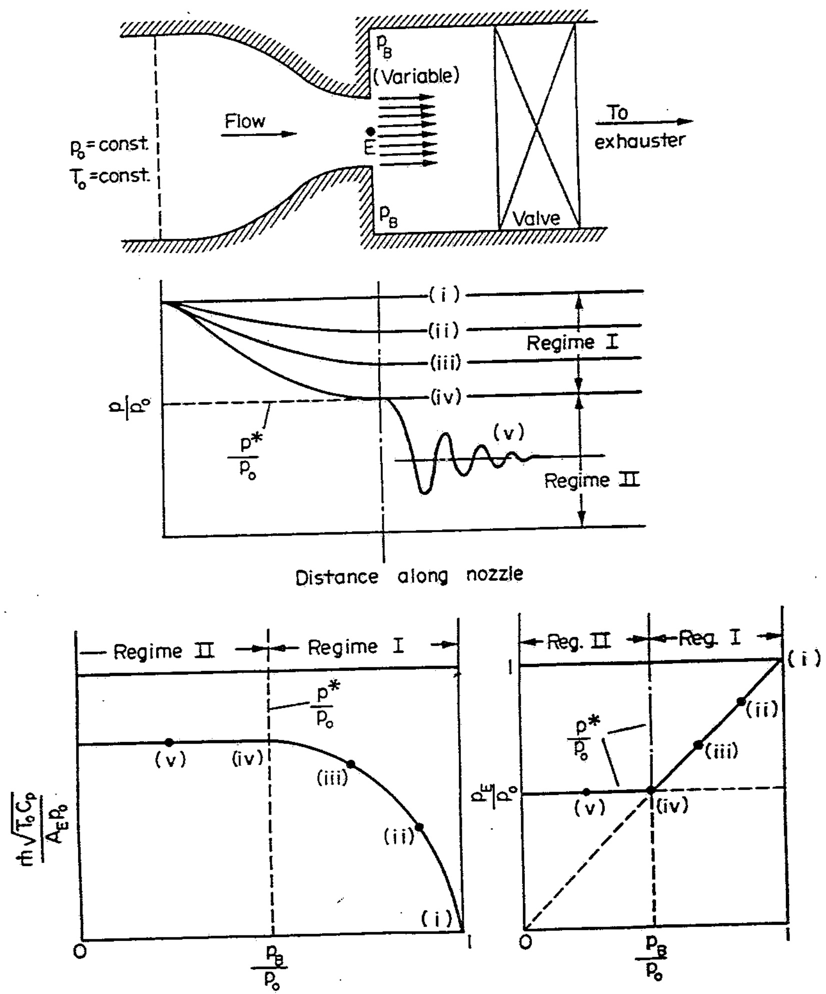


Fig. 3.1 Flow through a convergent duct.

