

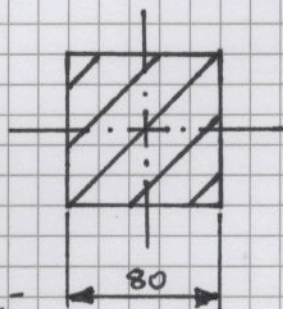
CARICO :  $q = 2 \text{ N/mm}$

MATERIALE : ACCIAIO  $E = 210 \text{ GPa}$

$\nu = 0.3$

$G = 80.769 \text{ GPa}$

SEZIONE : QUADRATA



$$A = 80^2 = 6400 \text{ mm}^2$$

$$I = \frac{80^4}{12} = 3413333.3333 \text{ mm}^4$$

$$\chi = \frac{6}{5} = 1.2$$

STATICITÀ E LABILITÀ

NODO	GDV
A	4
B	2
D	3

GDV :  $N^{\circ} \text{ASTE} \cdot 3 = 3 \cdot 3 = \underline{9}$

STRUTTURA ISOSTATICA

TOT. GDV = 9

La struttura è composta da 3 travi:

l'aste DCB è un unico corpo rigido che può essere schematizzato come un collegamento rigido tra D e B.



La struttura analizzata è quindi un arco a 3 cerniere chiuso e NON LABILE per definizione.

Quest'ultimo è vincolato da un conello in D e una cerniera in A, posti in modo

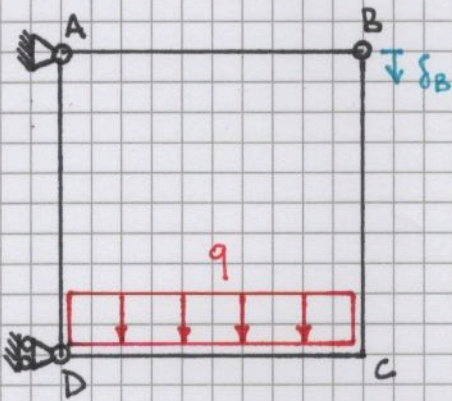
che le linee dei CE del conello non intersecano mai la cerniera A. Pertanto la struttura è NON LABILE.

# CALCOLO DELLO SPOSTAMENTO $\delta_B$ DEL PUNTO B

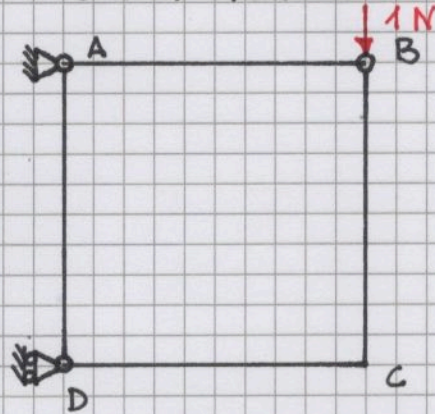
②

Per il calcolo dello spostamento del punto B si utilizzerà il principio dei Lavori Virtuali. Servirà costruire due strutture, una REALE e una VIRTUALE.

STRUTT. REALE



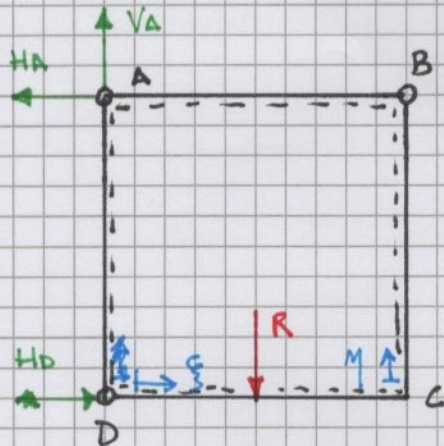
STRUTT. VIRTUALE



Analizziamole una alla volta.

