Università degli Studi di Cagliari

Corso di Laurea Magistrale in Ingegneria delle

Tecnologie per Internet

DIGITAL MODULATIONS

ASK, QAM

Digital Modulations

DIGITAL MODULATIONS

- **ASK** (AMPLITUDE SHIFT-KEYING)
- **FSK** (FREQUENCY SHIFT-KEYING)
- **PSK** (PHASE SHIFT-KEYING)

A digital signal can modulate the amplitude, frequency, or phase of a sinusoidal carrier wave.

Digital Modulations

- **NOTE:** Every *T* seconds a symbol among *m* possible ones is emitted by the source.

$$r = \frac{1}{\mathrm{T}}$$

- If there are 2 symbols (i.e., only 2 states) → $r \rightarrow r_b$

$r_b \rightarrow$ real bit transmission rate through the channel.

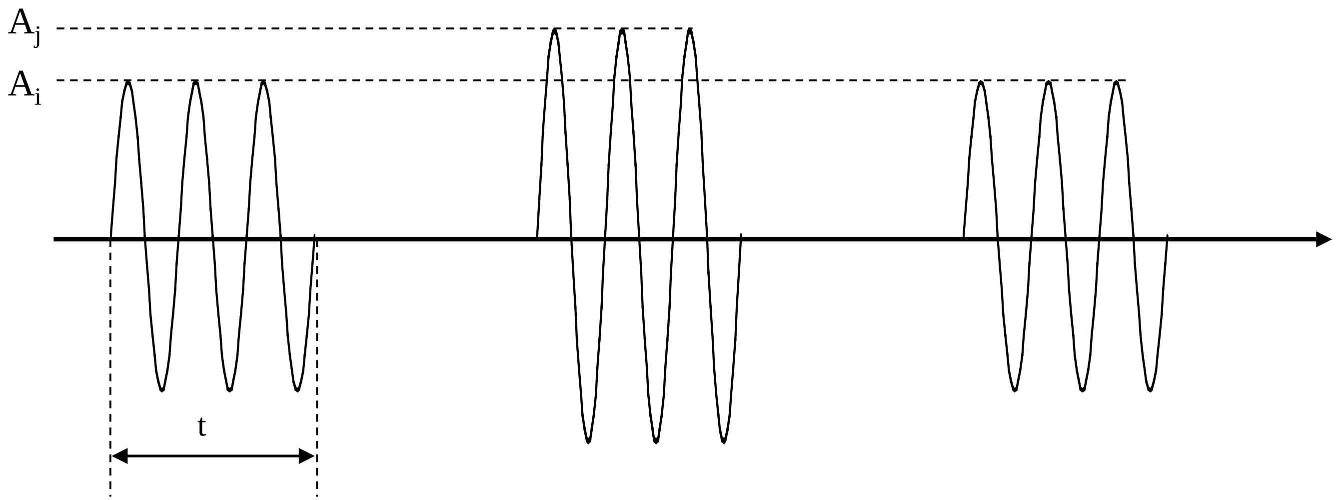
Digital Modulations

- ✓ The parameters to evaluate a digital modulation system performance are:
  - error probability (P_e)
  - signal/noise ratio (S/N)
  - spectral efficiency $r_b / B_T \left[\frac{Bit/s}{Hz} \right]$ (sent by the modulator)
  - hardware complexity

AMPLITUDE SHIFT KEYING (ASK)