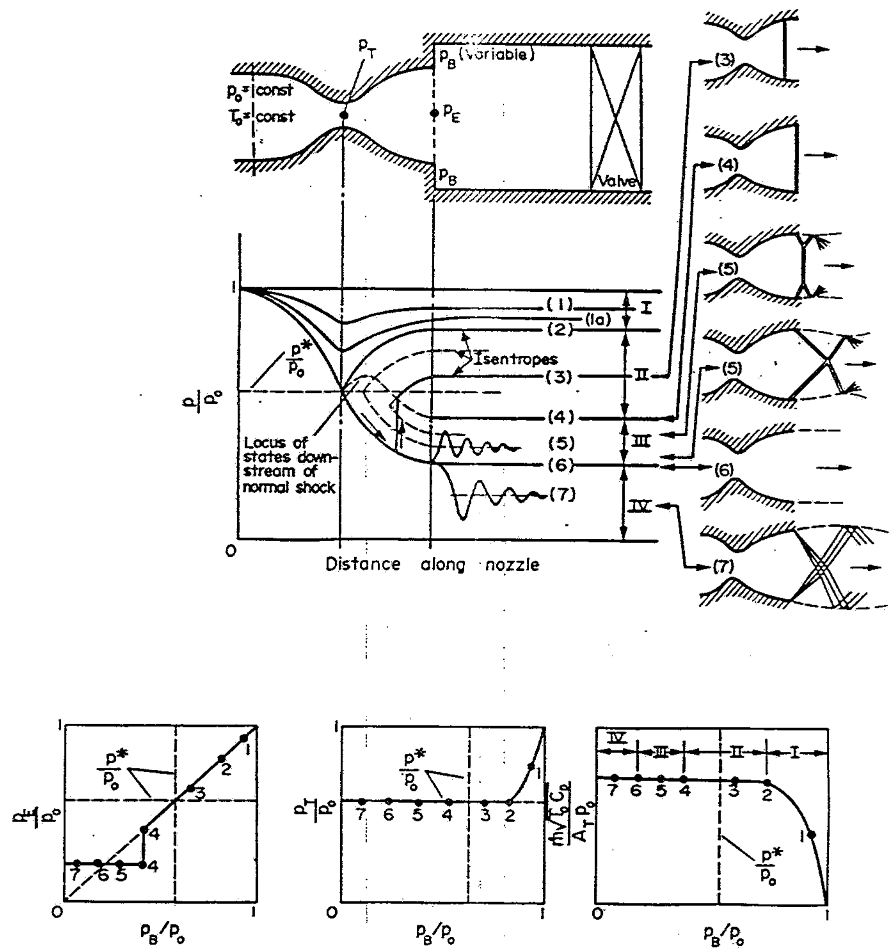
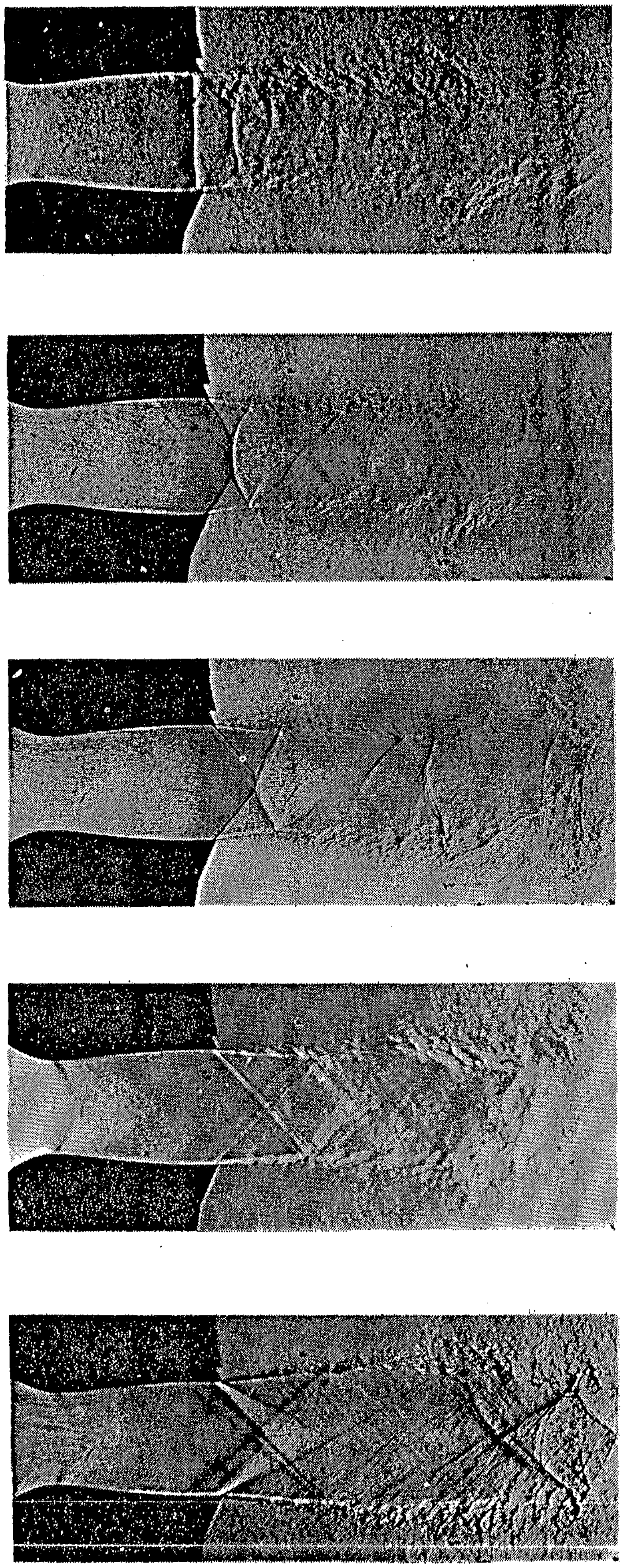


