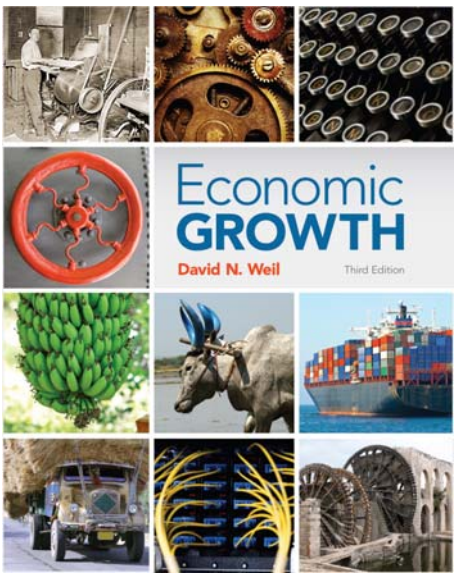
Chapter 3

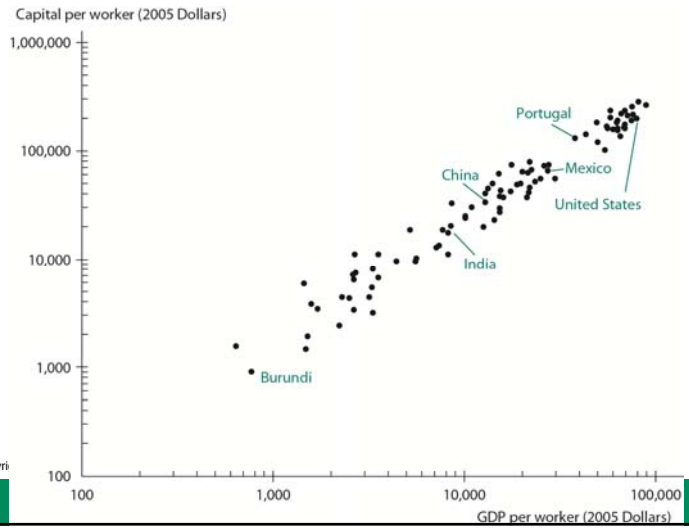
PHYSICAL CAPITAL



Nature of capital

- Capital is productive
- But it is has been produced itself…through investments. In other words capital is accumulated. Distinction between flows and stocks
- Capital depreciates
- Capital stock is made of machinery, tools, buildings, roads….
- It can private or public (mainly infrastructures)

An interesting stylised fact: GDP and Capital per Worker, 2009

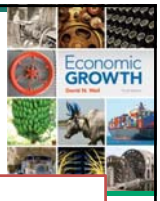


Source: Calculations based on Heston et al. (2010).

Early models of economic growth

- Harrod-Domar model
- Solow model

Capital's role in production



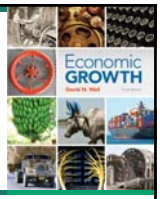
- Production function:
  - Y=F(K,L)
  - Y/L=F(K/L, 1)
  - y= f(k)

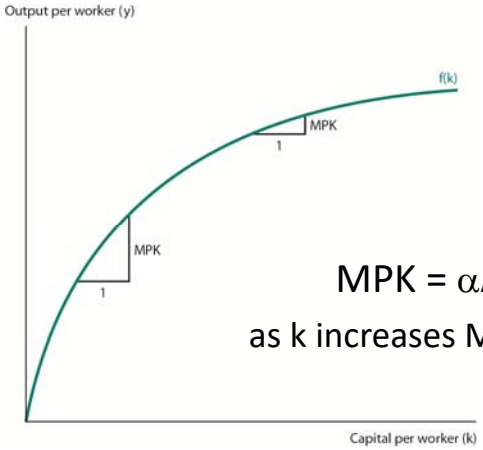
NOTE: in a competitive environment alpha is the capital factor share, in a Cobb-Douglas model this is constant

