Università degli Studi di Cagliari

Corso di Laurea Magistrale in Ingegneria delle

Tecnologie per Internet

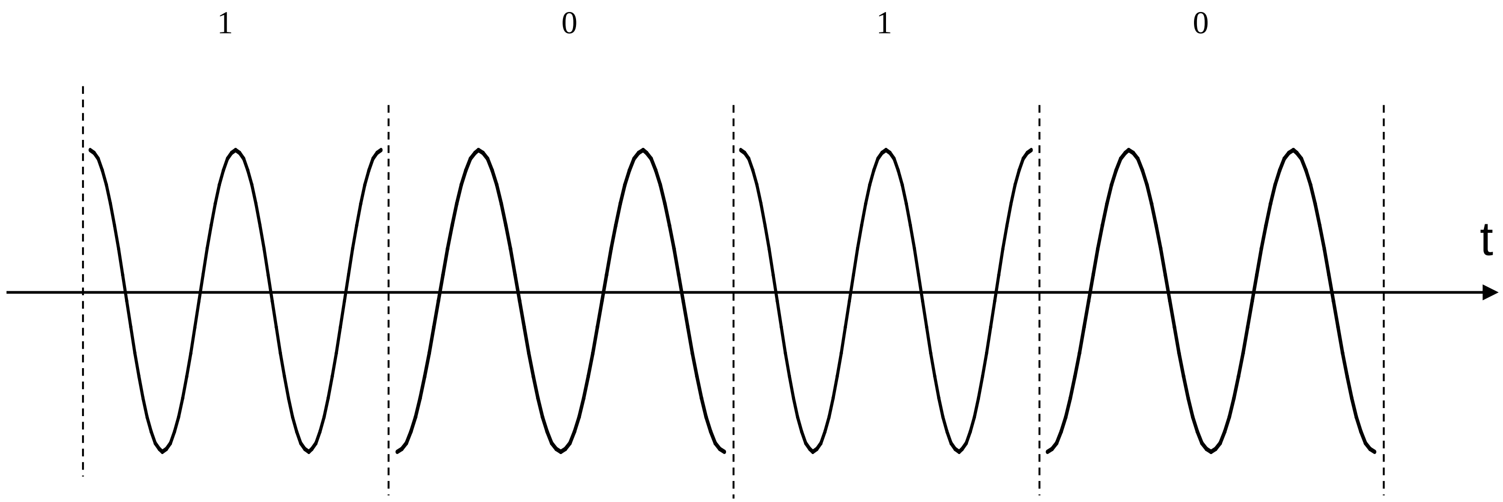
DIGITAL EXPONENTIAL MODULATIONS

PSK, FSK

PHASE SHIFT KEYING
PSK

Phase Shift Keying

✓ In the PSK (Phase Shift Keying) the information, in digital form, is contained in the carrier phase.



Phase Shift Keying

- In the M-ary PSK:

$$x_c(t) = \sum_{-\infty}^{+\infty} \cos(\omega_c t + \varphi_k + \vartheta) g_T(t - kT) \qquad a_k = 0,1,\ldots,M-1$$

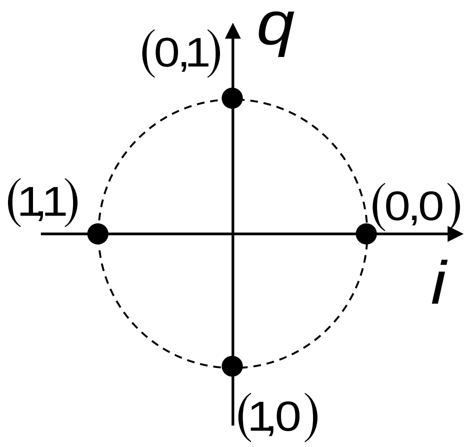
$$\varphi_k = \pi \frac{(2a_k + N)}{M} \qquad N = 0,1$$

