Sviluppo di Taylor pera una funzione reale di una variabile reale

```
Quit[]
```

```
f[x_] := Exp[Sqrt[x + 5]] + 3 Log[2 + x^2]; n = 3; x0 = -3; m = 6; a = x0 - m; b = x0 + m;
```

```
g[x_] = Table[D[f[x], {x, k}], {k, 0, n}];
```

```
v = g[x0]
```

$$\left\{e^{\sqrt{2}} + 3\,\text{Log}[11],\ -\frac{18}{11} + \frac{e^{\sqrt{2}}}{2\sqrt{2}},\ -\frac{42}{121} + \frac{e^{\sqrt{2}}}{8} - \frac{e^{\sqrt{2}}}{8\sqrt{2}},\ -\frac{108}{1331} - \frac{3\,e^{\sqrt{2}}}{32} + \frac{5\,e^{\sqrt{2}}}{32\sqrt{2}}\right\}$$

```
p[x_] = Sum[v[[k]] (x - x0)^(k - 1) / (k - 1)!, {k, 1, n + 1}]
```

$$e^{\sqrt{2}} + \left(-\frac{18}{11} + \frac{e^{\sqrt{2}}}{2\sqrt{2}}\right)(3 + x) + \frac{1}{2}\left(-\frac{42}{121} + \frac{e^{\sqrt{2}}}{8} - \frac{e^{\sqrt{2}}}{8\sqrt{2}}\right)(3 + x)^2 + \frac{1}{6}\left(-\frac{108}{1331} - \frac{3\,e^{\sqrt{2}}}{32} + \frac{5\,e^{\sqrt{2}}}{32\sqrt{2}}\right)(3 + x)^3 + 3\,\text{Log}[11]$$

-]
  Verifica delle proprietà del polinomio di Taylor. Analisi dell' errore

```
PF[x_] = Table[D[f[x], {x, k}] - D[p[x], {x, k}], {k, 0, n}];
```