SOLOW model: Cobb-Douglas PF

- $Y=F(K,L) = AK^{\alpha}L^{\beta}$
- Usually $\alpha+\beta=1$ (assumption of no economies of scale), in this case we can rewrite the function above as follows:
- $Y=F(K,L) = AK^{\alpha}L^{1-\alpha}$, which is also in per capita terms
- $y = A\,k^{\alpha}$

Solow model: A Production Function with Diminishing Marginal Product of Capital

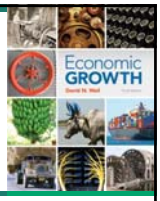




$MPK = \alpha AK^{\alpha-1}L^{1-\alpha}$

as k increases MPK goes down

Capital's role in production



In a competitive environment, where factors are paid according to marginal productivity,

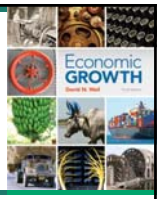
α, which is btwn 0 and 1, is the capital factor share, and in a Cobb-Douglas model this is constant:

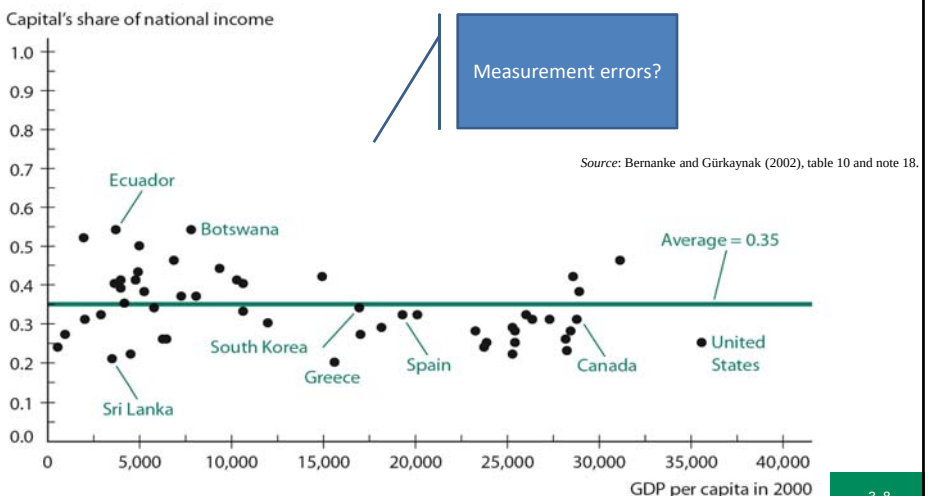
$$MPK = \alpha A K^{\alpha-1} L^{1-\alpha}$$

– Quota of income to capital:

$$(MPK*K)/Y = (\alpha A K^{\alpha-1} L^{1-\alpha} * K)/A K^{\alpha} L^{1-\alpha} = \alpha$$

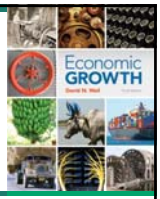
Capital's Share of Income (which is equal to α) in a Cross-Section of Countries





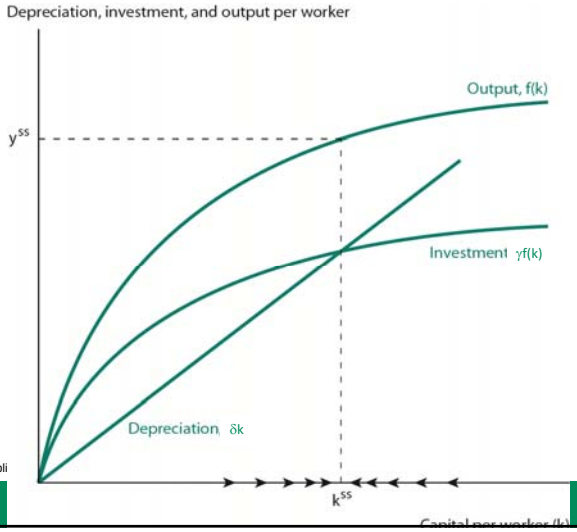
Source: Bernanke and Gürkaynak (2002), table 10 and note 18.