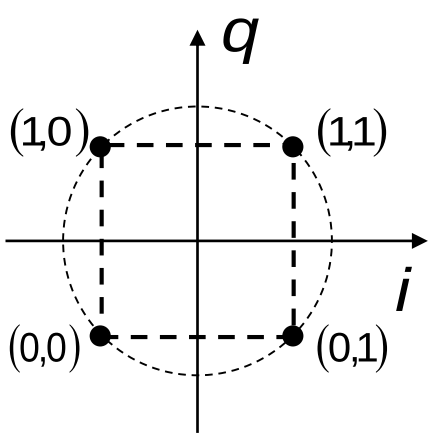
T → time M→ levels

$$T = T_b \log_2 \mathrm{M}$$

Phase Shift Keying

✓ Example





$M = 4 \qquad N = 0$

$M = 4 \qquad N = 1$

Phase Shift Keying

- Analyzing the PSK power spectra (supposed that $\theta=0$):

$$x_c(t) = A_C \sum_{-\infty}^{+\infty} \left(\cos\varphi_k \cos\omega_c t - \operatorname{sen}\varphi_k \operatorname{sen}\omega_c t \right) \cdot g_T(t - kT)$$

i component

$$x_i(t) = \sum_{-\infty}^{+\infty} \cos\varphi_k \cdot g_T(t - kT)$$

q component

$$x_q(t) = \sum_{-\infty}^{+\infty} \operatorname{sen}\varphi_k \cdot g_T(t - kT)$$

Phase Shift Keying

$$\left.\begin{aligned} m_a &= E\{\cos\varphi_k\} = E\{\mathrm{sen}\,\varphi_k\} = 0 \\ \sigma_a{}^2 &= E\{\cos^2\varphi_k\} = E\{\mathrm{sen}^2\varphi_k\} = \frac{1}{2} \end{aligned}\right\} \Rightarrow \quad Sx_i(t) = Sx_q(t)$$

thus **i** and **q** component are uncorrelated then:

$$S_{Xi}(f) = S_{Xq}(f) = \sigma_a{}^2 r |G(f)|^2 + (m_a r)^2 \sum_{+\infty}^{+\infty} |G(nr)|^2 \delta(f - nr)$$

Phase Shift Keying

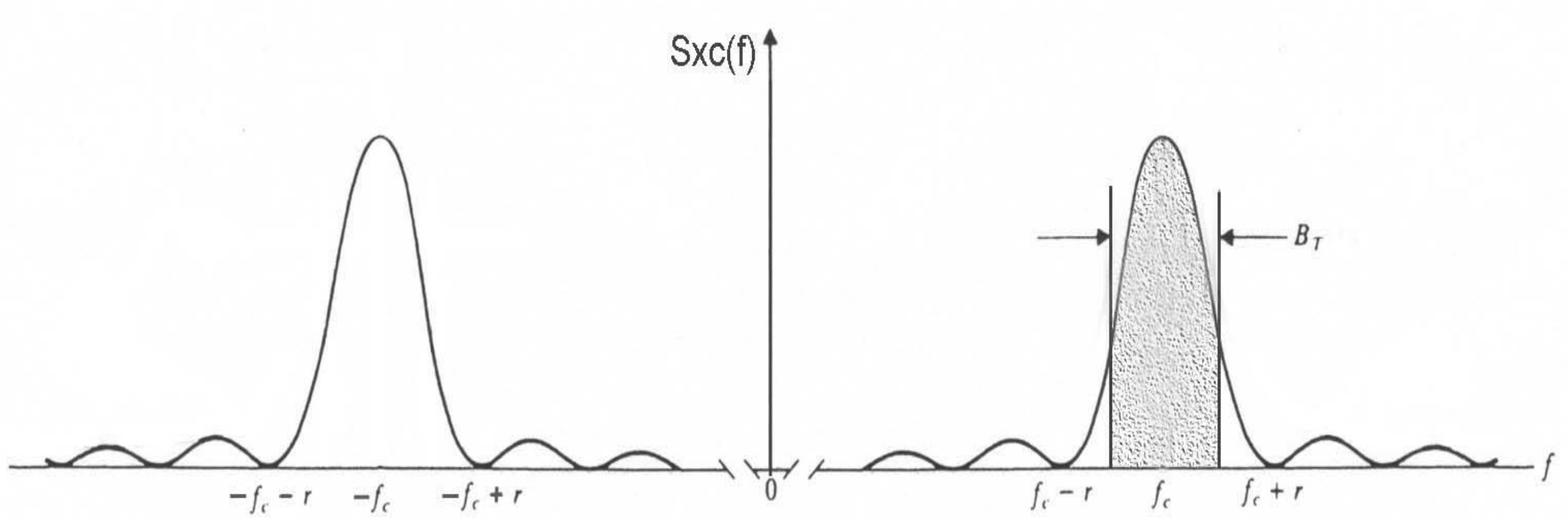
$$S_{Xi}(f) = S_{Xq}(f) = \frac{1}{2} r \left|G(f)\right|^2 = \frac{1}{2} r \frac{1}{r^2} sinc^2 \frac{\pi f}{r} = \frac{1}{2r} sinc^2 \frac{\pi f}{r}$$

