Università degli Studi di Cagliari

Corso di Laurea Magistrale in Ingegneria delle Tecnologie per Internet

MOLDULATIONS OVER WIRELESS CHANNEL

- ✓ PSK – reminder
- ✓ OQPSK
- ✓ π/4 QPSK
- ✓ CPFSK
- ✓ MSK
- ✓ Gaussian MSK

Introduction

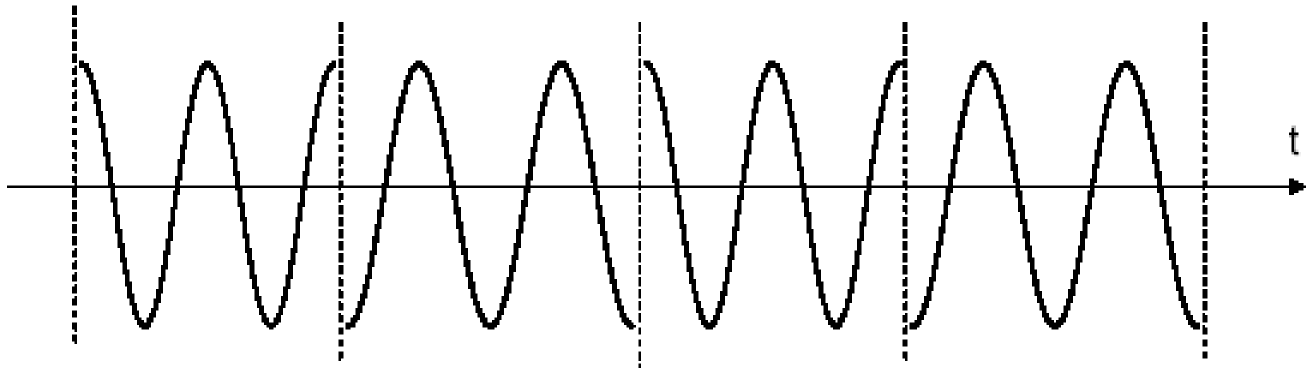
- ✓ The purpose is to describe modulations used for transmission over wireless channels.

- ✓ These are derived from PSK digital modulation, in the sense that they retain the single most attractive feature of PSK (its constant envelope), but reduce certain undesired effects.

PSK

Phase Shift Keying

- ✓ In the PSK modulation the information, in digital form, is contained in the carrier phase.



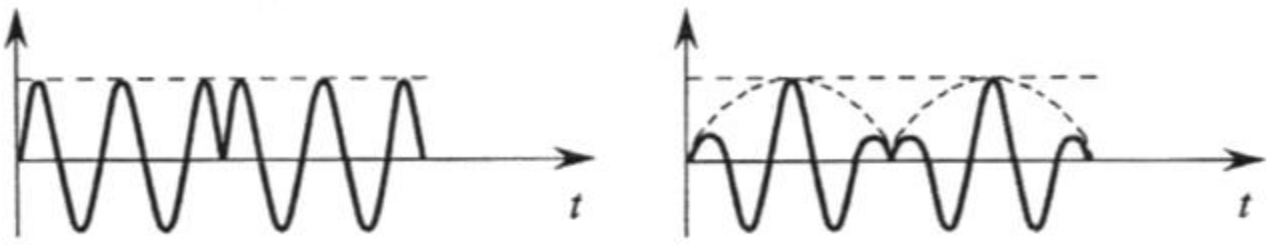
Phase Shift Keying

- Advantages:
  - Constant envelope ➡ constant transmission power
  - High spectral efficiency
  - No power loss on the carrier

- Disadvantages:
  - High sidelobes level in power spectrum
  - Filtering needed ➡ loss of constant envelope

Quadrature Phase Shift Keying - QPSK

- ✓ Phase reversals of $\pm\pi$ cause the envelope to go to zero momentarily. This may make us susceptible to non-linearity in power amplifier circuitry.
- ✓ Nonlinear amplifiers operated at saturation tend to restore the constant envelope of the signal, but enhancing the out-of-band spectral lobes.
- ✓ A solution to the above mentioned problem is the use of OQPSK.