Fig. 3.2 Flow through a convergent-divergent duct.



$P_E/P_B < 0.4$

$P_E/P_B = 0.66$

$P_E/P_B = 0.85$

$P_E/P_B = 1.00$

$P_E/P_B = 1.50$

FIG. 6-8. Photographs of flows from a convergent-divergent nozzle at various back pressure. P_E denotes the pressure at the exit from the nozzle, upstream of the waves. (Courtesy of the National Physical Laboratory, Teddington, Middlessex, England.)

M	$\frac{T}{T_o}$	$\frac{p}{P_o}$	$\frac{V}{\sqrt{c_p T_o}}$	$\frac{\dot{m}\sqrt{c_p T_o}}{A P_o}$	$\frac{A}{A^*}$	$\frac{I}{I^*}$
0.00	1.000	1.000	0.000	0.000	∞	∞
0.02	1.000	1.000	0.013	0.044	28.942	22.834
0.04	1.000	0.999	0.025	0.088	14.481	11.435
0.06	0.999	0.997	0.038	0.133	9.666	7.643
0.08	0.999	0.996	0.051	0.176	7.262	5.753
0.10	0.998	0.993	0.063	0.220	5.822	4.624
0.12	0.997	0.990	0.076	0.263	4.864	3.875
0.14	0.996	0.986	0.088	0.306	4.182	3.343
0.16	0.995	0.982	0.101	0.349	3.673	2.947
0.18	0.994	0.978	0.113	0.391	3.278	2.642
0.20	0.992	0.972	0.126	0.432	2.964	2.400
0.22	0.990	0.967	0.138	0.473	2.708	2.205
0.24	0.989	0.961	0.151	0.513	2.496	2.043
0.26	0.987	0.954	0.163	0.553	2.317	1.909
0.28	0.985	0.947	0.176	0.592	2.116	1.795
0.30	0.982	0.939	0.188	0.629	2.035	1.698
0.32	0.980	0.932	0.200	0.667	1.922	1.614
0.34	0.977	0.923	0.213	0.703	1.823	1.542
0.36	0.975	0.914	0.225	0.738	1.736	1.479
0.38	0.972	0.905	0.237	0.772	1.659	1.424
0.40	0.969	0.896	0.249	0.806	1.590	1.375
0.42	0.966	0.886	0.261	0.838	1.529	1.332
0.44	0.963	0.876	0.273	0.869	1.474	1.294
0.46	0.959	0.865	0.285	0.899	1.425	1.260
0.48	0.956	0.854	0.297	0.928	1.380	1.230
0.50	0.952	0.843	0.309	0.956	1.340	1.203
0.52	0.949	0.832	0.320	0.983	1.303	1.179
0.54	0.945	0.820	0.332	1.008	1.270	1.157
0.56	0.941	0.808	0.344	1.033	1.240	1.138
0.58	0.937	0.796	0.355	1.056	1.213	1.121
0.60	0.933	0.784	0.367	1.078	1.188	1.105
0.62	0.929	0.772	0.378	1.099	1.166	1.091
0.64	0.924	0.759	0.389	1.119	1.145	1.079
0.66	0.920	0.747	0.400	1.137	1.127	1.068
0.68	0.915	0.734	0.411	1.154	1.110	1.058
0.70	0.911	0.721	0.422	1.171	1.094	1.049
0.72	0.906	0.708	0.433	1.185	1.081	1.041
0.74	0.901	0.695	0.444	1.199	1.068	1.034