## ANALISI STRUTTURA REALE

### - ANALISI REAZIONI VINCOLARI A TERRA



$$R = 2 \cdot 400 = 800 \text{ N}$$

$$\rightarrow^+ \quad H_D - H_A = 0 \quad H_D = H_A$$

$$\uparrow^+ \quad V_A - R = 0 \quad V_A = 800 \text{ N}$$

$$\curvearrowleft^+ \quad -R \cdot 200 + H_D \cdot 400 = 0$$

$$\bullet H_D = \frac{R \cdot 200}{400} = 400 \text{ N}$$

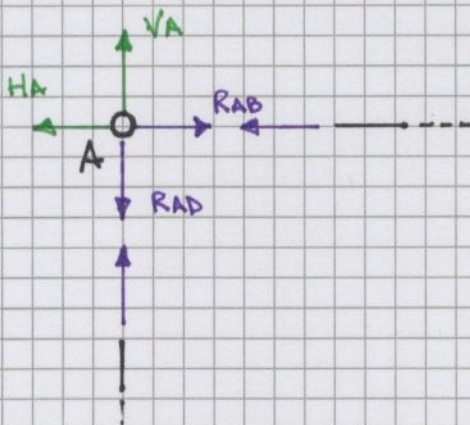
$$\bullet H_A = 400 \text{ N}$$

### - ANALISI REAZIONI VINCOLARI INTERNE

Le aste  $\overline{AD}$  e  $\overline{AB}$  sono BIELLE SCARICHE.

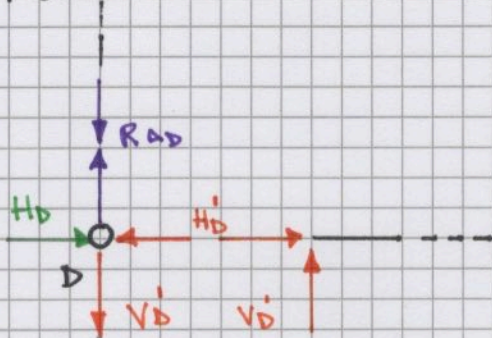
Procediamo all'equilibrio dei nodi A e D.

NO DO A.



$$\begin{cases} R_{AB} = H_A = 400 \text{ N} \\ R_{AD} = V_A = 800 \text{ N} \end{cases}$$

NODO D



$$\begin{cases} V_D' = \overbrace{800\text{N}}^{R_{AD}} \\ H_D' = H_D = 400\text{N} \end{cases}$$

- AZIONI INTERNE

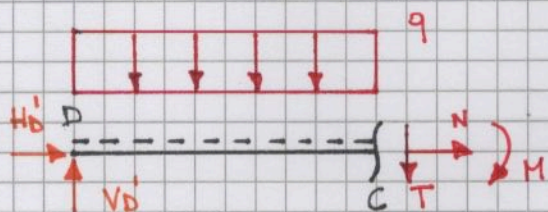
Nelle aste  $\overline{AB}$  e  $\overline{AD}$  è presente solo azione normale quindi:

$$N_{AD}^{(R)} = R_{AD} = +800\text{N}$$

$$N_{AB}^{(R)} = R_{AB} = +400\text{N}$$

A Per l'asta DCB è necessaria la solita procedura.