The Steady State of the Solow Model (with no population dynamics)

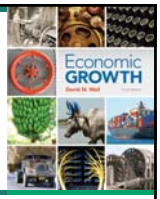


γ= saving rate

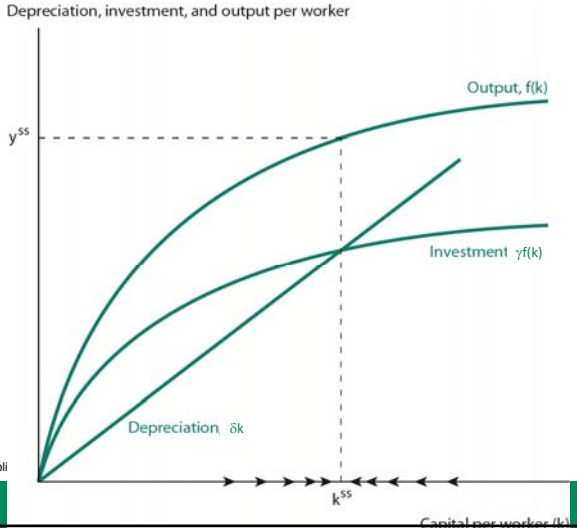
δ=depreciation rate



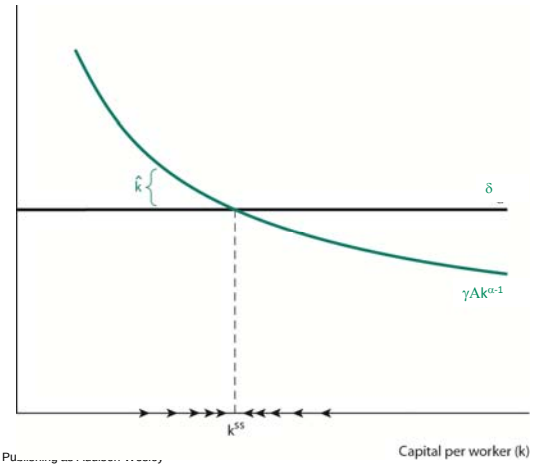
The Steady State of the Solow Model (with no population dynamics)



In the long run the SOLOW model predicts a zero growth equilibrium

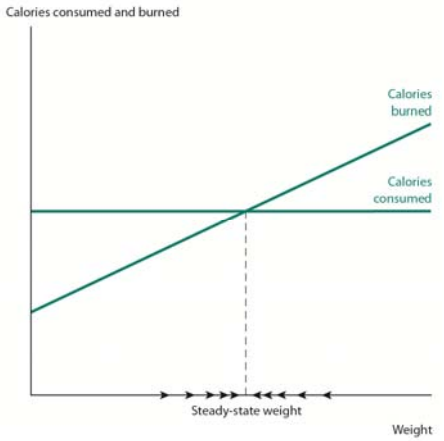


Speed of Convergence to the Steady State

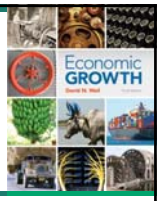


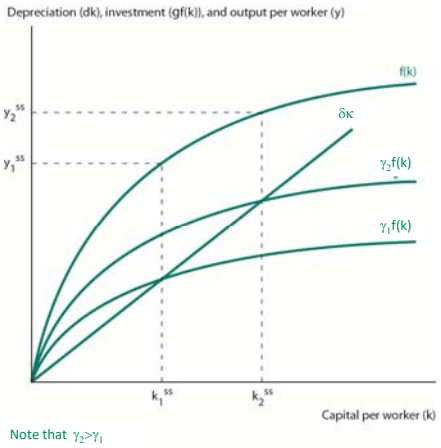
A non economic example: Determination of Steady-State Weight