ASK Modulation

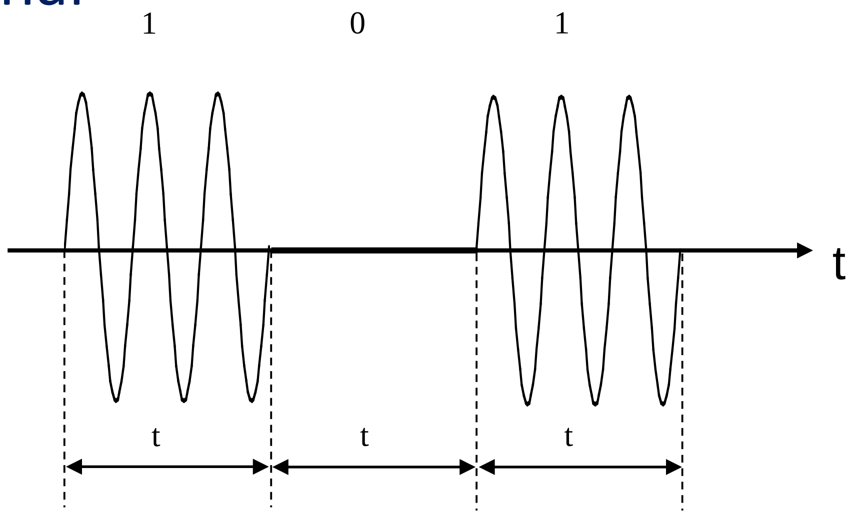
- ✓ The carrier is modulated by a digital signal with M discrete levels



- ✓ Particular case: M=2 → *OOK* (On–Off keying)

OOK Modulation

Modulated signal



$$x_C(t) = x(t) \cdot cos\omega_C t$$

$x_C(t)$ represents a sequence of pulses (Ex. PCM).

OOK Modulation

$$x_C(t) = x(t) \cdot cos\omega_C t$$

- ✓ Supposed the modulator input $x(t)$ be a binary signal.
- ✓ *OOK* could be seen as a mean to send bits on translated band

  (Ex: PCM on translated band).
- ✓ *OOK* could be generated simply by turning the carrier on and off.

ASK Modulation

- ✓ In general M-*ary ASK* waveform can be obtained from the general expression of pass-band signals:

$$x_C(t) = A_C\left[x_i(t)\cdot cos(\omega_C t+\theta) - x_q(t)\cdot sin(\omega_C t+\theta)\right]$$

- ✓ Since there are no phase reversals or other variations, we can set the *q* component of $x_C(t)$ equal to zero → $x_q(t) = 0$

$$x_C(t) = A_C x_i(t)\cdot cos(\omega_C t+\theta)$$

$$x_i(t) = \sum_k a_k g_T(t-kT) \qquad a_k = 0,1,\ldots,M-1$$

$g_T(t)$ waveform with period *T.*

ASK Modulation

- ✓ We will examine the ASK waveform power spectra.
- ✓ We will considerate the *equivalent lowpass spectrum* assuming that a_k symbols are uncorrelated.
- ✓ Then the PAM spectrum is:

$$S_{Xi}(f)=\sigma_a{}^2 r|G(f)|^2+(m_a r)^2\sum_{+\infty}^{+\infty}|G(nr)|^2\delta(f-nr)$$

ASK Modulation

- ✓ In *M-ary ASK* with *M* levels equally likely and unipolar:

$$m_a = \frac{1}{M}\sum_{0}^{M-1} i = \frac{1}{M}\left[(0+M-1)+(1+M-2)+(2+M-3)+\ldots\right] = \frac{(M-1)\cdot M}{2M} = \frac{(M-1)}{2}$$

$$\sigma_a{}^2 = \frac{1}{M}\sum_{0}^{M-1} i^2 - m_a{}^2 = \frac{1}{M}\left[\frac{M\cdot(M-1)\cdot(2M-1)}{6}\right] - \frac{(M-1)^2}{4} =$$

$$\sum_{0}^{M-1} i^2 \rightarrow ***$$

$$= \frac{(M-1)}{2}\left[\frac{2M-1}{3} - \frac{M-1}{2}\right] = \frac{(M^2-1)}{12}$$

ASK Modulation

- For NRZ pulses $G(f)=\frac{\mathrm{sinc}(\pi f/r)}{r}$

- Then, the $x(t)$ power spectra is:

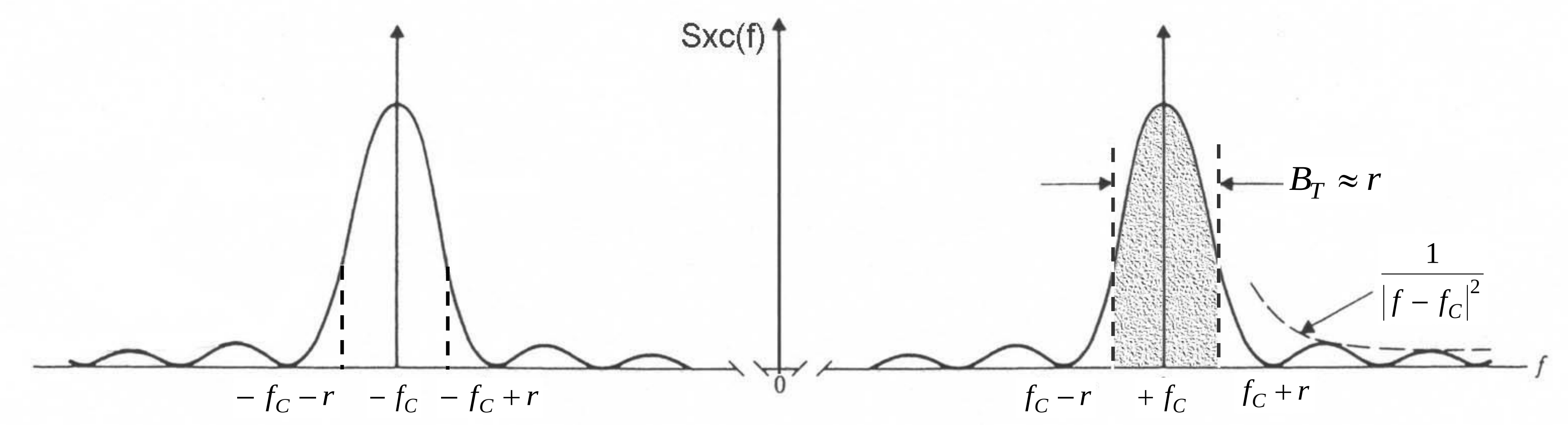
$$S_{Xi}(f)=\frac{M^2-1}{12}r\frac{\mathrm{sinc}^2(\pi f/r)}{r^2}+\frac{(M-1)^2}{4}r^2\frac{\delta(f)}{r^2}$$

- The ASK power spectrum is generated by a translation of $S_{xi}(f)$ at the carrier frequency.

$$x_C(t)=A_C x_i(t)\cdot cos(\omega_C t+\theta) \qquad S_{Xc}(f)=A_C S_{x_i}(f)*F[cos(\omega_C t)]$$

ASK Modulation

- ✓ The power spectrum of *M-ary ASK* is showed below:



ASK Modulation

- ✓ In theory, *sinc(πf/r)* is not bandlimited.
- ✓ But in practice the signal power is contained within the range $f_c \pm \frac{r}{2}$
- ✓ The spectrum has a second order roll-off ($|f - f_c|^{-2}$).

ASK Modulation

- ✓ The bandwidth of an ASK is: $B_T = r = \frac{1}{\mathrm{T}}$
- ✓ While the spectral efficiency is: $\frac{r_b}{B_T} \quad \left[\frac{bps}{Hz}\right]$
- ✓ For M-*ary ASK* $\begin{cases} B_T \approx r \\ r_b = r\,\mathbf{log}_2\,\mathrm{M} \end{cases} \Longrightarrow \frac{r_b}{B_T} = \mathbf{log}_2\,\mathrm{M} \quad {}^{bps}\!/_{Hz}$

ASK Modulation

- Example (*OOK*)

