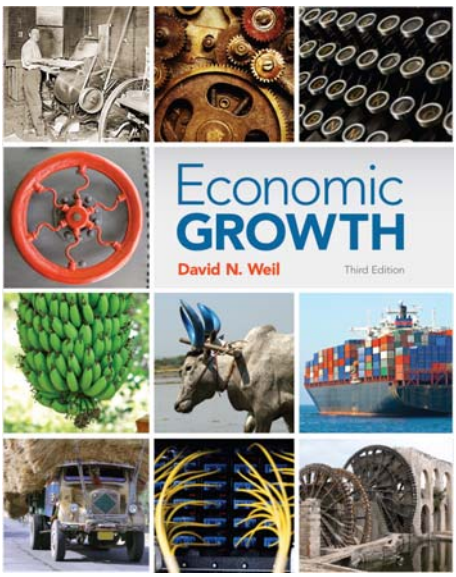
Chapter 3

PHYSICAL CAPITAL



A few things before starting

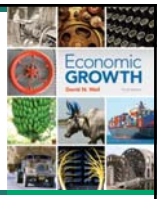
Database on WB indicators

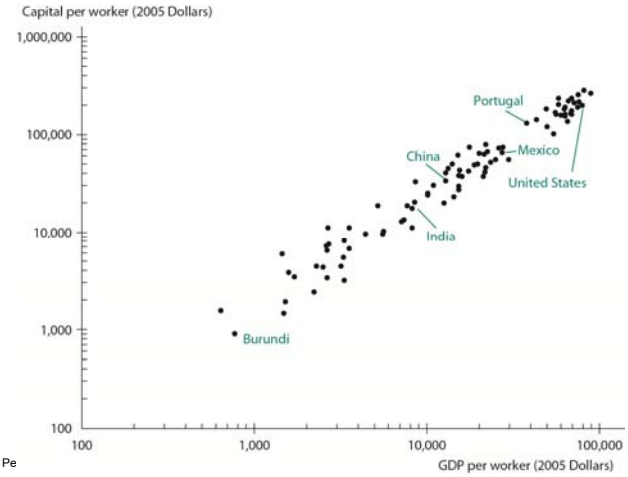
http://data.worldbank.org/products/wdi

Nature of capital

- Capital is productive
- But it is has been produced itself…through investments. In other words capital is accumulated. Distinction between flows and stocks
- Capital depreciates
- Capital stock is made of machinery, tools, buildings, roads….
- It can private or public (mainly infrastructures)

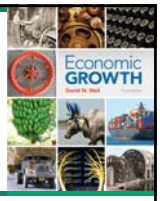
An interesting stylised fact: GDP and Capital per Worker, 2009





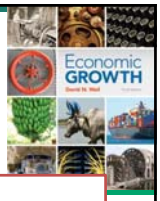
Source: Calculations based on Heston et al. (2010).

Early models of economic growth



- Harrod-Domar model
- Solow model

Capital's role in production



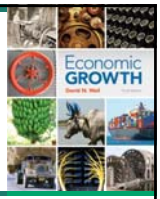
- Production function:
  - Y=F(K,L)
  - Y/L=F(K/L, 1)
  - y= f(k)

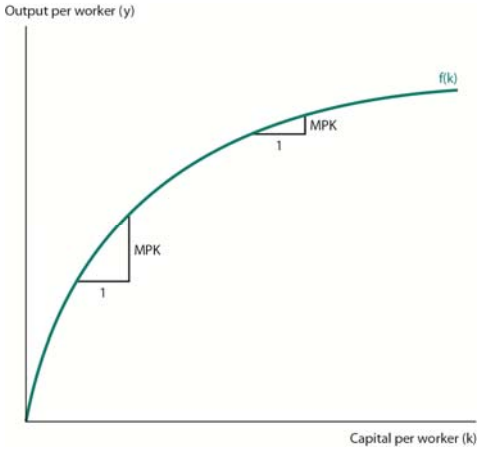
NOTE: in a competitive environment alpha is the capital factor share, in a Cobb-Douglas model this is constant