Think also about changes in the position of the two lines



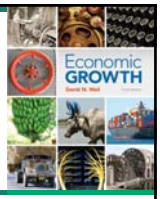
Effect of Increasing the Investment Rate on the Steady State





Note that $\gamma_2 > \gamma_1$

Solow model calibration



Using $y = A k^{\alpha}$

- $\Delta k = I\text{-}D = S\text{-}D = \gamma A k^{\alpha} - \delta k$

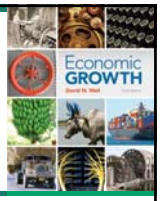
In steady state Δk is equal to zero, that is

- $0 = \gamma A k_{ss}^{\alpha} - \delta k_{ss}$

Which implies that

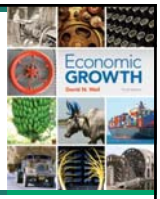
- $\gamma A k_{ss}^{\alpha} = \delta k_{ss} \Rightarrow \gamma A k_{ss}^{\alpha} / k_{ss} \delta = 1 \Rightarrow \gamma A k_{ss}^{\alpha-1} / \delta = 1$
- $k_{ss} = (\delta / \gamma A)^{1/(\alpha-1)} = (\gamma A / \delta)^{1/(1-\alpha)}$

Solow model calibration



- $y_{ss} = A\, k_{ss}^{\ \alpha} = A^{1/(1-\alpha)} (\gamma/\delta)^{\alpha/(1-\alpha)}$
- If we take the value of y of steady state for country *i* and country *j*, we can compare them in the following way
- $y^i_{ss} / y^j_{ss} = (\gamma^i/\gamma^j)^{\alpha/(1-\alpha)}$
- Now let us assume that country *i* has an investment rate of 20% and country *j* of 5%

Solow model calibration



- With alpha equal to 0.33 the previous formula gives a value of 2….

$$(\gamma^i/\gamma^j)^{\alpha/(1-\alpha)} = (20/5)^{0{,}33/(0{,}66)} = (4)^{1/(2)} = 2$$

- … do you remember the example of Silvania and Freedonia?

Predicted versus Actual GDP per Worker

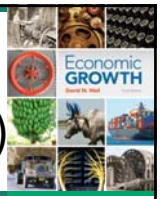
The Solow model as a theory of relative growth rates (difference btwn s.r. and l.r.)

The Solow model, in this version, does not provide a complete explanaton of growth rates since once a country reaches its steady state there is no longer growth!

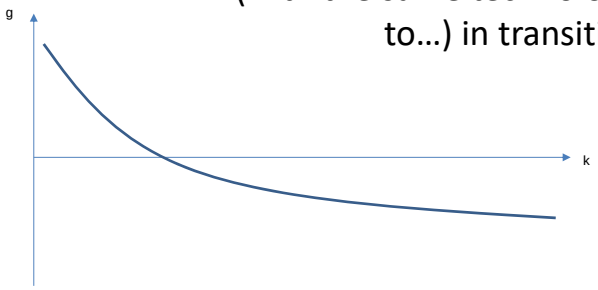
Despite this failing we may still ask whether the model has something to say about relative growth rates - that is why some countries grow faster than others…*

* Particularly serious since the model was built on the US case: steady 2% growth for two centuries

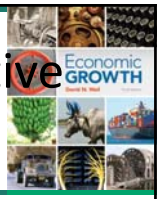
The Solow model as a theory of relative growth rates (difference btwn s.r. and l.r.)