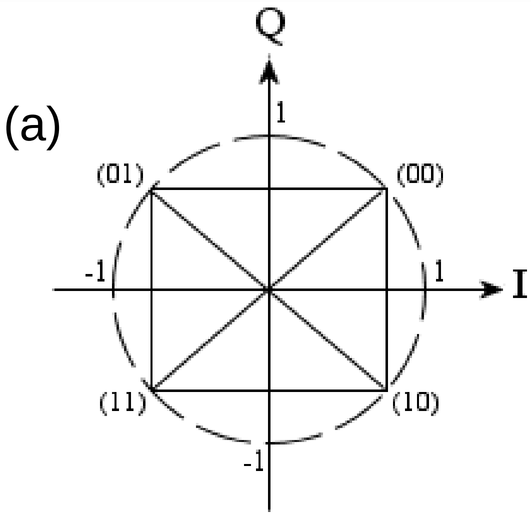
QPSK - ideal QPSK - filtered

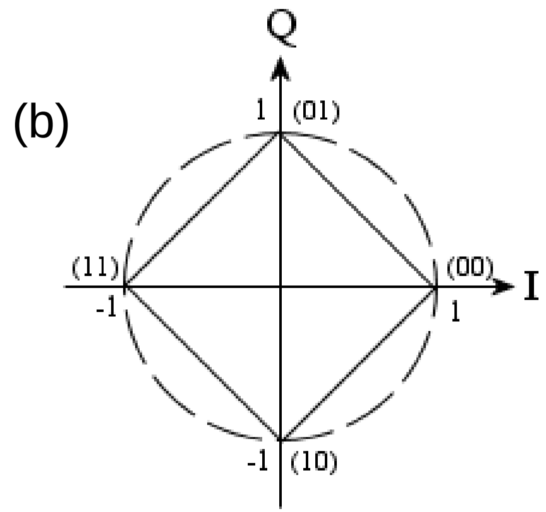
OFFSET QPSK

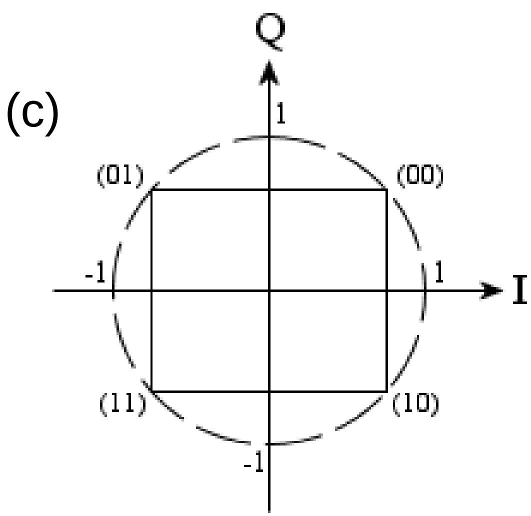
OQPSK

- ✓ Offset QPSK modulation is such that phase transitions of $\pm\pi$ are avoided.
- ✓ A reduction of the envelope fluctuations of QPSK signals is made possible by delaying the Q-channel digits by T_s seconds relative to the I-channel.
- ✓ This solution eliminates the possibility of $\pm\pi$ phase changes.
- ✓ Therefore, it may be expected that the undesired envelope variations due to filtering are greatly reduced, as is the dynamic range required from the power amplifier.

QPSK and OQPSK

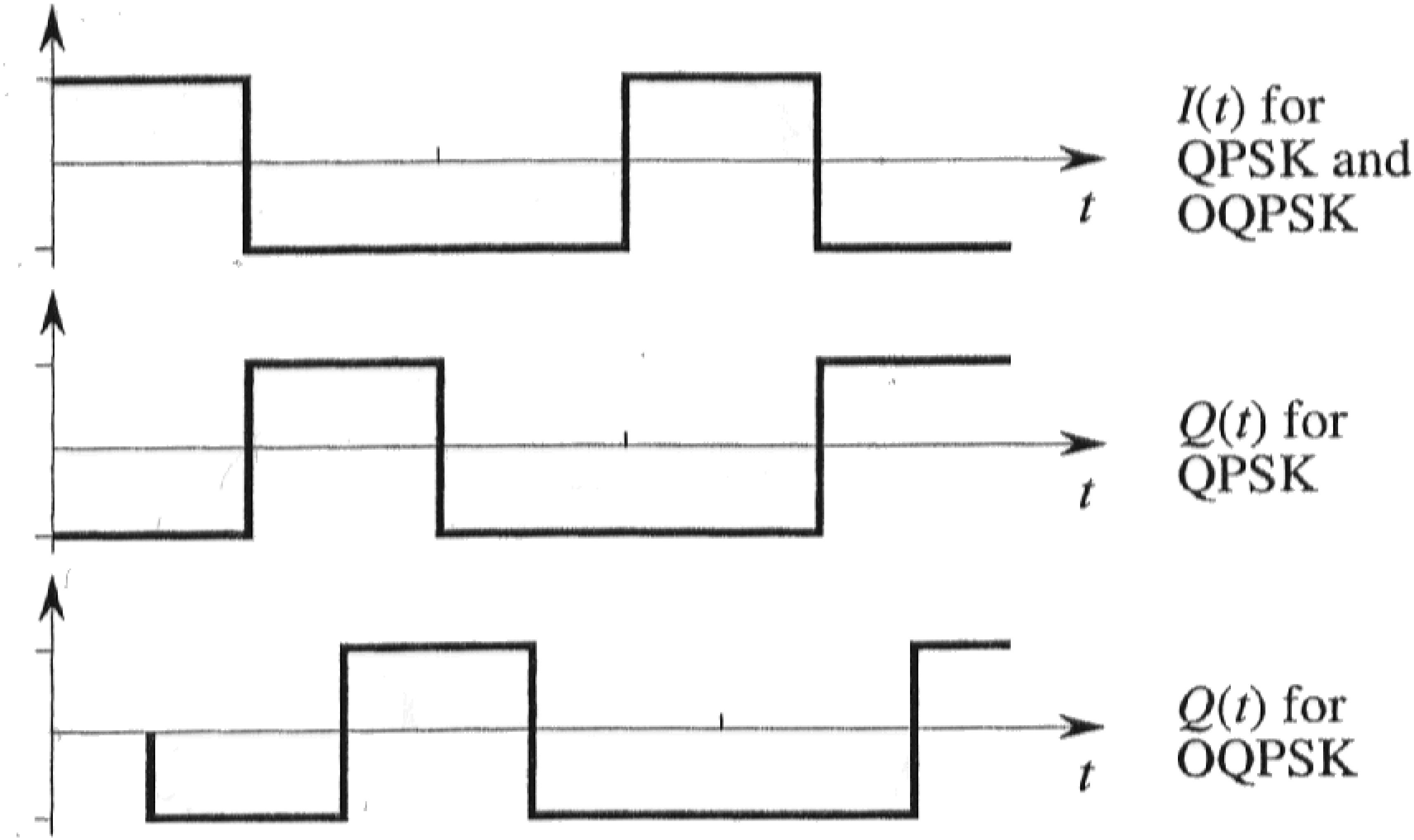






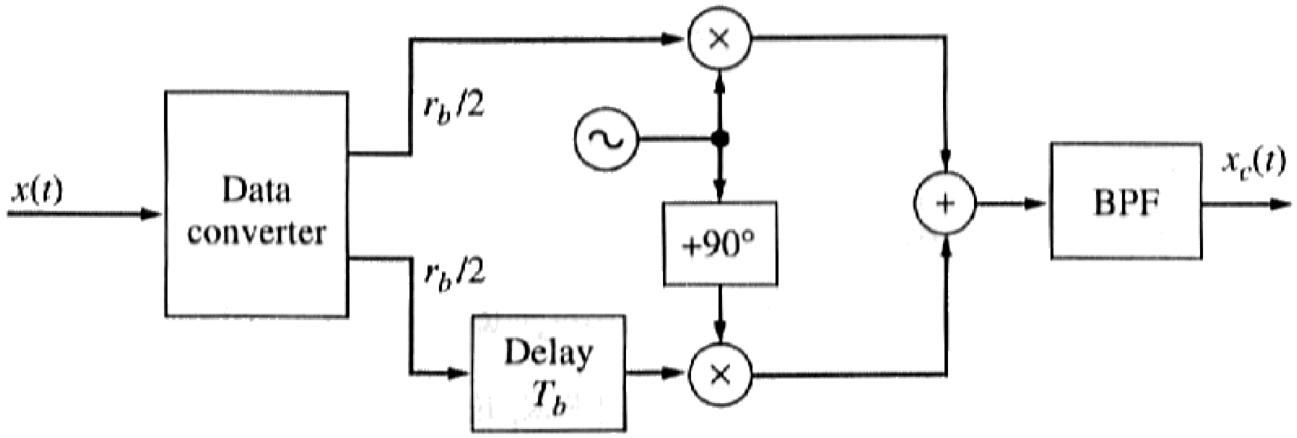
- (a) QPSK with N=1.
- (b) QPSK with N=0.
- (c) OQPSK with N=1:
    - Phase transitions about the origin are avoided.