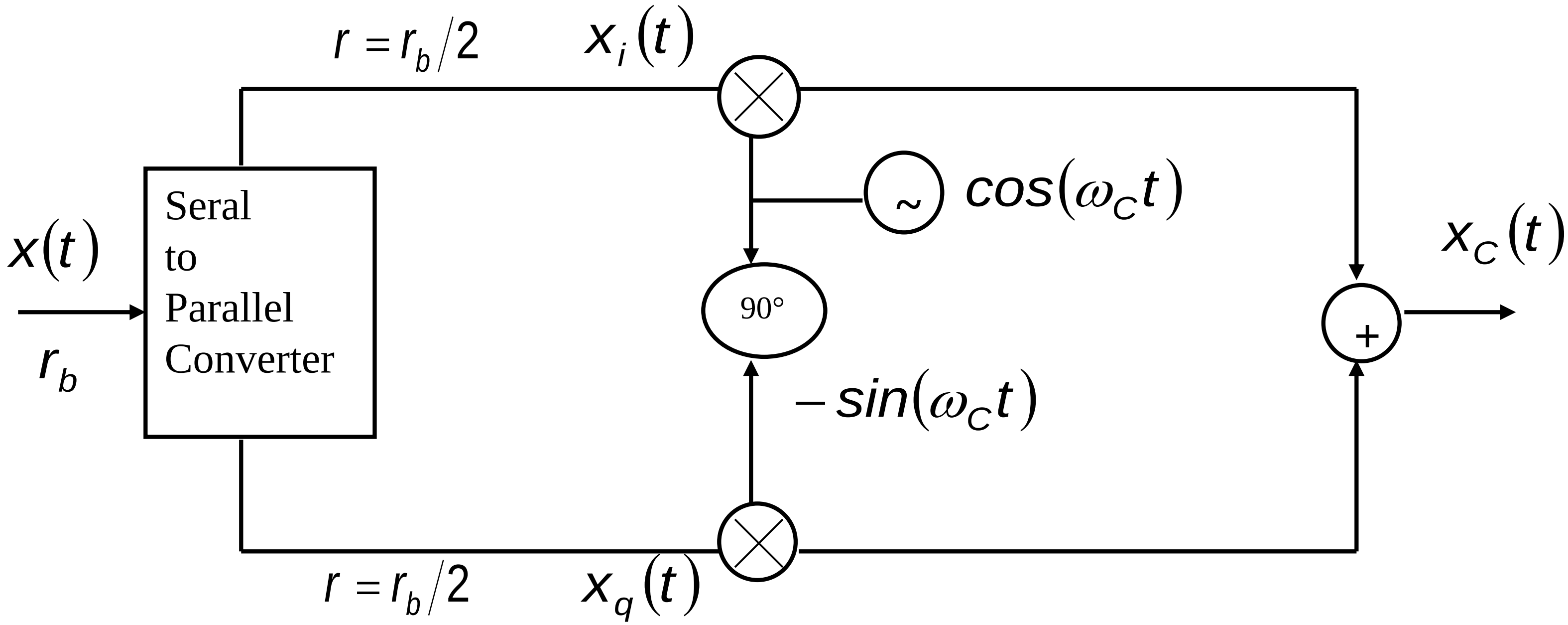
$$M = 2 \Rightarrow \frac{r_b}{B_T} = \log_2 2 = 1 \, {}^{bit}\!/_{Hz}$$

- The power spectrum shape is independent from M, while the spectral efficiency is dependent.

QUADRATURE AMPLITUDE MODLATION (QAM)

QAM

- ✓ In the QAM (Quadrature carrier AM) transmitter, the serial-to-parallel converter divides the input into two streams consisting of alternate bits $r = r_b/2$



QAM

$$T = \frac{1}{r} = 2T_b$$

- ✓ The modulated band-pass signal may be expressed in the quadrature carrier-form:

$$x_C(t) = A_C\left[x_i(t)\cdot cos(\omega_C t + \theta) - x_q(t)\cdot sin(\omega_C t + \theta)\right]$$

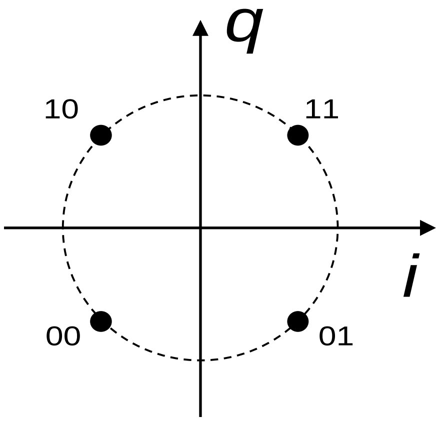
$$\left.\begin{aligned} x_i(t) &= \sum_k a_{2k} g_T(t - kT) \\ x_q(t) &= \sum_k a_{2k+1} g_T(t - kT) \end{aligned}\right\}$$

