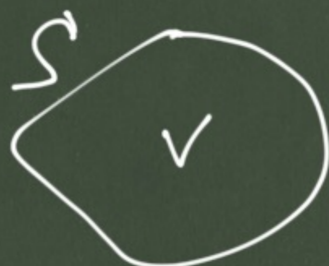
$$\oint_S \bar{E} \cdot d\bar{A} = \frac{Q}{\varepsilon_0} = \frac{1}{\varepsilon} \int_V d\bar{r}\, \rho(\bar{r})$$



$$\rho(r) = \rho_0 \left(\frac{r}{R}\right)^{\alpha} \quad r \le R$$

$$= 0 \quad r > R$$

$$\int_{S'} \bar{E}\cdot d\bar{A} = \int_S E\,\hat{r}\cdot\hat{r}\,dA = \int_S E\,dA$$

$$d\bar{A} = \hat{r}\,dA$$

$$= E\int_{S'} dA$$

$$= E\,4\pi r^2$$

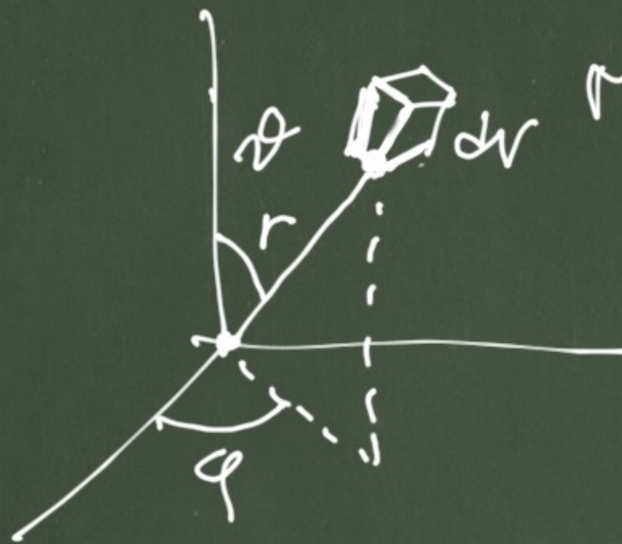
$$\int \rho(\bar{r})\,d\bar{r} \rightarrow$$ coord. sferiche

r, ϑ, φ

azimut $\varphi \in (0, 2\pi)$

polare $\vartheta \in (0, \pi)$

$r \in (0, \infty)$



$$dV = r^2 \sin\vartheta \, dr \, d\vartheta \, d\varphi$$

$$Q = \int d\bar{r}\, \rho(\bar{r}) = \frac{\rho_0}{R^\alpha} \int_0^R dr\, r^2 r^\alpha \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\vartheta\, d\vartheta$$

$$= \frac{\rho_0}{R^\alpha} \int_0^R r^{\alpha+2} dr \underbrace{\left(\varphi \Big|_0^{2\pi}\right)}_{2\pi} \underbrace{\left(-\cos\vartheta\right)_0^{\pi}}_{2}$$

$$= 4\pi \frac{\rho_0}{R^\alpha} \int_0^R r^{\alpha+2} dr$$

$$4\pi \frac{\rho_0}{R^\alpha} \int_0^{R} r^{\alpha+2}\,dr = \frac{4\pi\rho_0}{R^\alpha} \left.\frac{r^{\alpha+3}}{\alpha+3}\right|_0^R$$

$$= \frac{4\pi\rho_0}{R^\alpha} \frac{R^{\alpha+3}}{\alpha+3} = \frac{4\pi R^3}{\alpha+3}\rho_0$$

$$4\pi r^2 E = \frac{4\pi R^3}{\varepsilon_0(\alpha+3)}\rho_0$$

$$4\pi r^2 E = \frac{4\pi R^3}{\varepsilon_0(\alpha+3)}\rho_0$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\frac{4\pi R^3}{\alpha+3}\rho_0 \rightarrow Q^{(\alpha)}$$

$$= \frac{K_e Q^{(\alpha)}}{r^2}$$

FUORI

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Dentro

$$Q_d = \frac{4\pi\rho_0}{R^\alpha}\int_0^{r_g} dr\, r^{\alpha+2} = \frac{4\pi\rho_0}{(3+\alpha)R^\alpha}\left(r^{3+\alpha}\right)\Big|_0^{r_g}$$

$$= \frac{4\pi\rho_0\, r_g^{3+\alpha}}{(3+\alpha)R^\alpha}$$

$$= Q\left(\frac{r_g}{R}\right)^{3+\alpha}$$

$$E = \frac{k_e Q_d}{r^2} = \frac{k_e Q}{r^2}\left(\frac{r}{R}\right)^{3+\alpha}$$