0.76	0.896	0.682	0.455	1.212	1.057	1.028
0.78	0.892	0.669	0.466	1.223	1.047	1.023
0.80	0.887	0.656	0.476	1.234	1.038	1.019
0.82	0.881	0.643	0.487	1.243	1.030	1.015
0.84	0.876	0.630	0.497	1.251	1.024	1.011
0.86	0.871	0.617	0.508	1.259	1.018	1.008
0.88	0.866	0.604	0.518	1.265	1.013	1.006
0.90	0.861	0.591	0.528	1.270	1.009	1.004
0.92	0.855	0.578	0.538	1.274	1.006	1.002
0.94	0.850	0.566	0.548	1.277	1.003	1.001
0.96	0.844	0.553	0.558	1.279	1.001	1.001
0.98	0.839	0.541	0.568	1.281	1.000	1.000
1.00	0.833	0.528	0.577	1.281	1.000	1.000
1.02	0.828	0.516	0.587	1.281	1.000	1.000
1.04	0.822	0.504	0.596	1.279	1.001	1.001
1.06	0.817	0.492	0.606	1.277	1.003	1.001
1.08	0.811	0.480	0.615	1.274	1.005	1.002
1.10	0.805	0.468	0.624	1.271	1.008	1.003
1.12	0.799	0.457	0.633	1.267	1.011	1.004
1.14	0.794	0.445	0.642	1.262	1.015	1.006
1.16	0.788	0.434	0.651	1.256	1.020	1.007
1.18	0.782	0.423	0.660	1.250	1.025	1.009
1.20	0.776	0.412	0.669	1.243	1.030	1.011
1.22	0.771	0.402	0.677	1.236	1.037	1.013
1.24	0.765	0.391	0.686	1.228	1.043	1.015
1.26	0.759	0.381	0.694	1.220	1.050	1.017
1.28	0.753	0.371	0.703	1.211	1.058	1.019
1.30	0.747	0.361	0.711	1.201	1.066	1.022
1.32	0.742	0.351	0.719	1.192	1.075	1.024
1.34	0.736	0.342	0.727	1.181	1.084	1.027
1.36	0.730	0.332	0.735	1.171	1.094	1.029
1.38	0.724	0.323	0.743	1.160	1.104	1.032
1.40	0.718	0.314	0.750	1.149	1.115	1.035
1.42	0.713	0.305	0.758	1.138	1.126	1.037
1.44	0.707	0.297	0.766	1.126	1.138	1.040
1.46	0.701	0.289	0.773	1.114	1.150	1.043
1.48	0.695	0.280	0.781	1.102	1.163	1.046
1.50	0.690	0.272	0.788	1.089	1.176	1.049
1.52	0.684	0.265	0.795	1.077	1.190	1.052
1.54	0.678	0.257	0.802	1.064	1.204	1.055
1.56	0.673	0.250	0.809	1.051	1.219	1.058
1.58	0.667	0.242	0.816	1.038	1.234	1.060
1.60	0.661	0.235	0.823	1.025	1.250	1.063
1.62	0.656	0.228	0.830	1.011	1.267	1.066
1.64	0.650	0.222	0.836	0.998	1.284	1.069
1.66	0.645	0.215	0.843	0.985	1.301	1.072
1.68	0.639	0.209	0.849	0.971	1.319	1.075
1.70	0.634	0.203	0.856	0.958	1.338	1.079
1.72	0.628	0.197	0.862	0.944	1.357	1.082
1.74	0.623	0.191	0.869	0.931	1.376	1.085
1.76	0.617	0.185	0.875	0.917	1.397	1.088
1.78	0.612	0.179	0.881	0.904	1.418	1.091