TRATTO DC  $0 < \xi < 400$



$$\bullet N_{DC}^{(R)} = -H_D' = -400\text{N}$$

$$\bullet T_{DC}^{(R)} = V_D' - q\xi$$

$$T(0) = V_D' = 800\text{N}$$

$$T(400) = 0$$

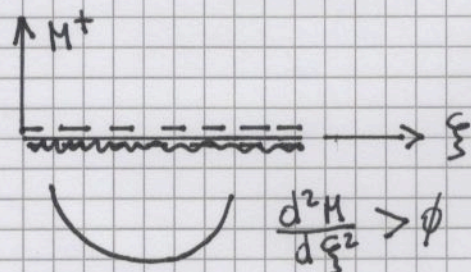
$$T=0 \text{ in } \xi = 400\text{ mm}$$

$$\bullet M_{DC}^{(R)} = -V_D' \xi + q \frac{\xi^2}{2}$$

$$M(0) = 0$$

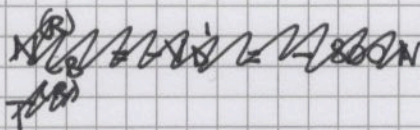
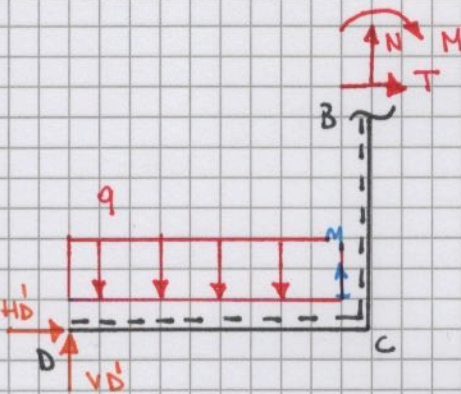
$$M(400) = -160'000\text{ N}\cdot\text{mm}$$

$\frac{d^2M}{d\xi^2} > 0 \rightarrow$  concavità verso la dir. POSITIVA DEI MOMENTI



# ANALISI STRUTTORA REALE

- TRATTO CB  $0 < \eta < 400$



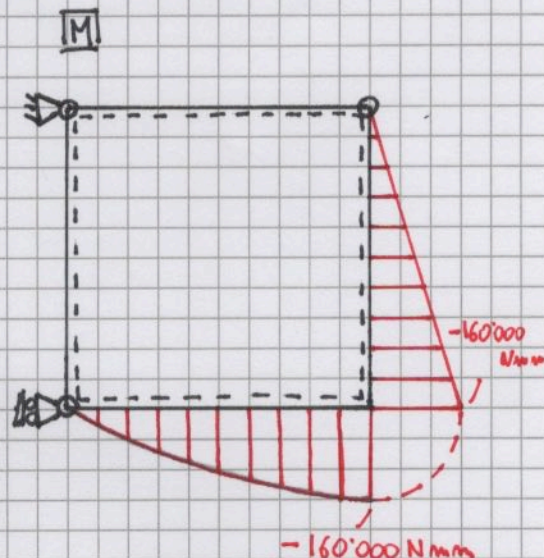
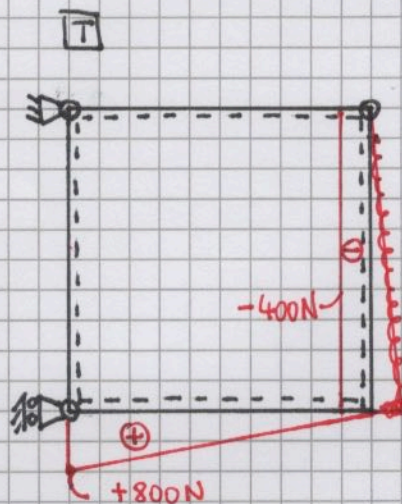
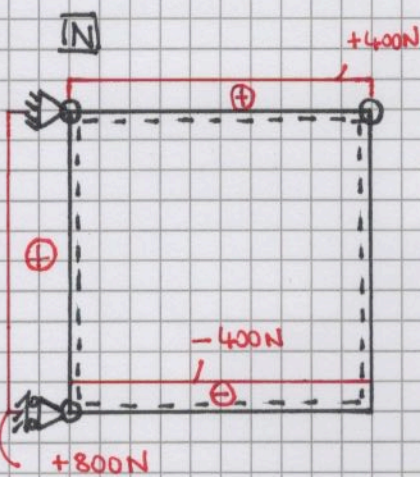
$$N_{CB}^{(R)} = -V_D' + q \cdot 400 = \phi$$

$$T_{CB}^{(R)} = -H_D' = -400 \text{ N}$$

$$M_{CB}^{(R)} = -V_D'(400) + H_D' \cdot \eta + q \cdot (400)(200) = 400\eta - 160'000$$

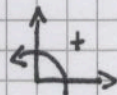
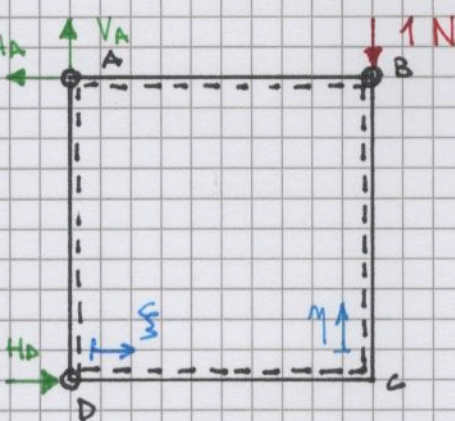
$$M(\phi) = -160'000 \text{ N}\cdot\text{mm} \quad M(400) = \phi$$