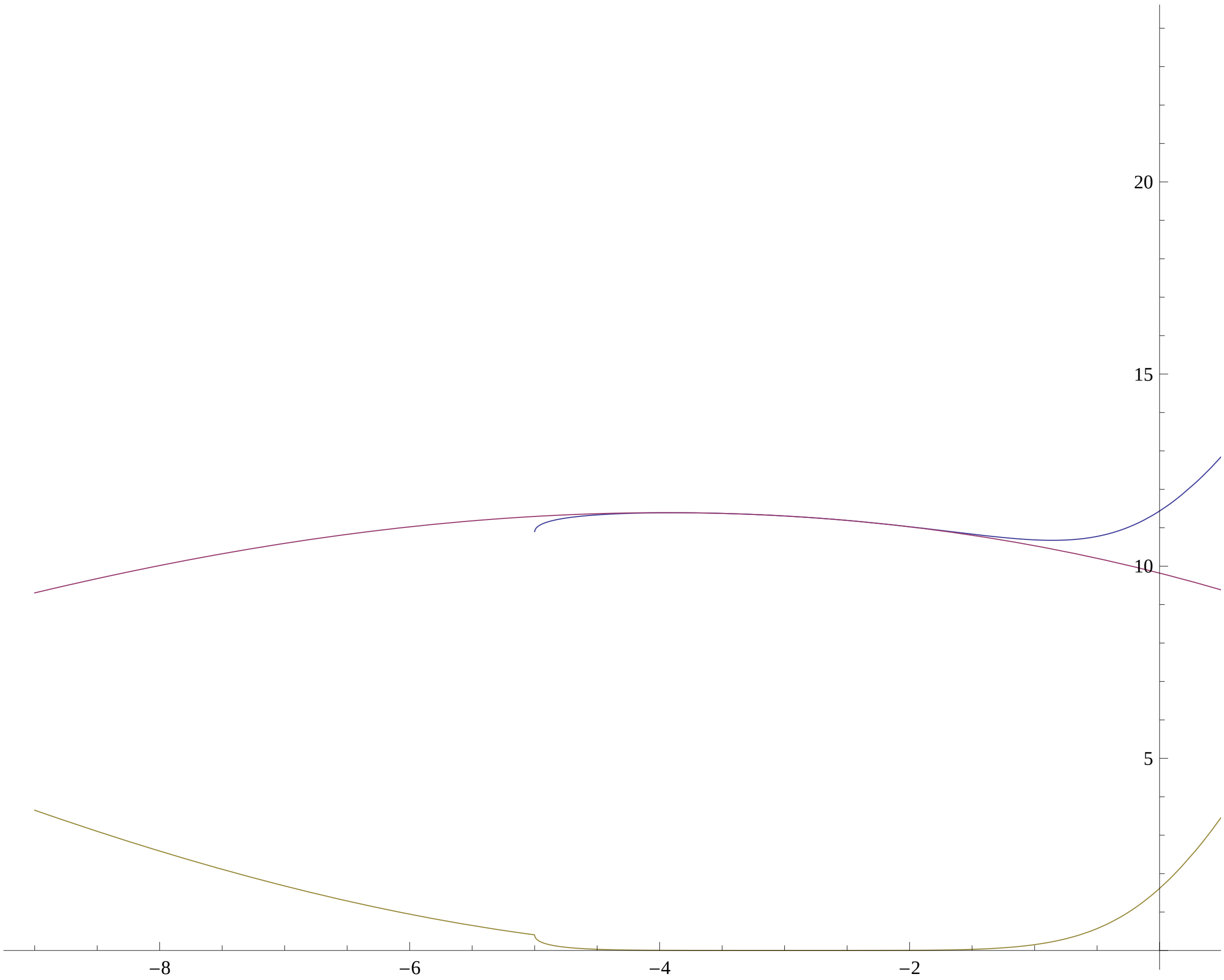
```
PF[x0]
```

{0, 0, 0, 0}

```
Error[x_] = Abs[p[x] - f[x]]
```

$$\text{Abs}\left[e^{\sqrt{2}} - e^{\sqrt{5+x}} + \left(-\frac{18}{11} + \frac{e^{\sqrt{2}}}{2\sqrt{2}}\right)(3 + x) + \frac{1}{2}\left(-\frac{42}{121} + \frac{e^{\sqrt{2}}}{8} - \frac{e^{\sqrt{2}}}{8\sqrt{2}}\right)(3 + x)^2 + \frac{1}{6}\left(-\frac{108}{1331} - \frac{3\,e^{\sqrt{2}}}{32} + \frac{5\,e^{\sqrt{2}}}{32\sqrt{2}}\right)(3 + x)^3 + 3\,\text{Log}[11] - 3\,\text{Log}\left[2 + x^2\right]\right]$$

```
Plot[{f[x], p[x], Error[x]}, {x, a, b}]
```



Errore secondo Peano:

```
Limit[Error[x] / (x - x0)^n, x → x0]
```

```
0
```

Errore secondo Lagrange:

Questo implica

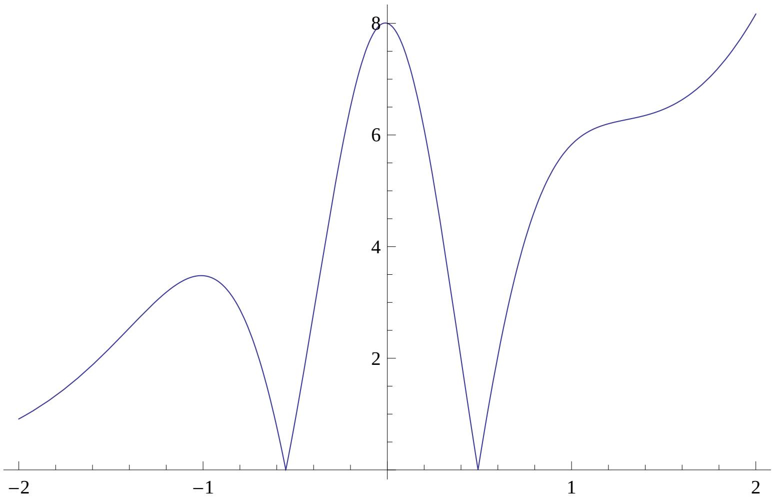
$$| f(x) - p(x) | \leq \frac{M\, | x - x_0 |^{n+1}}{(n+1)!}$$ dove e

```
tolerancia = 0.5
```

0.5

```
fError[x_] = D[f[x], {x, n + 1}];
```

```
Plot[Abs[fError[x]], {x, a, b}]
```




```
FindMaximum[{Abs[fError[x]], a ≤ x ≤ b}, {x, 2}]
```

{8.16683, {x → 2.}}

```
Cota = N[8.2 * (tolerancia) ^ (n + 1) / (n + 1) !]
```

0.0213542

```
Abs[f[x0 + 0.4] - p[x0 + 0.4]]
0.00795885
```