QAM

- ✓ The **i** and **q** components are independent, but they have the same pulse shape and the same statistics, namely:

$$m_a = \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot(-1) = 0$$

$$\sigma_a^{\,2} = 1$$



using PAM power spectra

expression : $S_{Xi}(f) = S_{Xq}(f) = r\left|G(f)\right|^2$

where $G(f)$ is $F\{\,\Pi(t)\}$

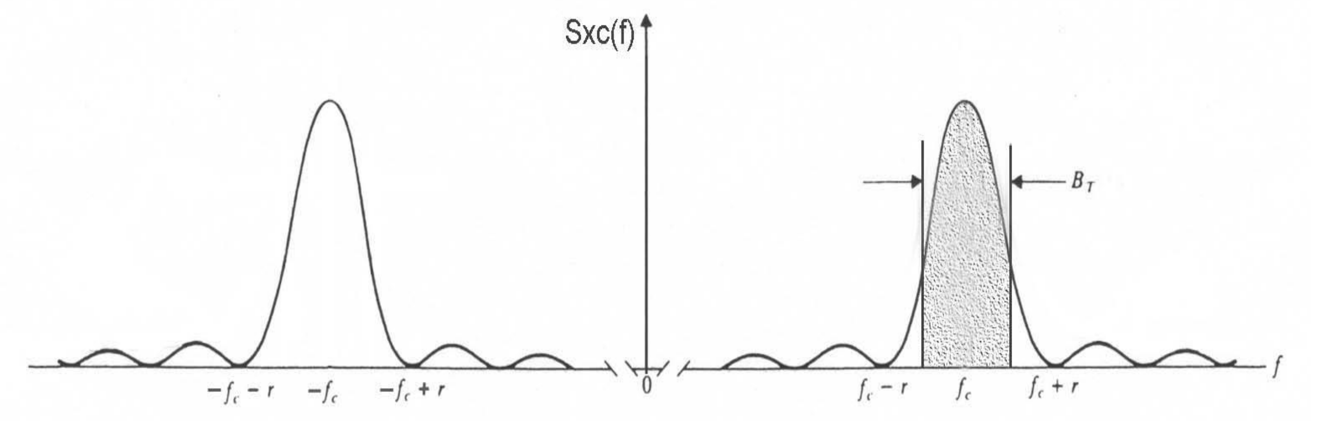
$$\left|G(f)\right|^2 = \frac{sinc^2(\pi f/r)}{r^2}$$

QAM

- The QAM spectrum is:

$$S_{Xc}(f) = \frac{A_C^2}{4}\left\{2S_{Xi}(f - f_c) + 2S_{Xq}(f + f_c)\right\} \qquad S_{Xi} = S_{Xq}$$

then:



as the ASK bandwidth :