QPSK and OQPSK



Modulator

- ✓ 4 symbols ➡ $T = 2T_s$



OQPSK - Analytic Form

- ✓ For QPSK:

$$x_c(t) = A_c \Sigma_k \cos(\omega_c t + \phi_k) \Pi\left(\frac{t-kT}{T}\right)$$

$$= A_c \Sigma_k \left[\cos(\omega_c t)\cos(\phi_k) - \sin(\omega_c t)\sin(\phi_k)\right] \Pi\left(\frac{t-kT}{T}\right)$$

OQPSK - Analytic Form

- ✓ As we have seen, the Q component is delayed of T_s seconds relative to the I component. So, in the OQPSK case:

$$x_c(t) = A_c \Sigma_k \left[\cos(\omega_c t)\cos(\phi_k)\Pi\left(\frac{t-kT}{T}\right) - \sin(\omega_c t)\sin(\phi_k)\Pi\left(\frac{t-kT-T_s}{T}\right) \right]$$

$$x_c(t) = A_c\left[x_i(t)\cos(\omega_c t) - x_q(t)\sin(\omega_c t)\right]$$

OQPSK - Analytic Form

- Reminder:

$$\phi_k = \pi \frac{2a_k + N}{M} \qquad N = 0,1$$

$$a_k = 0,1,2,3$$

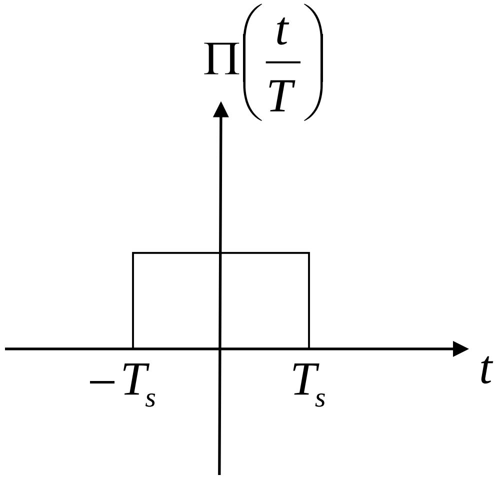
$$x_i(t) = \Sigma_k \cos(\phi_k)\Pi\left(\frac{t-kT}{T}\right) = \Sigma_k I_k \Pi\left(\frac{t-kT}{T}\right)$$

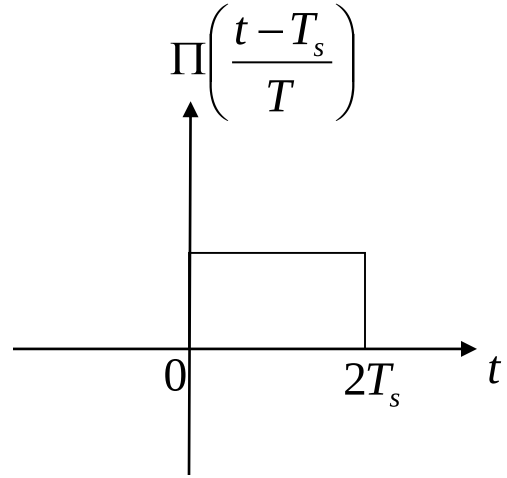
$$x_q(t) = \Sigma_k \sin(\phi_k)\Pi\left(\frac{t-kT-T_s}{T}\right) = \Sigma_k Q_k \Pi\left(\frac{t-kT-T_s}{T}\right)$$

OQPSK - Analytic Form

- ✓ In our case $T = 2T_s$, so:

$$\Pi\left(\frac{t}{T}\right) \begin{cases} 1, & -T_s \le t < T_s \\ 0, & elsewhere \end{cases}$$





OQPSK - Power Spectral Density

- ✓ The power spectral density of the transmitted signal is not affected by the delay incurred by one of the quadrature components, and hence is the same as for QPSK.

$$m_a = E\{I_k\} = E\{Q_k\} = 0$$

$$\sigma_a^2 = E\{I_k^{\,2}\} = E\{Q_k^{\,2}\} = \frac{1}{2}$$

$$F(f) = \Im\left\{\Pi\left(\frac{t}{2T_s}\right)\right\} = 2T_s \mathrm{sinc}(\pi f\, 2T_s)$$