SOLOW model: Cobb-Douglas PF

- $Y=F(K,L) = AK^{\alpha}L^{\beta}$
- Usually $\alpha+\beta=1$ (assumption of no economies of scale), in this case we can rewrite the function above as follows:
- $Y=F(K,L) = AK^{\alpha}L^{1-\alpha}$, which is also in per capita terms
- $y = A\,k^{\alpha}$

Solow model: A Production Function with Diminishing Marginal Product of Capital





Capital's role in production

In a competitive environment, where factors are paid ~~according to marginal productivity,~~

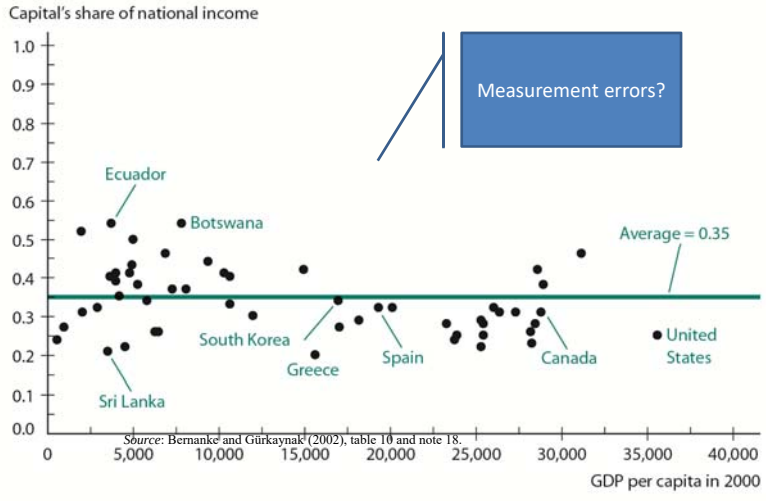
alpha is the capital factor share, and in a Cobb-Douglas model this is constant:

$$MPK = \alpha A K^{\alpha-1} L^{1-\alpha}$$

– Quota of income to capital:

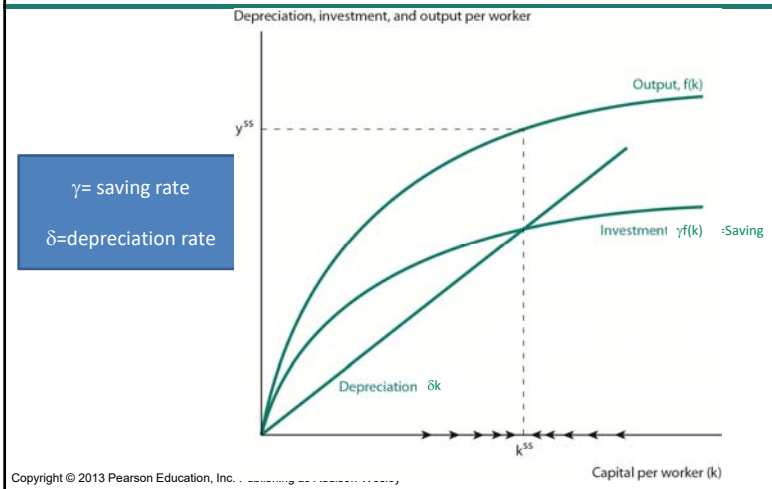
– $(MPK*K)/Y = (\alpha A K^{\alpha-1} L^{1-\alpha} * K)/A K^{\alpha} L^{1-\alpha} = \alpha$

Capital's Share of Income (which is equal to α) in a Cross-Section of Countries



Source: Bernanke and Gürkaynak (2002), table 10 and note 18.