$$S_{X_C}(f) = \frac{A_C^{\,2}}{4}\left[S_{Xi}(f)(f - f_C) + S_{Xi}(f)(f + f_C)\right] =$$

$$S_{X_C}(f) = \frac{A_C^{\,2}}{4}\frac{1}{r}\left[sinc^2 \frac{\pi(f - f_C)}{r} + sinc^2 \frac{\pi(f + f_C)}{r}\right]$$

✓ The same shape as ASK spectrum without the carrier-frequency impulse.

Phase Shift Keying



$$B_T \approx r$$

$$r_b = r \log_2 \mathrm{M}$$

$$\Longrightarrow \frac{r_b}{B_T} \cong \log_2 \mathrm{M} \quad bps/Hz$$

as the ASK case.

Phase Shift Keying

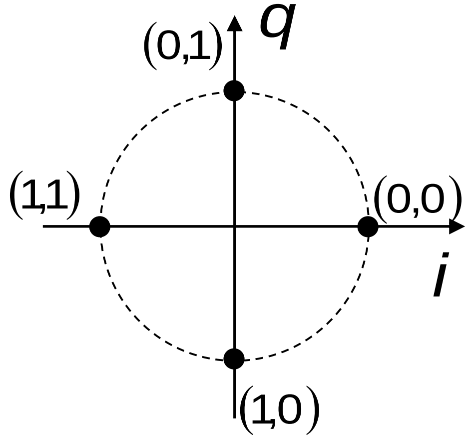
- ✓ PSK has better power efficiency than ASK (the discrete carrier component is absent).

- ✓ Example (QPSK)

  Quaternary or Quadriphase PSK

$$M = 4 \qquad N = 0$$

$$\varphi_k = \pi \frac{2a_k + N}{M} \qquad a_k = 0,1,2,3$$

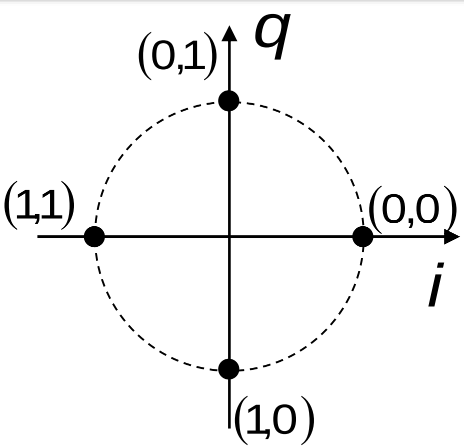


QPSK signal constellation

Phase Shift Keying

$$i \rightarrow \cos\varphi_k$$
$$q \rightarrow \mathrm{sen}\,\varphi_k$$

$$\varphi_k = \pi \frac{2a_k}{M} \quad a_k = 0,1,2,3$$



$$a_k = 0 \Rightarrow \varphi_k = 0 \Rightarrow \cos\varphi_k = 1 \Rightarrow \mathrm{sen}\,\varphi_k = 0$$

$$a_k = 1 \Rightarrow \varphi_k = \frac{\pi}{2} \Rightarrow \cos\varphi_k = 0 \Rightarrow \mathrm{sen}\,\varphi_k = 1$$

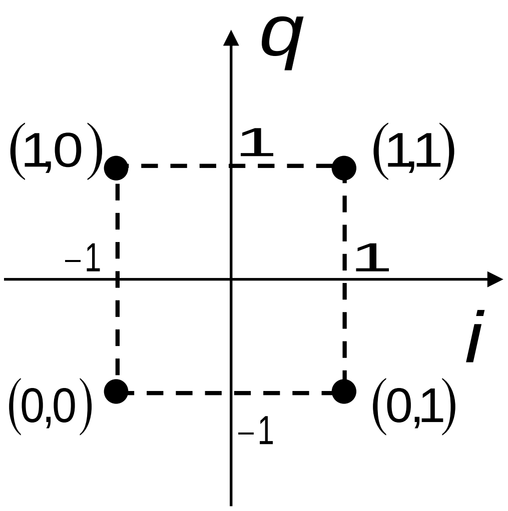
$$a_k = 2 \Rightarrow \varphi_k = \pi \Rightarrow \cos\varphi_k = -1 \Rightarrow \mathrm{sen}\,\varphi_k = 0$$

$$a_k = 3 \Rightarrow \varphi_k = \frac{3\pi}{2} \Rightarrow \cos\varphi_k = 0 \Rightarrow \mathrm{sen}\,\varphi_k = -1$$