OQPSK - Power Spectral Density

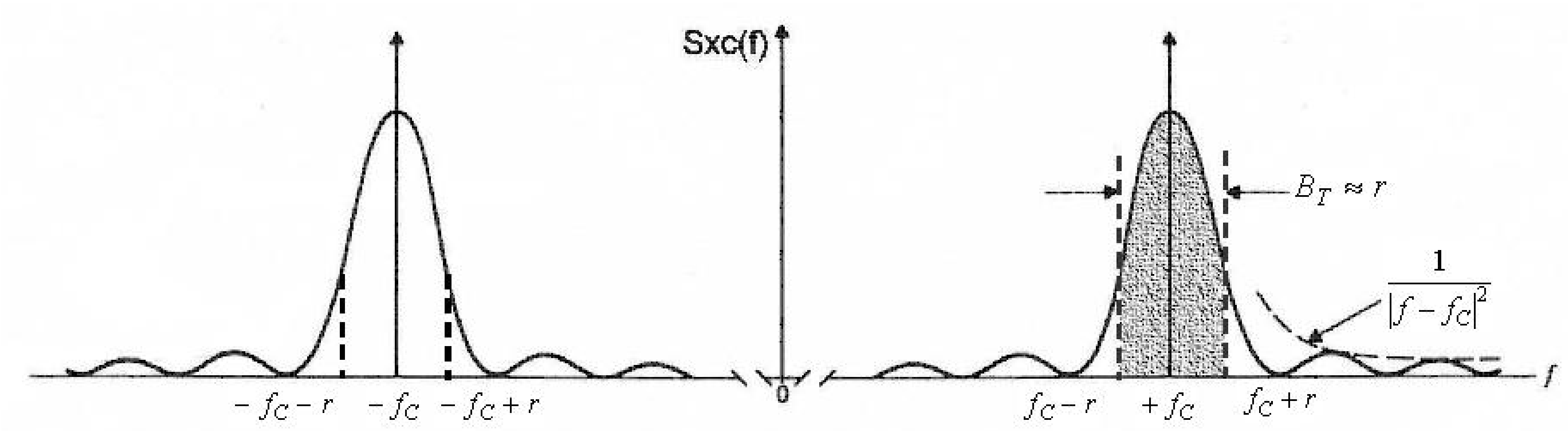
$$S_{xi}(f) = \frac{\sigma_a^2}{T}\left|F(f)\right|^2 = \frac{1}{4T_s} 4T_s^2 \mathrm{sinc}^2\left(\pi f\, 2T_s\right)$$

$$S_{xq}(f) = \frac{\sigma_a^2}{T}\left|F(f)e^{-j2\pi f T_s}\right|^2 = \frac{\sigma_a^2}{T}\left|F(f)\right|^2\left|e^{-j2\pi f T_s}\right|^2 = S_{xi}(f)$$

$$S_x(f) = S_{xi}(f) = T_s\, \mathrm{sinc}^2\left(\pi f\, 2T_s\right)$$

$$S_{xc}(f) = \frac{A_c^2}{4}\left[2S_x(f+f_c) + 2S_x(f-f_c)\right]$$

OQPSK - Power Spectral Density



$$B_T \approx r$$

$$r_b = r \log_2 M$$

$$\Rightarrow \frac{r_b}{B_T} \cong \log_2 M \quad bps/Hz$$

OQPSK-Receiver and Error Performance

- ✓ The coherent receiver for OQPSK is identical to that of QPSK, with the only change being the delay of the I-stream by T_s so that the two pulses carrying the even and odd symbols are realigned in time.

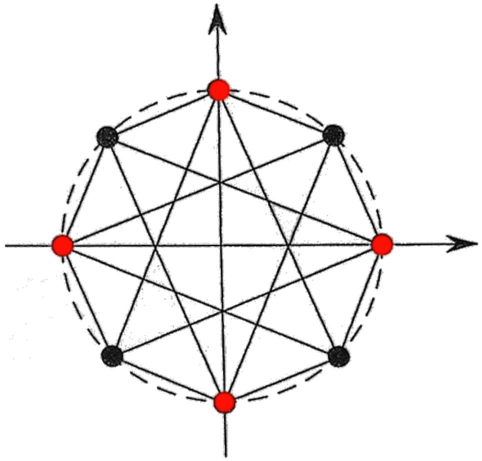
- ✓ Consequently, the error performance of this modulation is identical to that of QPSK.

π/4 QPSK

π/4 QPSK

- ✓ This is a modulation scheme derived from QPSK, offering a trade off between the MSK and OQPSK in terms of phase transitions.

- ✓ π/4 QPSK uses 8 phases to carry 2 information bits per modulated symbol and has a maximum phase transition of 135°.



$\pi/4$ QPSK

- ✓ The idea here is to use two different QPSK signal constellation shifted by $\pi/4$ and to move from one to the other in every symbol interval.
- ✓ So in each interval we have a phase transition of at least $\pi/4$.

π/4 QPSK – Analytic Form

$$x_i(t) = \Sigma_k \cos(\phi_k + k\frac{\pi}{4})\Pi\left(\frac{t-kT}{T}\right) = \Sigma_k I_k \Pi\left(\frac{t-kT}{T}\right)$$

$$x_q(t) = \Sigma_k \sin(\phi_k + k\frac{\pi}{4})\Pi\left(\frac{t-kT}{T}\right) = \Sigma_k Q_k \Pi\left(\frac{t-kT}{T}\right)$$

$$\phi_k \in \left(0, \pm\pi/2, \pi\right)$$

π/4 QPSK

Power Density Spectrum and Demodulator

- ✓ We can consider this system as two independent forms of QPSK, so it has the same power spectrum.

$$S_x(f) = S_{xi}(f) = T_s \,\mathrm{sinc}^2\left(\pi f \, 2T_s\right)$$

- ✓ Coherent demodulation can be achieved by feeding a standard QPSK demodulator with a received signal sequence shifted by π/4 every $2T_s$.

Continuous-Phase Modulation

CPM

- ✓ A general family of modulations, which retains the basic feature of PSK (and FSK) of having a constant envelope, while decreasing the spectrum occupancy of the latter by smoothing the phase transitions of the transmitted signal.

$$x_c(t) = A_c \cos\left(2\pi f_c t + \vartheta(t, a_k)\right)$$

$$\vartheta(t, a_k) = 2\pi f_d \int_0^t \sum_k a_k g(\tau) d\tau = 2\pi f_d \int_0^t \sum_k a_k \Pi\left(\frac{\tau - T/2 - kT}{T}\right) d\tau$$

CPM

- To improve PSK spectral efficiency we can avoid discontinuities in the phase function. One may think of smoothing out the phase discontinuities, which is precisely the idea underlying continuous-phase modulation (CPM).

- The signal we are going to obtain is a special case of CPM called CPFSK. The most general form of CPM will be described later, it allows to use different shape-pulses with a duration greater than T.