1.80	0.607	0.174	0.887	0.890	1.439	1.094
1.82	0.602	0.169	0.893	0.877	1.461	1.096
1.84	0.596	0.164	0.899	0.863	1.484	1.099
1.86	0.591	0.159	0.904	0.850	1.507	1.102
1.88	0.586	0.154	0.910	0.837	1.531	1.105
1.90	0.581	0.149	0.916	0.824	1.555	1.108
1.92	0.576	0.145	0.921	0.811	1.580	1.111
1.94	0.571	0.140	0.927	0.798	1.606	1.114
1.96	0.566	0.136	0.932	0.785	1.633	1.117
1.98	0.561	0.132	0.938	0.772	1.660	1.120
2.00	0.556	0.1278	0.943	0.759	1.687	1.123
2.02	0.551	0.1239	0.948	0.747	1.716	1.126
2.04	0.546	0.1201	0.953	0.734	1.745	1.128
2.06	0.541	0.1164	0.958	0.722	1.775	1.131
2.08	0.536	0.1128	0.963	0.709	1.806	1.134
2.10	0.531	0.1094	0.968	0.697	1.837	1.137
2.12	0.527	0.1060	0.973	0.685	1.869	1.139
2.14	0.522	0.1027	0.978	0.674	1.902	1.142
2.16	0.517	0.0996	0.983	0.662	1.935	1.145
2.18	0.513	0.0965	0.987	0.650	1.970	1.147
2.20	0.508	0.0935	0.992	0.639	2.005	1.150
2.22	0.504	0.0906	0.996	0.628	2.041	1.153
2.24	0.599	0.0878	1.001	0.617	2.078	1.155
2.26	0.495	0.0851	1.005	0.606	2.115	1.158
2.28	0.490	0.0825	1.010	0.595	2.154	1.160
2.30	0.486	0.0800	1.014	0.584	2.193	1.163
2.32	0.482	0.0775	1.018	0.574	2.233	1.165
2.34	0.477	0.0751	1.022	0.563	2.274	1.168
2.36	0.473	0.0728	1.027	0.553	2.316	1.170
2.38	0.469	0.0706	1.031	0.543	2.359	1.173
2.40	0.465	0.0684	1.035	0.533	2.403	1.175
2.42	0.461	0.0663	1.039	0.523	2.448	1.177
2.44	0.456	0.0643	1.043	0.514	2.494	1.180
2.46	0.452	0.0623	1.046	0.504	2.540	1.182
2.48	0.448	0.0604	1.050	0.495	2.588	1.184
2.5	0.444	0.0585	1.054	0.486	2.637	1.187
2.6	0.425	0.0501	1.072	0.442	2.896	1.198
2.7	0.407	0.0430	1.089	0.402	3.183	1.208
2.8	0.389	0.0368	1.105	0.366	3.500	1.218
2.9	0.373	0.0317	1.120	0.333	3.850	1.228
3.0	0.357	0.0272	1.134	0.303	4.235	1.237
3.1	0.342	0.0234	1.147	0.275	4.657	1.245
3.2	0.328	0.0202	1.159	0.250	5.121	1.253
3 3	0.315	0.0175	1.171	0.228	5.629	1.260
3.4	0.302	0.0151	1.182	0.207	6.184	1.268
3.5	0.290	0.0131	1.192	0.189	6.790	1.274
3.6	0.278	0.0114	1.201	0.172	7.450	1.281
3.7	0.268	0.0099	1.210	0.157	8.169	1.287
3.8	0.257	0.0086	1.219	0.143	8.951	1.292
3.9	0.247	0.0075	1.227	0.131	9.799	1.298