$$B_T = r = \frac{1}{T}$$

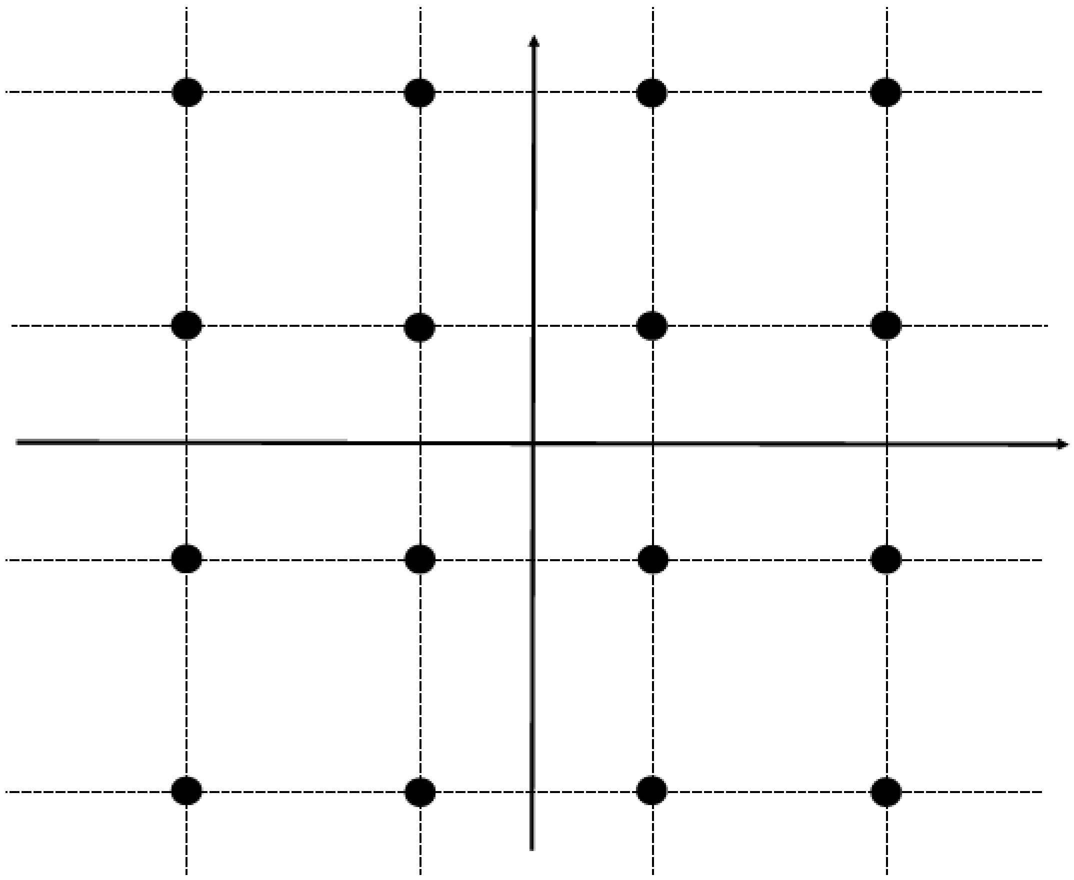
QAM

- While the spectral efficiency is:

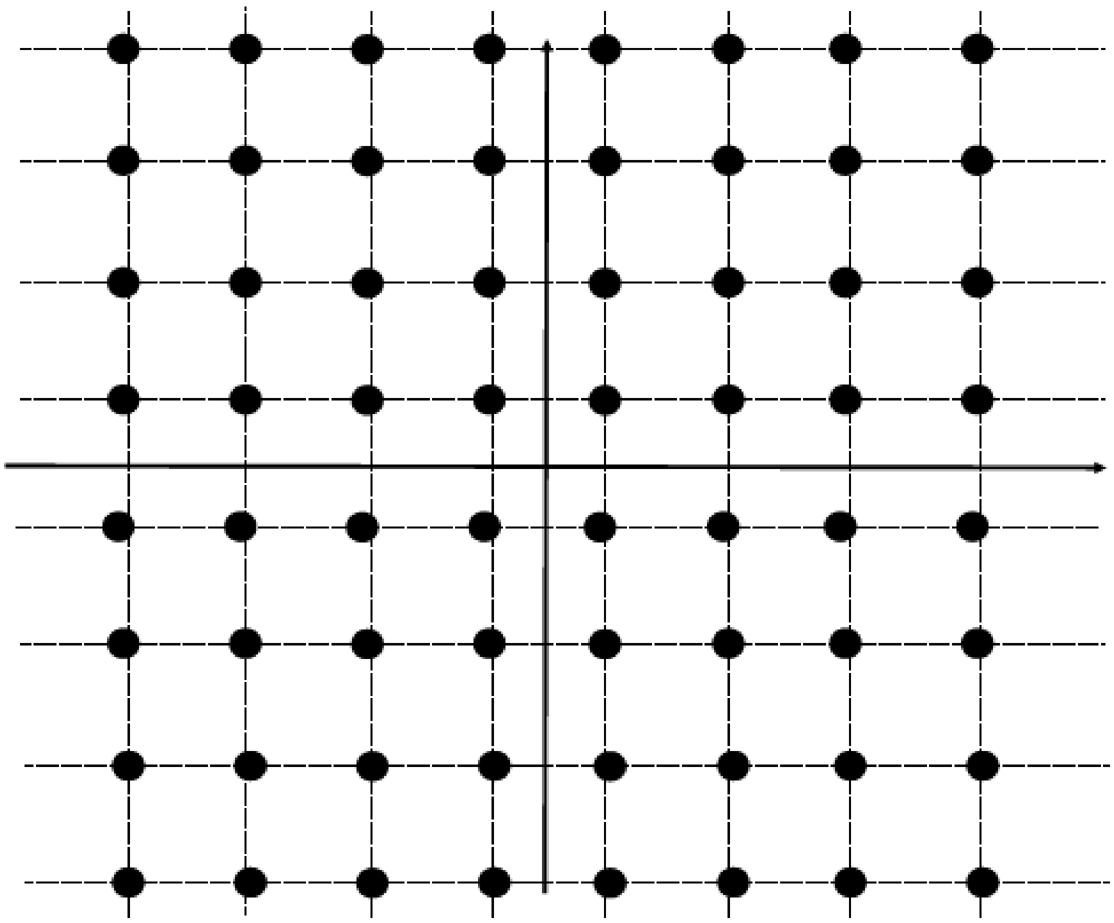
$$\frac{r_b}{B_T} \qquad \Rightarrow \qquad B_T = r = \frac{r_b}{2} \qquad \Rightarrow \qquad r = \frac{r_b}{r_b/2} = 2$$