CPFSK

$$\int_0^t \sum_k a_k \Pi\left(\frac{\tau - T/2 - kT}{T}\right) d\tau = \sum_k a_k \int_0^t \Pi\left(\frac{\tau - T/2 - kT}{T}\right) d\tau =$$

$$= a_0 t \qquad 0 < \mathrm{t} < \mathrm{T}$$

$$= a_0 T + a_1 (t - T) \qquad \mathrm{T} < \mathrm{t} < 2\mathrm{T}$$

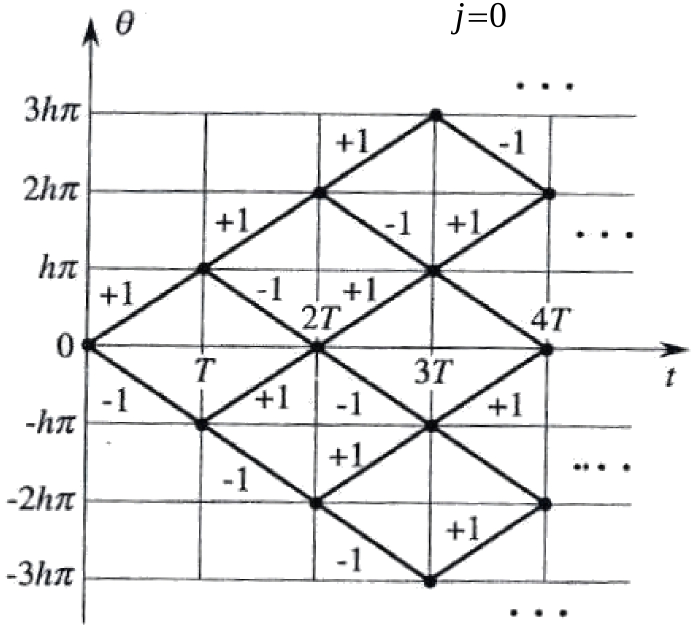
$$so,\ \vartheta(t, a_k) = 2\pi f_d \left[\sum_{j=0}^{k-1} a_j T + a_k (t - kT) \right] \quad k\mathrm{T} < \mathrm{t} < (\mathrm{k}+1)\mathrm{T}$$

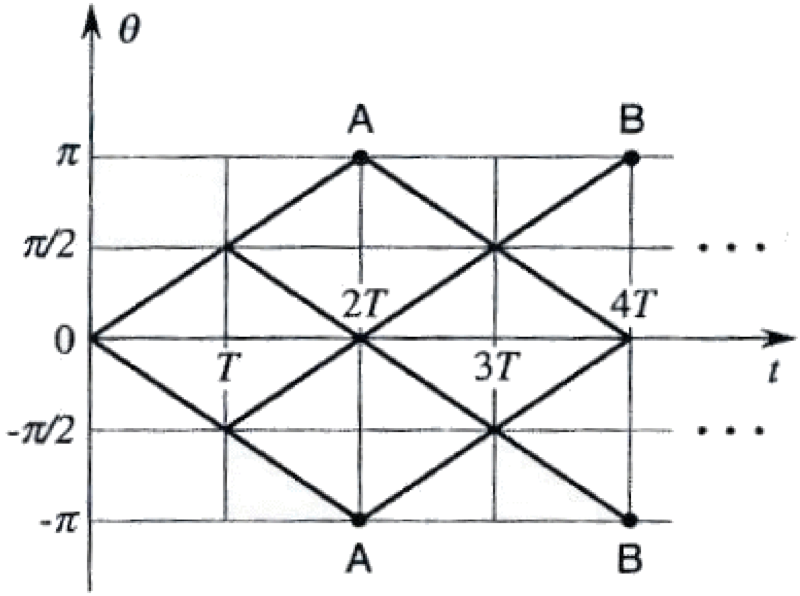
✓ Note that the frequency pulse area equals T.

CPFSK

$$x_c(t) = A_c \sum_{k=0}^{\infty} \cos\left(2\pi f_c t + \phi_k + 2\pi f_d a_k (t - kT)\right) \Pi\left(\frac{t - T/2 - kT}{T}\right)$$

where $\phi_k = 2\pi f_d T \sum_{j=0}^{k-1} a_j = h\pi \sum_{j=0}^{k-1} a_j$





CPFSK

Time varying vs time invariant trellises

- ✓ Those phase trellises are time varying, in the sense that the phase trajectories in the even numbered symbol intervals are not time translations of those in odd numbered symbol interval.

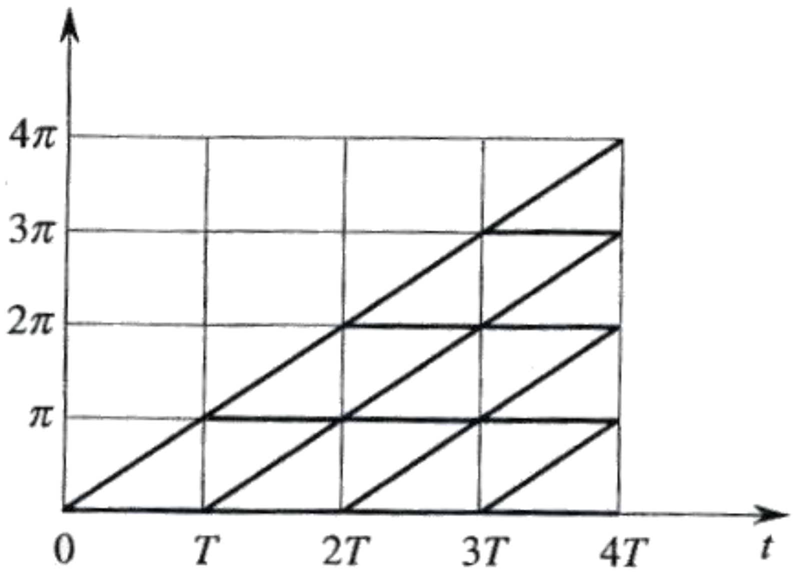
- ✓ It's important for many application (i.e., for Viterbi algorithm) to have a time-invariant trellis.