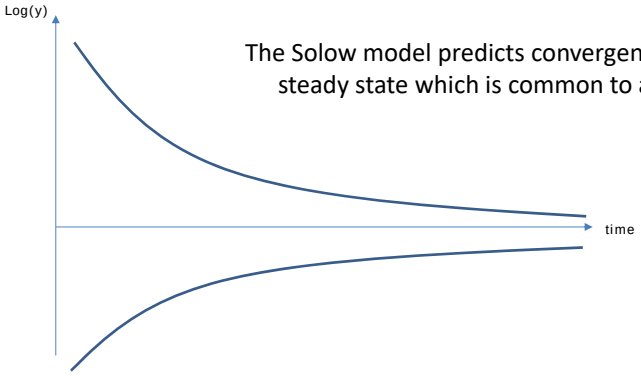


The key is to think about countries (with the same technology, or access to…) in transition



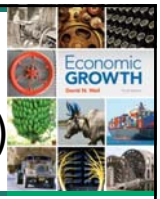
The Solow model as a theory of relative growth rates





The Solow model predicts convergence towards the steady state which is common to all countries

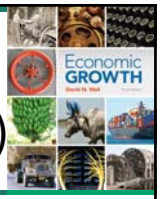
The Solow model as a theory of relative growth rates (difference btwn s.r. and l.r.)



The key is to think about countries which are not in s.s.

- *If two countries have the same rate of investment but different levels of income, the country with lower income will have higher growth*
- *If two countries have the same level of income but different rates of investment, then the country with a higher rate of investment will have higher growth*
- *A country that raises its level of investment will experience an increase in its rate of income growth*

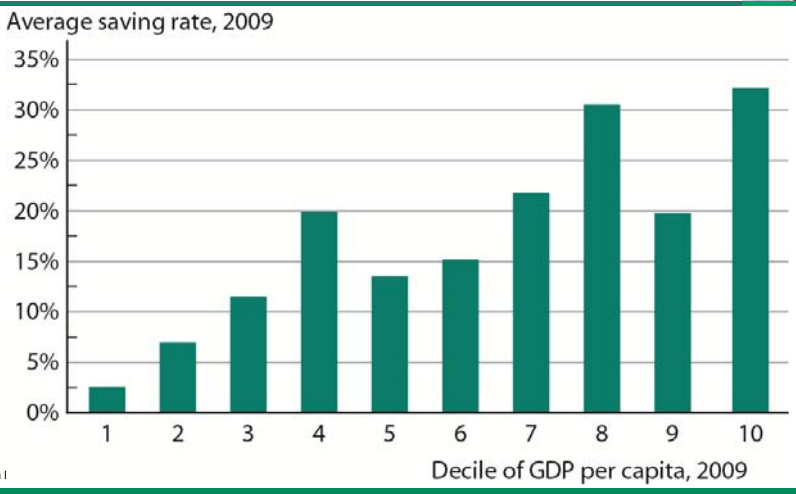
The Solow model as a theory of relative growth rates (difference btwn s.r. and l.r.)



The key is to think about countries which are not in s.s.

- *All the predictions above hold true only if the two countries have no other difference…, that is the same production function, the same productivity, and all the same parameters that are determinants of the steady state*

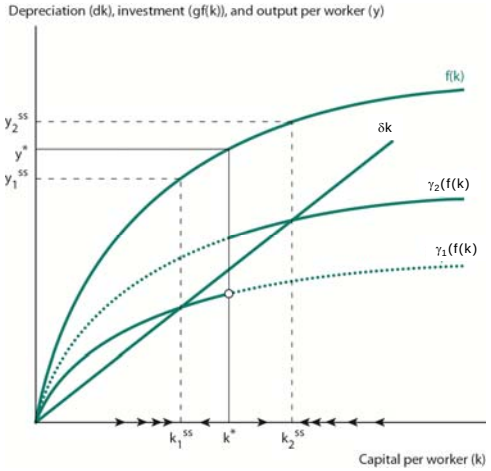
Saving Rate by Decile of Income per Capita



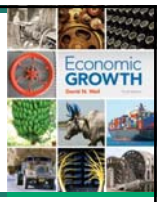
Solow Model with Saving Dependent on Income Level

Multiple equilibria=

Poverty traps



The rise and fall of capital revisited



- The belief that capital accumulation is the key ingredient for economic growth reached its peak after WWII (see Arthur Lewis and Soviet Union's success)
- Policies were designed accordingly
- Now economist have discarded the idea that development depends mainly on capital accumulation