$\gamma = 1.400$

M	$\frac{T}{T_o}$	$\frac{p}{p_o}$	$\frac{V}{\sqrt{c_p T_o}}$	$\frac{\dot{m}\sqrt{c_p T_o}}{A\, p_o}$	$\frac{A}{A^*}$	$\frac{I}{I^*}$
4.0	0.238	0.659E-02	1.234	0.120E 00	10.719	1.303
4.5	0.198	0.346E-02	1.266	0.773E-01	16.562	1.325
5.0	0.167	0.189E-02	1.291	0.512E-01	25.000	1.342
5.5	0.142	0.107E-02	1.310	0.347E-01	36.869	1.355
6.0	0.122	0.633E-03	1.325	0.241E-01	53.180	1.365
6.5	0.106	0.385E-03	1.337	0.170E-01	75.134	1.374
7.0	0.093	0.242E-03	1.347	0.123E-01	104.143	1.381
7.5	0.082	0.155E-03	1.355	0.903E-02	141.842	1.387
8.0	0.072	0.102E-03	1.362	0.674E-02	190.110	1.391
8.5	0.065	0.690E-04	1.368	0.510E-02	251.087	1.396
9.0	0.058	0.474E-04	1.372	0.392E-02	327.190	1.399
9.5	0.052	0.331E-04	1.377	0.304E-02	421.132	1.402
∞	0	0	1.414	0	∞	1.429

TABLE 2.3

$\gamma \equiv k$

Isentropic flow of a perfect gas with no external work transfer

From (2.5) $T_o = T\left(1 + \frac{\gamma-1}{2} M^2\right) = \text{constant}$

$$\frac{T}{T_o} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} \qquad (2.5s)$$

$$\frac{p}{p_o} = \left(\frac{T}{T_o}\right)^{\frac{\gamma}{\gamma-1}} \qquad p_o = p\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = \text{constant}$$

$\frac{T_o}{T^*} = \frac{k+1}{2}$ $\quad \frac{p_o}{p^*} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$

$$\frac{p}{p_o} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho}{\rho_o} = \left(\frac{p}{p_o}\right)^{\frac{1}{\gamma}} \qquad \rho_o = \rho\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{1}{\gamma-1}} = \text{constant} \qquad (2.7s)$$

From (2.8) $V/V^* = M^* = M\sqrt{\frac{\gamma+1}{2}} \Big/ \sqrt{1 + \frac{\gamma-1}{2} M^2}$

$$\frac{V}{\sqrt{c_p T_o}} = \sqrt{\gamma-1}\; M\left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{2}} \qquad (2.8s)$$

From (2.9) $$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{\gamma+1}\right)\left(1 + \frac{\gamma-1}{2} M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \qquad (2.9s)$$

From (2.10) $$\frac{I}{\dot{m}\sqrt{c_p T_o}} = \frac{\sqrt{\gamma-1}}{\gamma}\;\frac{1+\gamma M^2}{M}\left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{2}}$$

$$\frac{I}{I^*} = \frac{1+\gamma M^2}{M\sqrt{2(\gamma+1)}\sqrt{1 + \frac{\gamma-1}{2} M^2}} \qquad (2.10s)$$

From (2.11) $$\frac{\left(\frac{\dot{m}}{A}\right)}{\left(\frac{\dot{m}}{A}\right)^*} = \frac{M\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{-\frac{1}{2}(\gamma+1)}{(\gamma-1)}}}{\left(\frac{\gamma+1}{2}\right)^{\frac{-\frac{1}{2}(\gamma+1)}{(\gamma-1)}}}$$

but $$\left(\frac{\dot{m}}{A}\right)^* = \frac{p_o}{\sqrt{c_p T_o}}\;\frac{\gamma}{\sqrt{\gamma-1}}\left(\frac{\gamma+1}{2}\right)^{\frac{-\frac{1}{2}(\gamma+1)}{(\gamma-1)}}$$

hence, $$\frac{\dot{m}\sqrt{c_p T_o}}{A\, p_o} = \frac{\gamma}{\sqrt{\gamma-1}}\; M\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{-\frac{1}{2}(\gamma+1)}{(\gamma-1)}} \qquad (2.11s)$$