Phase Shift Keying

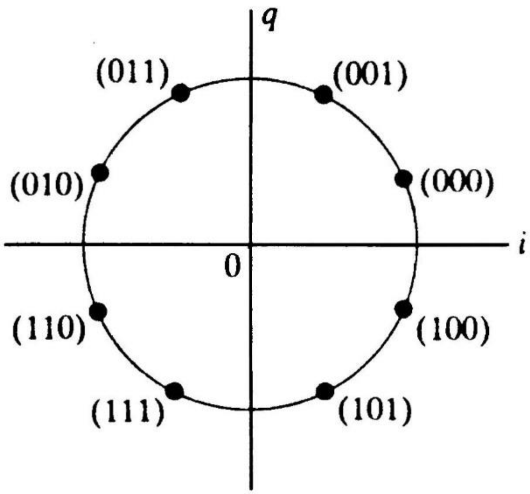
✓ Examples:

Binary QAM



$M = 4 \qquad N = 1$

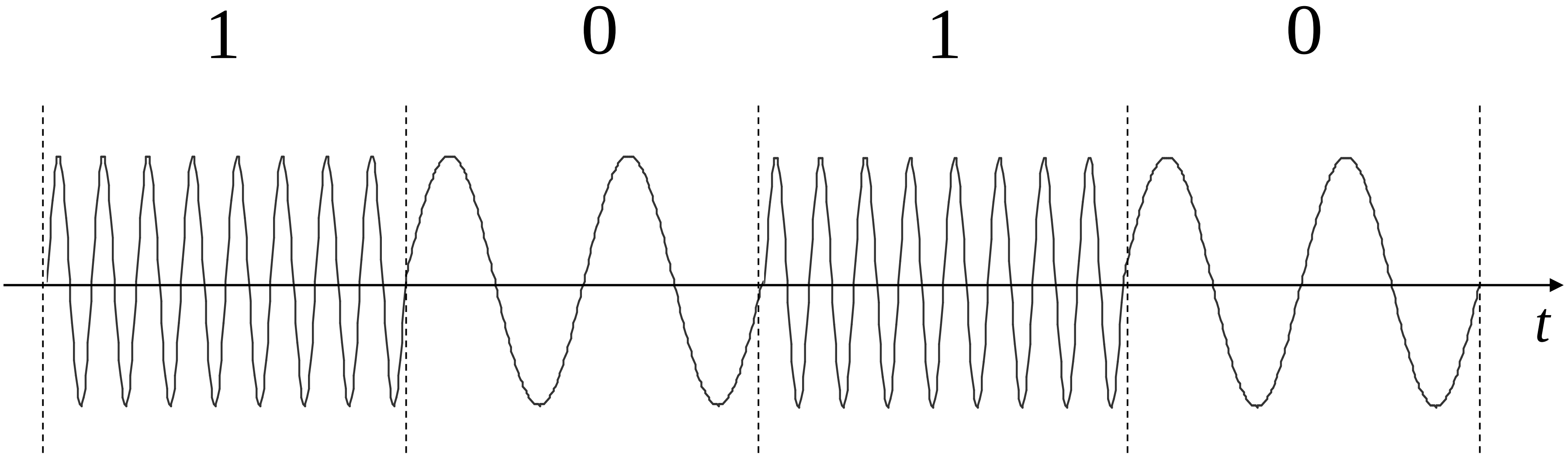
8-PSK



$M = 8 \qquad N = 1$

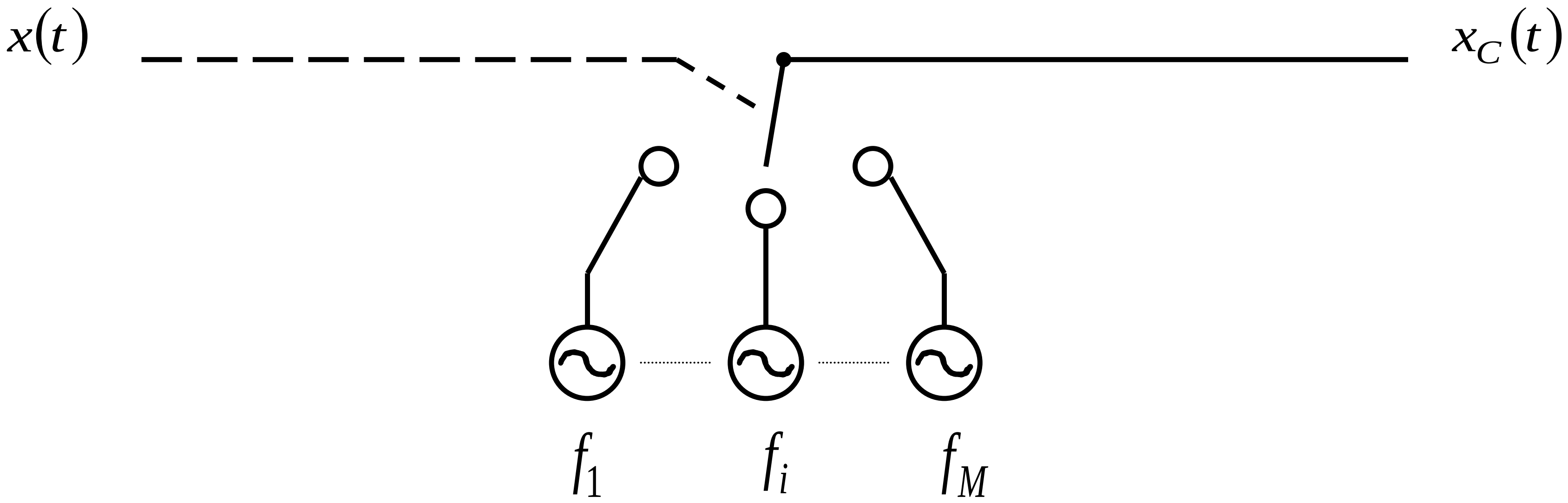
Frequency Shift Keyng (FSK)

- ✓ FSK : Frequency Shift Keying.
- ✓ Digital information is contained in the carrier frequency.



Frequency Shift Keyng (FSK)

- ✓ An M-ary FSK may be represented with a basic design as:



Frequency Shift Keyng (FSK)

- ✓ The digital signal x(t) controls a switch that selects the modulated frequency from a bank of M oscillators

- ✓ Problem: the modulated signal is discontinuous at every switching instant t=kT

Frequency Shift Keyng (FSK)

- ✓ Let all oscillators have the same amplitude A_c and phase ϑ and let their frequencies be related to a_k by:

$$f_k = f_k + f_\Delta a_k \qquad a_k = \pm 1, \pm 3, \ldots, \pm(M-1) \qquad M \quad even$$

- ✓ Then:

$$x_c(t) = A_C \sum_k \cos(\omega_c t + \vartheta + \omega_\Delta a_k t) \cdot g_T(t - kT) \qquad \omega_\Delta = 2\pi f_\Delta$$

Frequency Shift Keyng (FSK)

- ✓ Continuity of $x(t)$ at $t = kT$ is assured if $2\omega_{\Delta}T = 2\pi N$ where *N* is an integer.

- ✓ Condition to have continuity at every switching instant

$$\cos[\omega_c(t+T)+\vartheta+\omega_{\Delta}a_k(t+T)] = \cos[\omega_c(t+T)+\vartheta+\omega_{\Delta}a_{k+1}(t+T)]$$

$$\Rightarrow \quad \omega_{\Delta}a_kT - \omega_{\Delta}a_{k+1}T = 2\pi N \qquad N \quad integer \quad \Rightarrow$$

$$\Rightarrow \quad (a_k - a_{k+1})\omega_{\Delta}T = 2\pi N \qquad a_k = \pm1, \pm3, \pm5, \ldots$$

Frequency Shift Keyng (FSK)

- ✓ $(a_k - a_{k+1})$ minimum value is 2 (if there is switching):

$$\Rightarrow \qquad 2\omega_{\Delta}T = 2\pi N$$

Frequency Shift Keyng (FSK)

- We will analyze a version of binary FSK known as ***Sunde's FSK***.