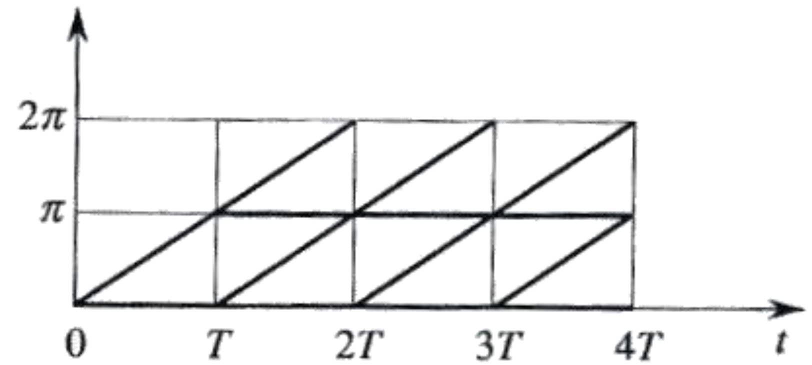
$$\phi_k' = h\pi \sum_{j=0}^{k-1} a_j + h\pi (M-1)\frac{t}{T}$$

CPFSK

Time varying vs time invariant trellises

- ✓ By doing this little change we obtain T-translations invariant trellises.





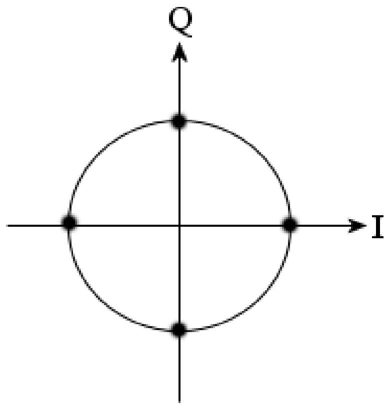
Minimum Shift-Keying

MSK as CPFSK

- ✓ We shall now show that MSK can be viewed as a special form of FSK.

$$a_k = \pm 1, \quad h = 0.5 \qquad \phi_k \Big|_{k=0} = 0$$

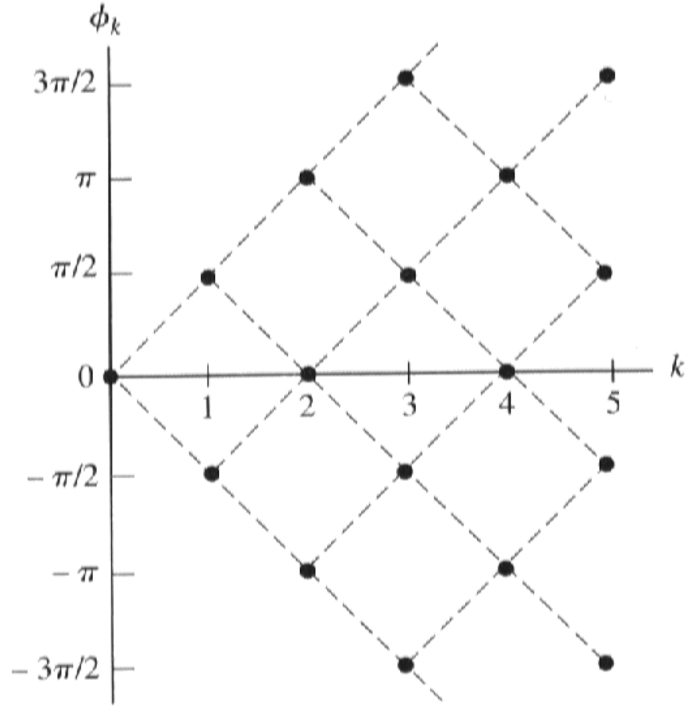
$$\phi_k = \frac{\pi}{2} \sum_{j=0}^{k-1} a_j$$



- ✓ Where $a_k = +1$ for input bit 1, $a_k = -1$ for input bit 0 and the phase shift ϕ_k depends on the previous digits. This phase shift ensures phase continuity for all t (MSK is a binary CPFSK).

MSK Phase Trellis

- ✓ We can draw the possible variations of ϕ_k



MSK Phase Trellis

- ✓ This pattern clearly reveals that:
  - ➢ $\phi_k = 0, \pm\pi, \pm 2\pi, \ldots$ for even values of k
  - ➢ $\phi_k = \pm\pi/2, \pm 3\pi/2, \ldots$ for odd values of k

- ✓ This means that:

$$\cos(\phi_k) = \pm 1, \ \ k \text{ even}$$

$$\sin(\phi_k) = \pm 1, \ \ k \text{ odd}$$

$$\Rightarrow \quad m_a = 0, \ \ \sigma_a^2 = 1$$

MSK vs OQPSK

$$x_c(t) = A_c \sum_{k=0}^{\infty} \cos\left(2\pi f_c t + \phi_k + \frac{\pi}{2T} a_k (t - kT)\right) \Pi\left(\frac{t - T/2 - kT}{T}\right)$$

- ✓ It can be proved that:

$$x_i(t) = \sum_{k\ even} \cos\phi_k\, f(t - kT_s)$$

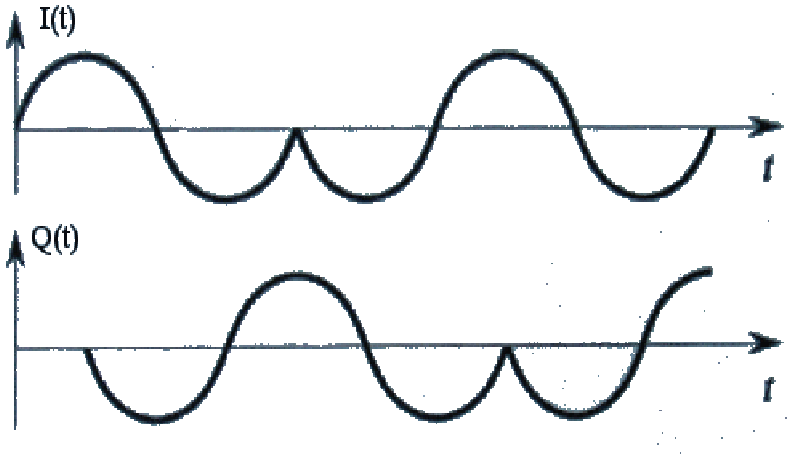
$$x_q(t) = \sum_{k\ odd} \sin\phi_k\, f(t - kT_s)$$

MSK

- So this modulation can be viewed as a modification of OQPSK obtained by shaping the transmitted pulse with a half-sinusoid to improve bandwidth efficiency by reducing sidelobes level.