- Doubled because, in fact, two sources transmit in the same band.

16-QAM

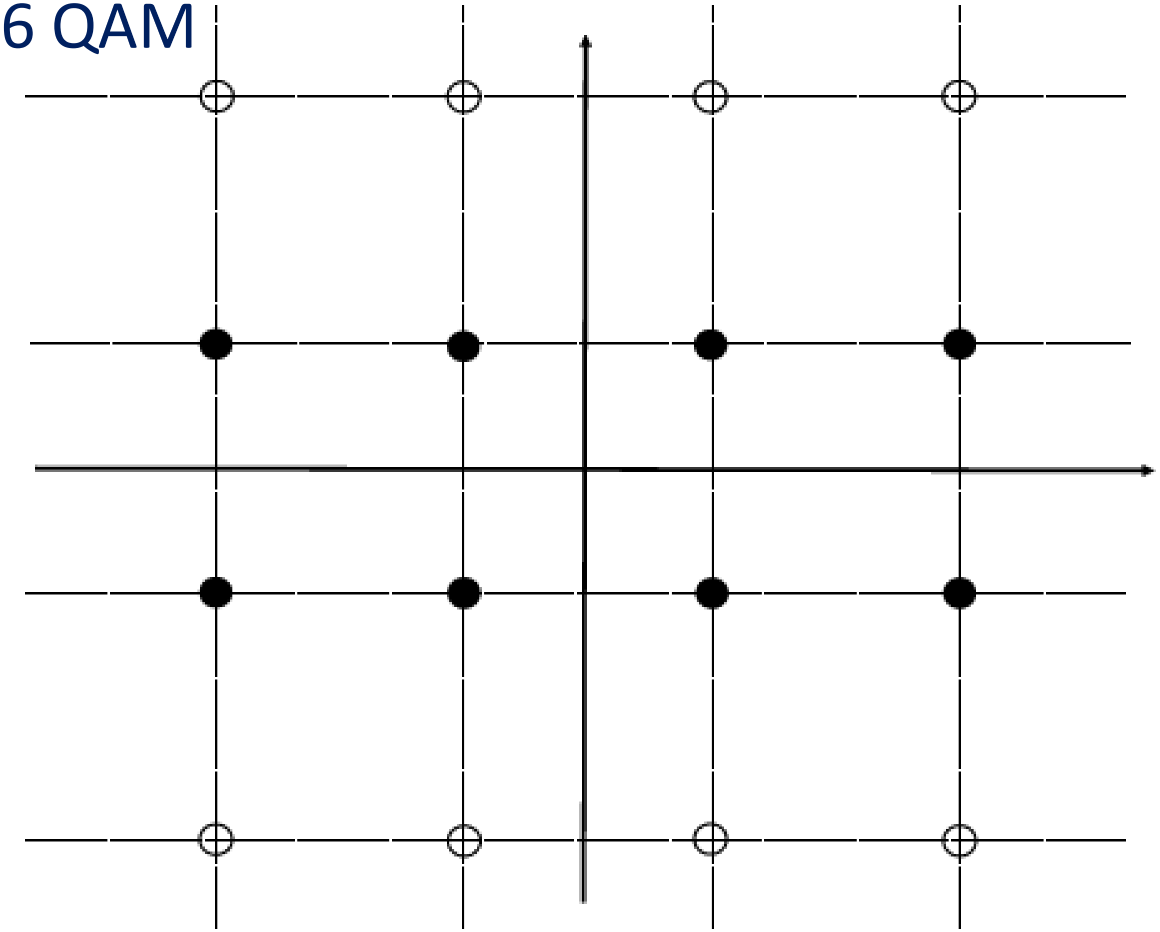


