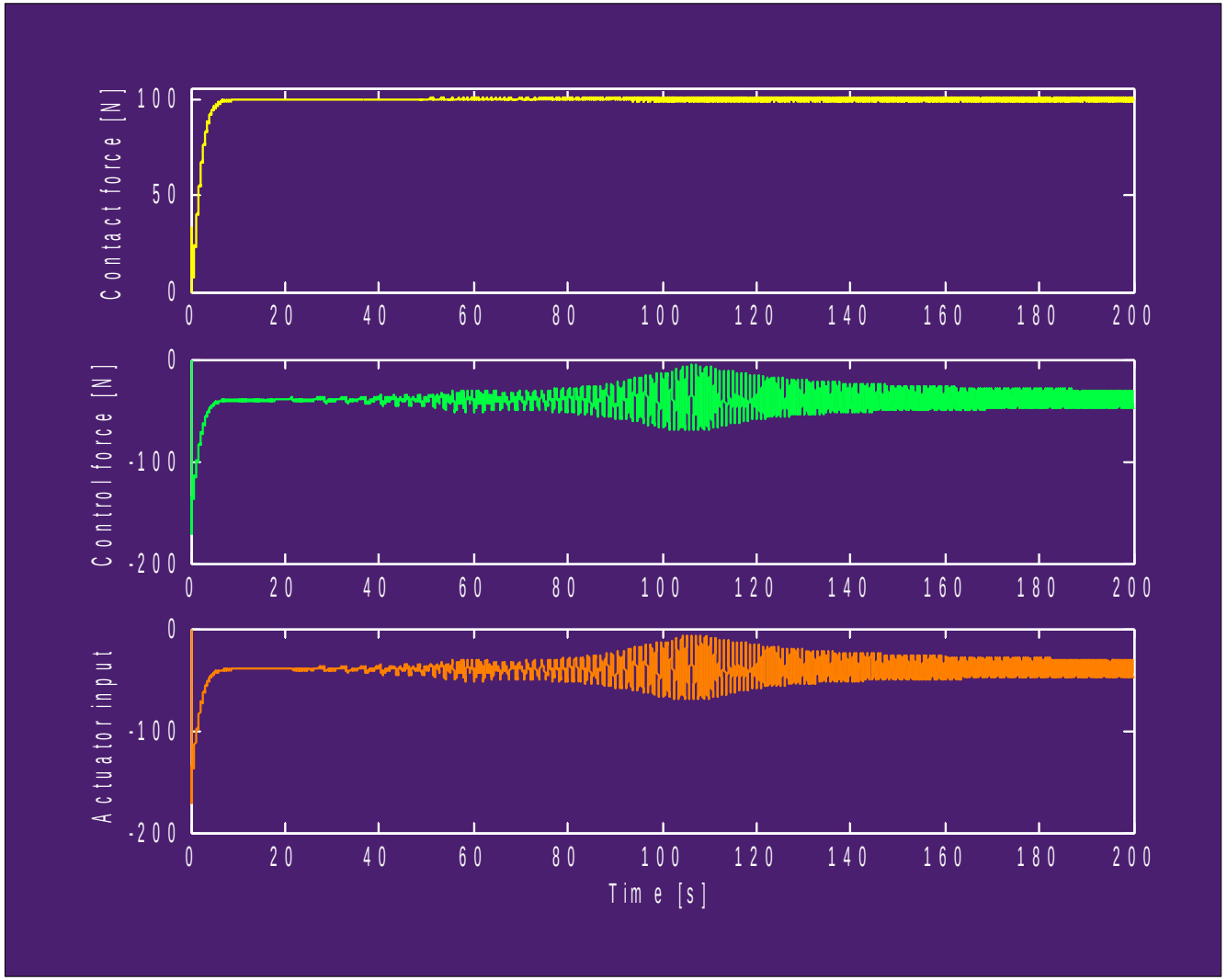
No actuator dynamics

$$U_M = 3000$$

$$\beta = 0.90$$

L5 – Contact Force Regulation

Simulations



Upper frame control

Actuator

&

shaping pre-filter

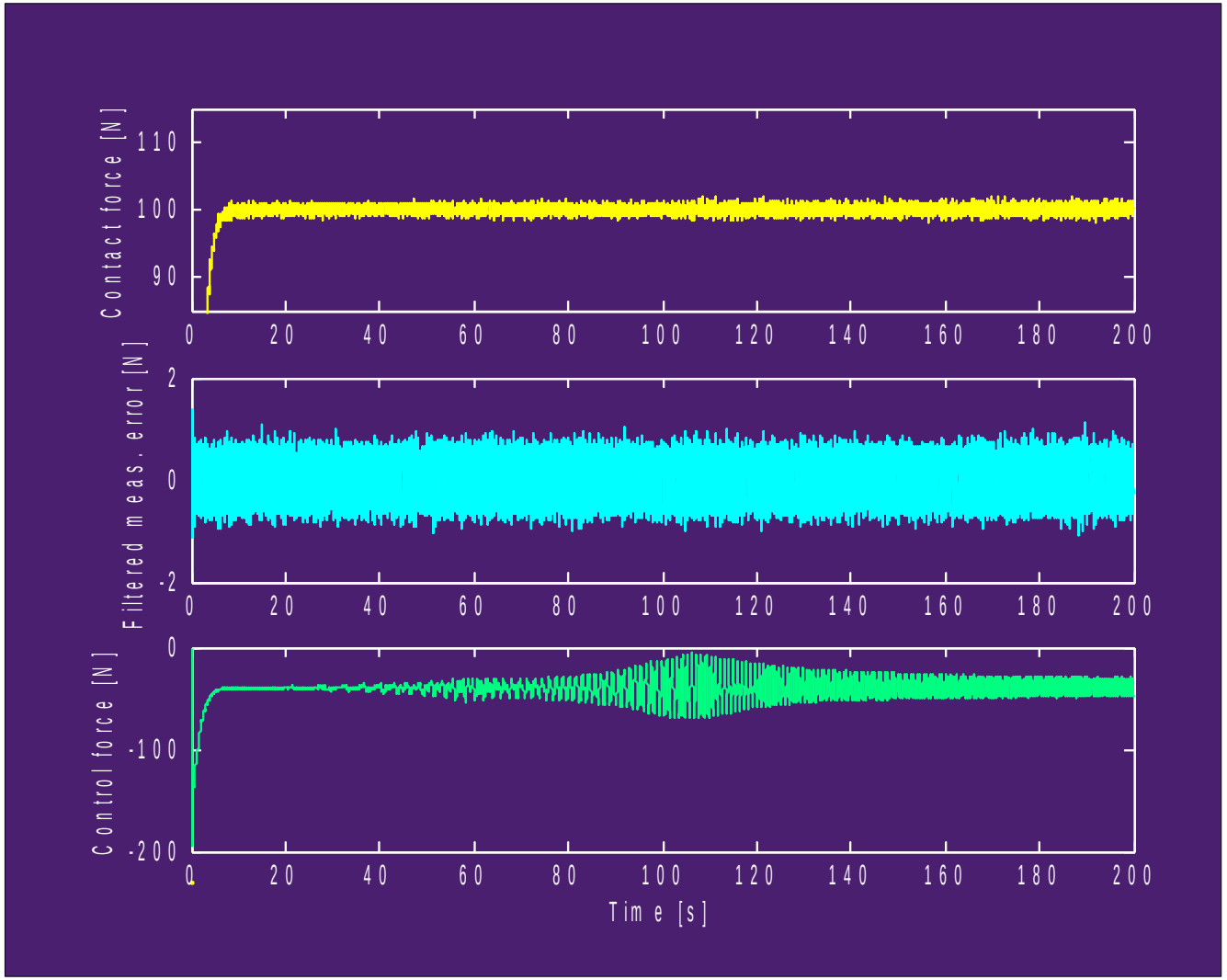
τ = 0.1 s

$$U_M = 3000$$

$$\beta = 0.90$$

L5 – Contact Force Regulation

Simulations



Upper frame control

Actuator with

measurement noise ±2N

&

shaping pre-filter

τ = 0.1 s

&

measurement filter

τ = 0.001 s

$$U_M = 3000$$

$$\beta = 0.90$$

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