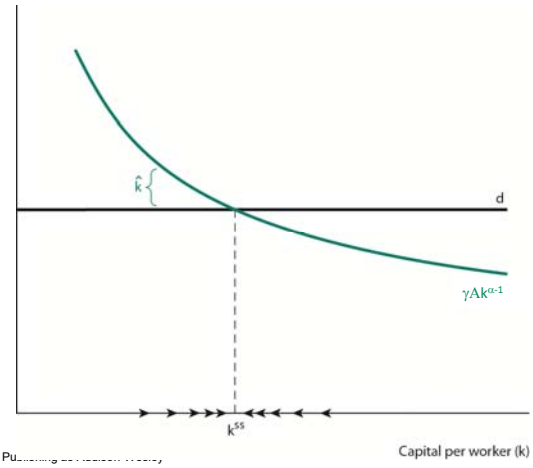
The Steady State of the Solow Model (with no population dynamics)



γ= saving rate

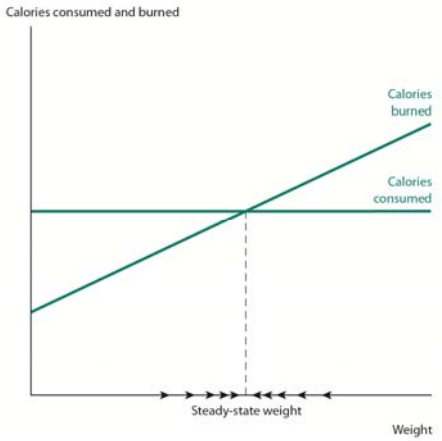
δ=depreciation rate

Speed of Convergence to the Steady State

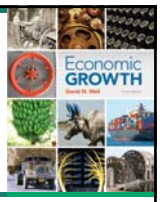


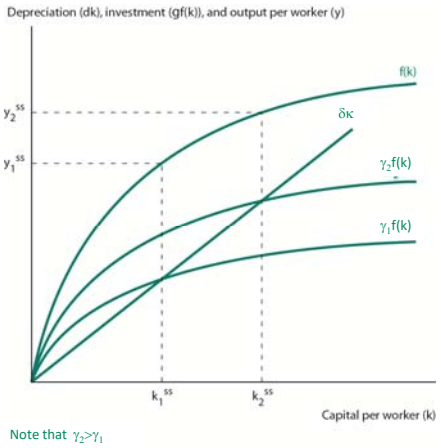
A non economic example: Determination of Steady-State Weight

Think also about changes in the position of the two lines



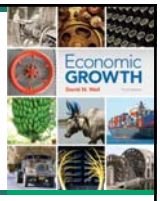
Effect of Increasing the Investment Rate on the Steady State





Note that $\gamma_2 > \gamma_1$

Solow model calibration



Using $y = A k^{\alpha}$

- $\Delta k = \gamma A k^{\alpha} - \delta k$

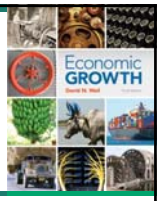
In steady state Δk is equal to zero, that is

- $0 = \gamma A k_{ss}^{\alpha} - \delta k_{ss}$

Which implies that

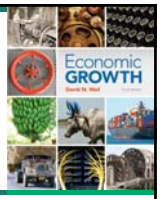
- $\gamma A k_{ss}^{\alpha} = \delta k_{ss} \Rightarrow \gamma A k_{ss}^{\alpha} / k_{ss} \delta = 1 \Rightarrow \gamma A k_{ss}^{\alpha-1} / \delta = 1$
- $k_{ss} = (\delta / \gamma A)^{1/(\alpha-1)} = (\gamma A / \delta)^{1/(1-\alpha)}$

Solow model calibration



- $y_{ss} = A\, k_{ss}^{\alpha} = A^{1/(1-\alpha)} (\gamma/\delta)^{\alpha/(1-\alpha)}$
- If we take the value of y of steady state for country *i* and country *j*, we can compare them in the following way
- $y^i_{ss} / y^j_{ss} = (\gamma^i/\gamma^j)^{\alpha/(1-\alpha)}$
- Now let us assume that country *i* has an investment rate of 20% and country *j* of 5%

Solow model calibration



- With alpha equal to 0.33 the previous formula gives a value of 2….
- … do you remember the example of Silvania and Freedonia?