$$= k_e Q r\left(\frac{r^\alpha}{R^{3+\alpha}}\right)$$

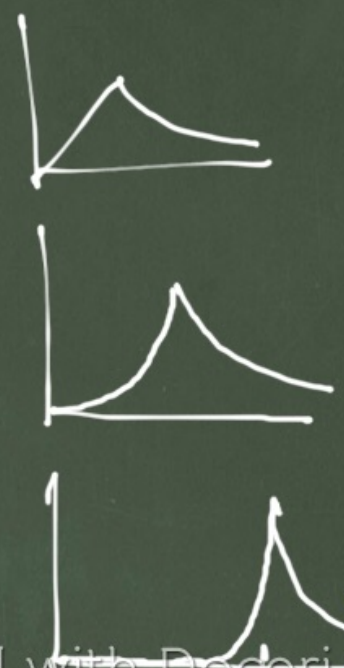
$$E = \frac{k_e Q_\ell}{r^2} = \frac{k_e Q}{r^2}\left(\frac{r}{R}\right)^{3+\alpha}$$

$$= k_e Q r \left(\frac{r^\alpha}{R^{3+\alpha}}\right)$$

$$\alpha = 0 \rightarrow \frac{k_e Q r}{R^3}$$

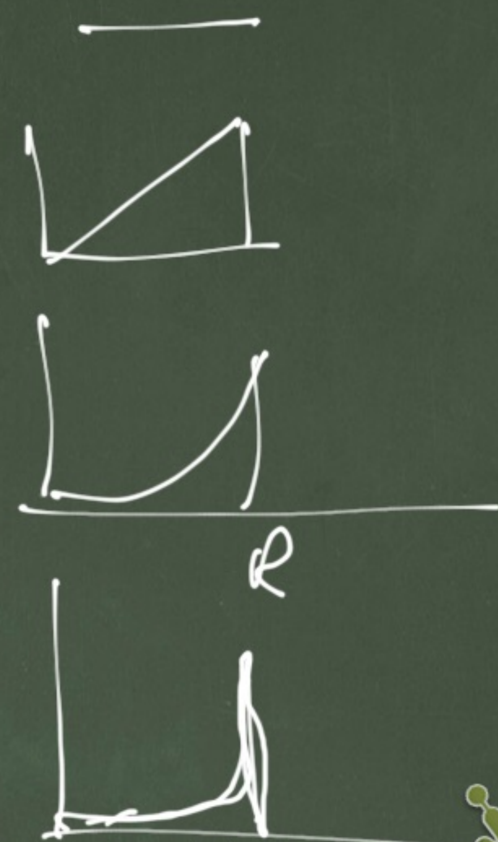
$$\alpha = 1 \rightarrow \frac{k_e Q r^2}{R^4}$$

$$\alpha = 50 \rightarrow \frac{k_e Q r^{51}}{R^{53}}$$



$$\rho \propto \left(\frac{P}{R^{\alpha}}\right)$$

α	ρ
0	
1	
2	
50	



$$\frac{d f(x)}{dx}$$

divergenza

opera su vettore

produce uno scalare

$$\operatorname{div} \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\bar{A} = (A_x, A_y, A_z)$$

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$$\boxed{\oint_S \bar{A}\cdot d\bar{A} = \oint_V \mathrm{div}\,\bar{A}\, dV}$$

$$\oint_S \bar{E}\, d\bar{A} = \frac{1}{\varepsilon_0}\int_V dV\, \rho$$

$$= \int_V \mathrm{div}\,\vec{E}\, dV$$

$$\text{GAUSS} \equiv \boxed{\mathrm{div}\,\bar{E} = \frac{\rho}{\varepsilon_0}} \Longrightarrow \text{I eq. Maxwell}$$

gradiente → pendenza

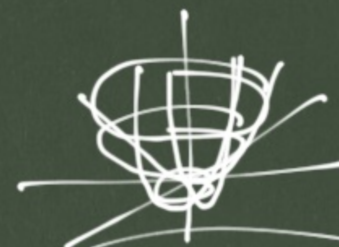
div. → flusso

rotore → rotazione

→ Enzo Tonti

Grad., rot, div

$f = x^2 + y^2$



$$\hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} = \hat{x}\,2x + \hat{y}\,2y$$

$$\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) f =$$

grad f

$\operatorname{div}\bar{E} = \rho/\varepsilon_0$? sfera carica con $\rho = \rho_0$

$$4\pi r^2 E = \frac{\rho}{\varepsilon_0}\frac{4\pi r^3}{3} \rightarrow E = \frac{r\rho_0}{3\varepsilon_0} \equiv E_r$$

E ha solo comp. r ; usiamo coord sferiche

$$\underbrace{\operatorname{div}\bar{E}}_{} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \dots = \frac{\rho}{\varepsilon_0}$$

$$= \frac{1}{r^2}\frac{\rho_0}{3\varepsilon_0}\frac{\partial}{\partial r}(r^3) = \frac{3}{r^2}\frac{\rho_0}{3\varepsilon_0} r^2 = \frac{\rho_0}{\varepsilon_0}$$

☺

<u>noto</u> ρ, <u>trovare E</u>

$$\mathrm{div}\,\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 E_r\right) = \frac{\rho_0}{\varepsilon_0}$$

$$r^2 E_r = \frac{1}{\varepsilon_0}\int r^2 \rho_0 = \frac{\rho_0 r^3}{3\varepsilon_0} \rightarrow E_r = \frac{\rho_0 r}{3\varepsilon_0} \;!$$

<u>GIUSTO</u> !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