64-QAM



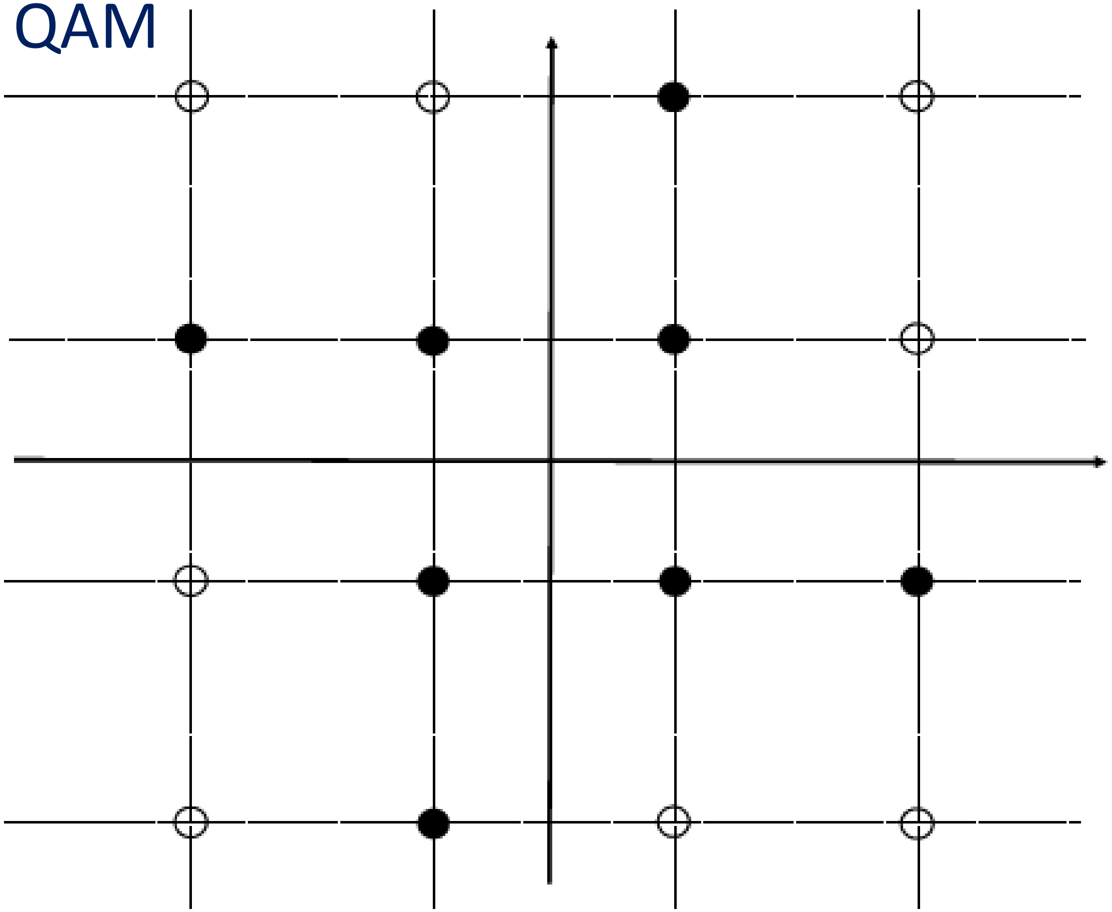
8-QAM

8 QAM ⊆ 16 QAM



8-QAM (a second choice)

8 QAM ⊆ 16 QAM



32-QAM