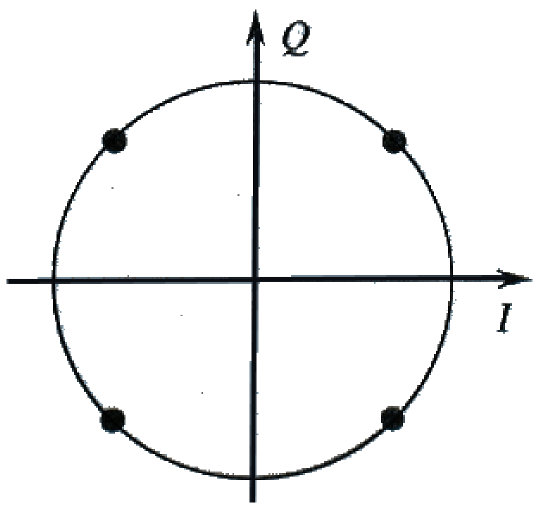
$$f(t) = \cos\left(\frac{\pi t}{2T_s}\right)\Pi\left(\frac{t}{2T_s}\right)$$

$$f(t-T_s) = \sin\left(\frac{\pi t}{2T_s}\right)\Pi\left(\frac{t-T_s}{2T_s}\right)$$

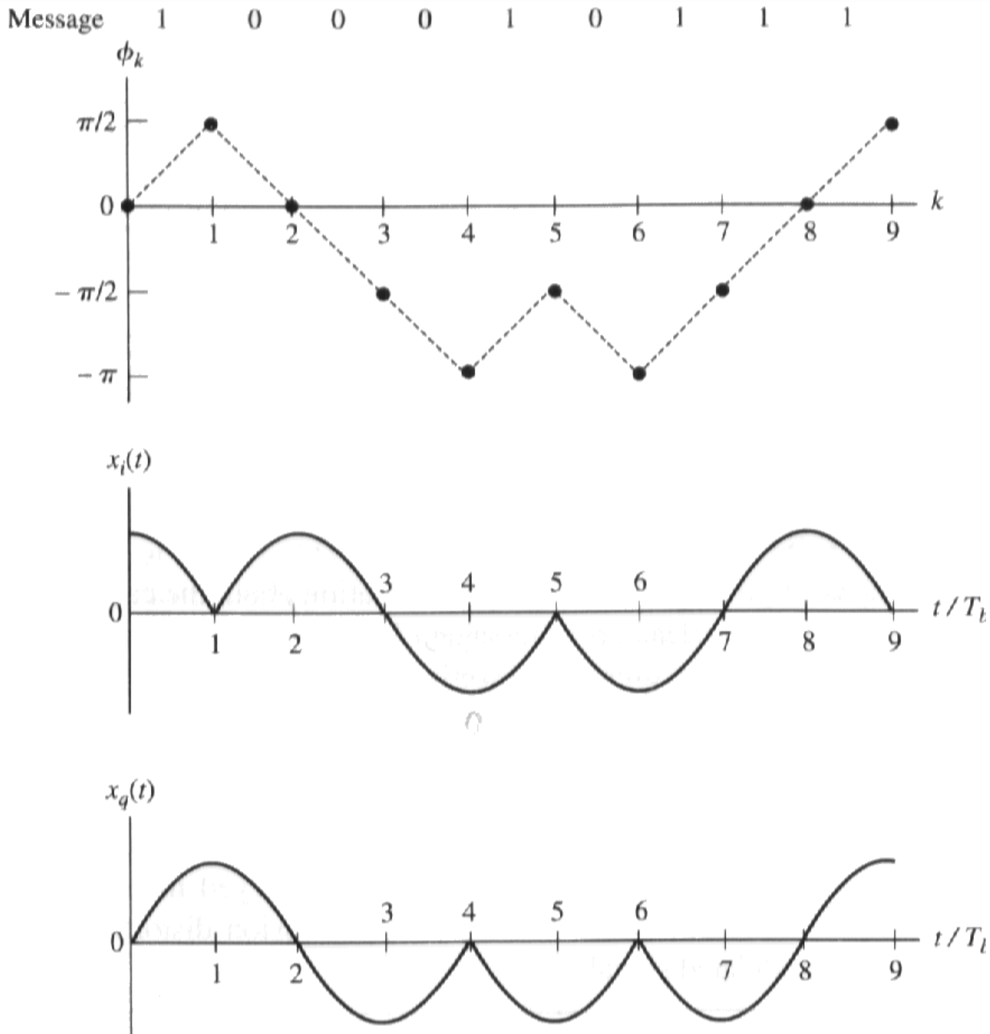


MSK

- ✓ Shaping waveforms causes phase transitions as shown in figure. MSK avoids phase transitions about the origin, too.



MSK


Message 1 0 0 0 1 0 1 1 1
ϕ_k
$\pi/2$
0
$-\pi/2$
$-\pi$
k
1 2 3 4 5 6 7 8 9
$x_i(t)$
t/T_b
$x_q(t)$
t/T_b

MSK - Analytic Form

- Phase and quadrature components:

$$x_i(t) = \sum_k \cos(\phi_k) f(t - kT) = \sum_k I_k f(t - kT)$$

$$x_q(t) = \sum_k \sin(\phi_k) f(t - kT - T_s) = \sum_k Q_k f(t - kT - T_s)$$

MSK - Analytic Form

$$x_i(t) = \sum_k \cos(\phi_k)\cos\left(\frac{\pi t}{2T_s}\right)\Pi\left(\frac{t-kT}{2T_s}\right) = \cos\left(\frac{\pi t}{2T_s}\right)\Delta_i$$

$$x_q(t) = \sin\left(\frac{\pi t}{2T_s}\right)\Delta_q$$

$$\Delta_i = \sum_k \cos(\phi_k)\Pi\left(\frac{t-kT}{2T_s}\right), \quad \Delta_q = \sum_k \sin(\phi_k)\Pi\left(\frac{t-kT-T_s}{2T_s}\right)$$