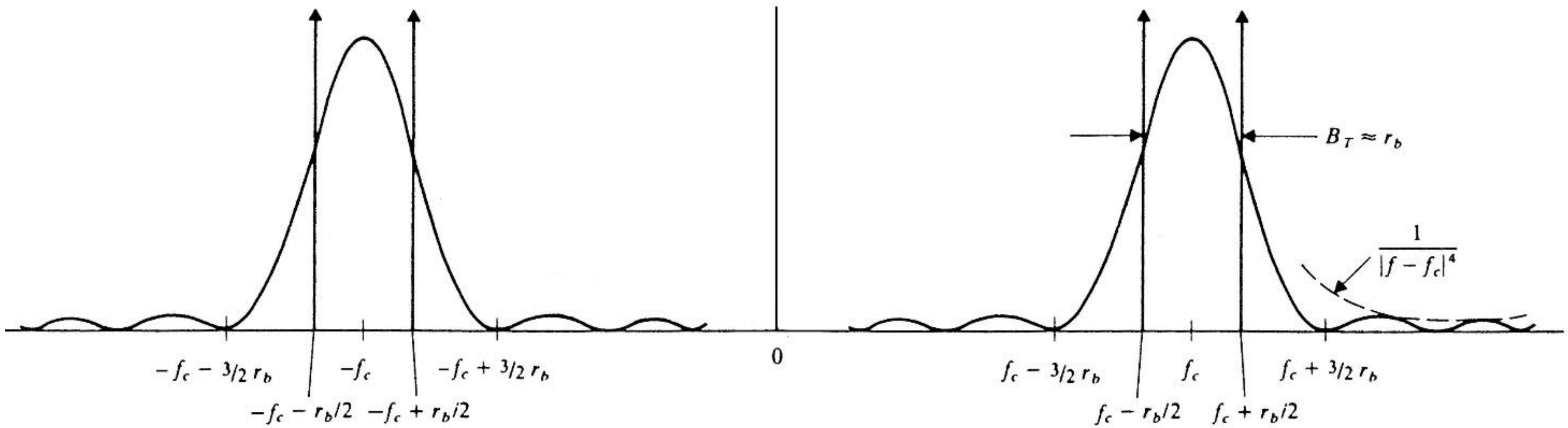
$$M = 2 \qquad a_k = \pm 1 \qquad T = T_b = \frac{1}{r_b} \qquad N = 1 \qquad f_\Delta = \frac{r_b}{2}$$

- Let's see if the condition to have continuity is verified:

$$2\omega_\Delta T = 2\pi N \qquad N \;\; integer \qquad \Rightarrow \qquad 2 \cdot 2\pi \cdot \frac{r_b}{2} T_b = 2\pi N \qquad \Rightarrow$$

$$\Rightarrow \qquad 2\pi = 2\pi N \qquad \Rightarrow \qquad N = 1 \qquad OK!$$

Frequency Shift Keyng (FSK)



$$B_T \approx r_b = 2 f_\Delta$$

Frequency Shift Keyng (FSK)

- ✓ The spectrum has a fourth order $\left(1/f^4\right)$ roll-off, more rapid than ASK and PSK.
- ✓ The central lobe is 50% wider than the central lobe of a binary ASK or PSK spectrum.

Frequency Shift Keyng (FSK)

- A special case is M-*ary orthogonal FSK*, in which the M keyed frequencies are equally spaced by:

$$2f_\Delta = \frac{1}{2T} = \frac{r}{2}$$

- Without attempting the spectral analysis, we can summarize that:

$$B_T \geq M \cdot 2f_\Delta = M\frac{r}{2}$$

Frequency Shift Keyng (FSK)

$$r_b = r\,\mathbf{log}_2\,\mathrm{M}$$

- ✓ The spectral efficiency is given by:

$$\frac{r_b}{B_T} \le \frac{r\,\mathbf{log}_2\,\mathrm{M}}{\mathrm{M}\,{}^{r}\!/_{2}} = \frac{2\,\mathbf{log}_2\,\mathrm{M}}{\mathrm{M}} \qquad {}^{bps}\!/_{Hz} \qquad \begin{cases} \dfrac{r_b}{B_T} = 1 & M = 2,\ 4 \\ \dfrac{r_b}{B_T} \le 1 & M > 4 \end{cases}$$

- ✓ The FSK spectral efficiency is worse than ASK and M-ary PSK.