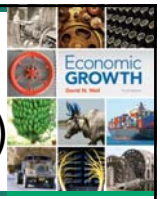
Predicted versus Actual GDP per Worker

The Solow model as a theory of relative growth rates (difference btwn s.r. and l.r.)

The Solow model does not provide a complete explanaton of growth rates since once a country reaches its steady state there is no longer growth!

Despite this failing we may still ask whether the model has something to say about relative growth rates - that is why some countries grow faster than others…

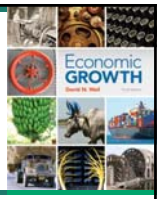
The Solow model as a theory of relative growth rates (difference btwn s.r. and l.r.)

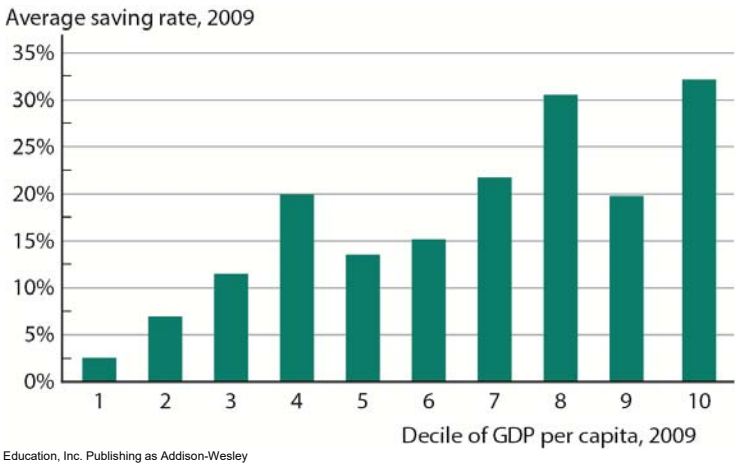


The key is to think about countries which are not in s.s.

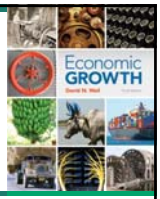
- *If two countries have the same rate of investment but different levels of income, the country with lower income will have higher growth*
- *If two countries have the same level of income but different rates of investment, then the country with a higher rate of investment will have higher growth*
- *A country that raises its level of investment will experience an increase in its rate of income growth*

Saving Rate by Decile of Income per Capita

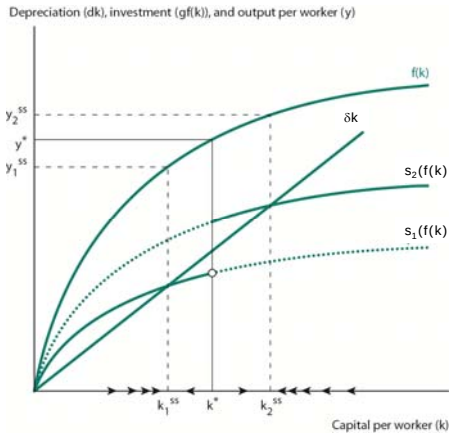




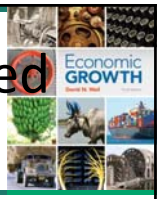
Solow Model with Saving Dependent on Income Level



Multiple equilibria= Poverty traps



The rise and fall of capital revisited



- The belief that capital accumulation is the key ingredient for economic growth reached its peak after WWII (see Arthur Lewis and Soviet Union's success)
- Policies were designed accordingly
- Now economist have discarded the idea that development depends mainly on capital accumulation