MSK - Analytic Form

- We can write, after using some simple trigonometry (Werner):

$$x_c(t) = A_c\left[x_i(t)\cos(\omega_c t) - x_q(t)\sin(\omega_c t)\right] =$$

$$= A_c\left[\cos\left(\frac{\pi t}{2T_s}\right)\cos(\omega_c t)\Delta_i - \sin\left(\frac{\pi t}{2T_s}\right)\sin(\omega_c t)\Delta_q\right] =$$

$$= \frac{A_c}{2}\Delta_i\left[\cos\left(2\pi\left(f_c - \frac{1}{4T_s}\right)\right) + \cos\left(2\pi\left(f_c + \frac{1}{4T_s}\right)\right)\right] +$$

$$- \frac{A_c}{2}\Delta_q\left[\cos\left(2\pi\left(f_c - \frac{1}{4T_s}\right)\right) - \cos\left(2\pi\left(f_c + \frac{1}{4T_s}\right)\right)\right]$$

MSK - Considerations

- ✓ The expression of the modulated signal shows that MSK can be interpreted as a form of frequency-shift keying with frequencies

$$\left(f_c - \frac{1}{4T_s}\right), \quad \left(f_c + \frac{1}{4T_s}\right)$$

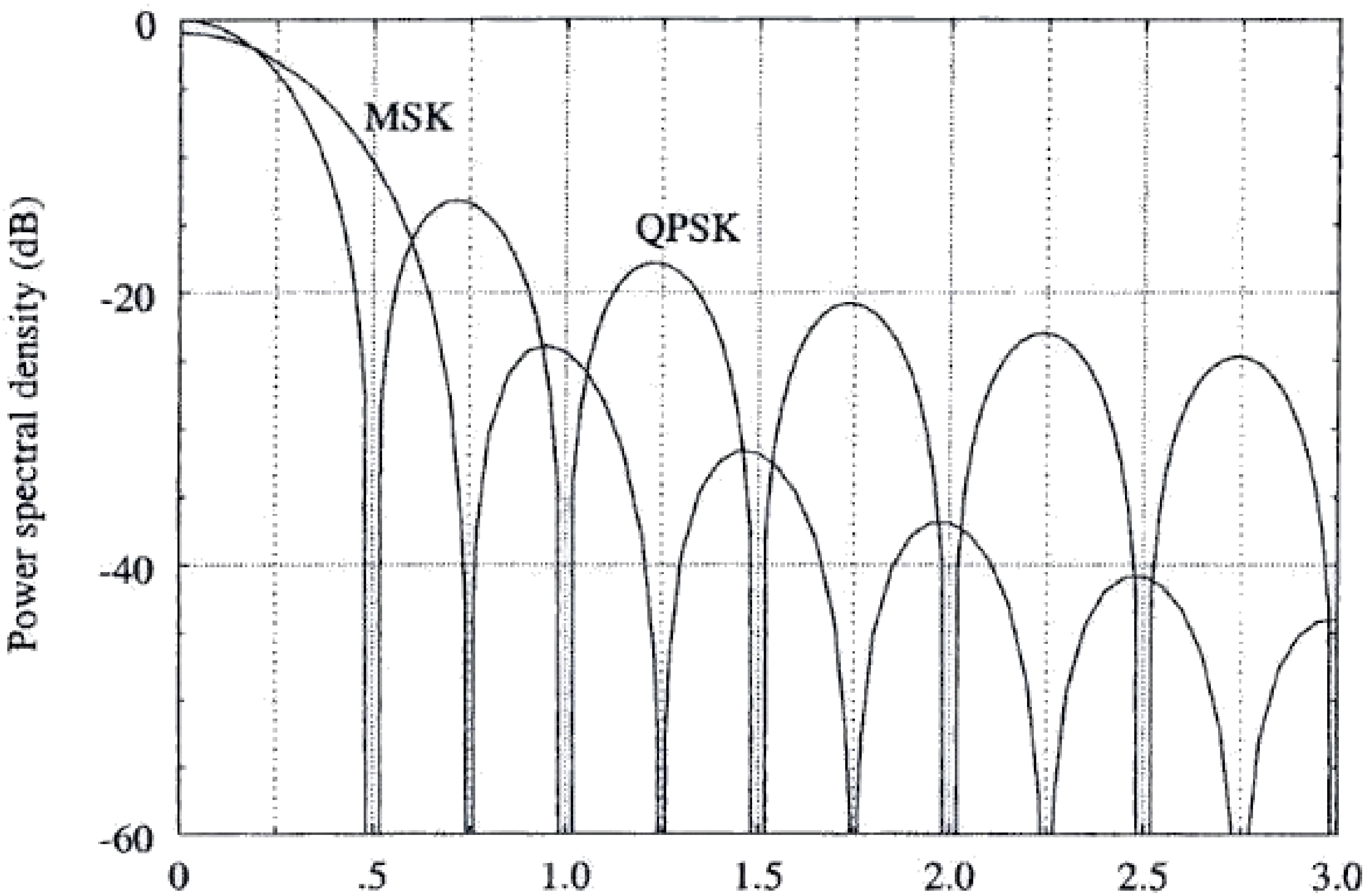
- ✓ The frequency spacing is the smallest for orthogonality. This is the reason why this modulation scheme is called minimum-shift keying.

MSK - Power Spectral Density

- ✓ In the following figure we compare MSK and QPSK power spectra. It can be noticed how the main lobe of the MSK is wider than that of QPSK and its sidelobes decrease more steadily than those of QPSK (OQPSK and QPSK have the same spectrum).

$$F(f) = T_s \frac{4}{\pi} \frac{\cos(2\pi f T_s)}{1 - 16 f^2 T_s^2} \qquad m_a = 0, \ \sigma_a^2 = 1$$

$$S_x(f) = \frac{\sigma_a^2}{T} \left| F(f) \right|^2 = T_s \frac{8}{\pi^2} \left(\frac{\cos(2\pi f T_s)}{1 - 16 f^2 T_s^2} \right)^2$$


0
-20
-40
-60
Power spectral density (dB)
MSK
QPSK
0
.5
1.0
1.5
2.0
2.5
3.0