## DIA GRAMMI AZIONI INTERNE STRUTTORA REALE



## ANALISI STRUTTORA VIRTUALE

- CALCOLO REAZIONI A TERRA



$$\uparrow^+ \quad V_A - 1 = \phi \cdot V_A = 1 \text{ N}$$

$$\rightarrow^+ \quad H_D - H_A = \phi \cdot H_A = H_D$$

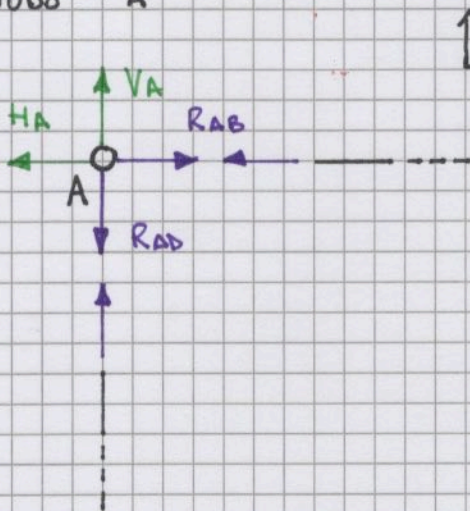
$$\uparrow^+ \quad H_D \cdot 400 - 1 \cdot 400 = \phi$$

$$\bullet H_D = 1 \text{ N}$$

$$\bullet H_A = 1 \text{ N}$$

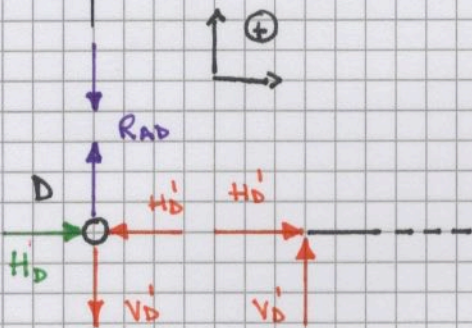
- CALCOLO REAZIONI INTERNE

NODO A



$$\begin{cases} R_{AD} = V_A = 1N \\ R_{AB} = H_A = 1N \end{cases}$$

NODO D



$$\begin{cases} H_D' = H_D = 1N \\ V_D' = R_{AD} = 1N \end{cases}$$

- AZIONI INTERNE

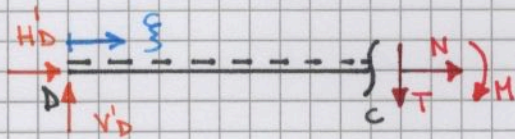
Come nel caso precedente le aste AB e AD è presente solo azione normale, poiché sono BIELLE SCARICHE.

$$N_{AD}^{(v)} = + R_{AD} = +1N$$

$$N_{AB}^{(v)} = + R_{AB} = +1N$$

TRATTO DC  $0 < \xi < 400$

TRATTO CB  $0 < \eta < 400$

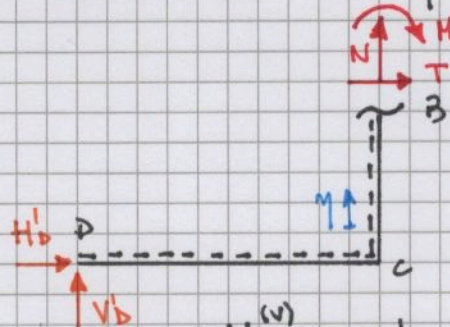


$$N_{DC}^{(s)} = -H_D' = -1N$$

$$T_{DC}^{(s)} = V_D' = 1N$$

$$M_{DC}^{(s)} = -V_D' \cdot \xi = -\xi$$

$$M(\phi) = \phi \quad M(400) = -400 \text{ N}\cdot\text{mm}$$



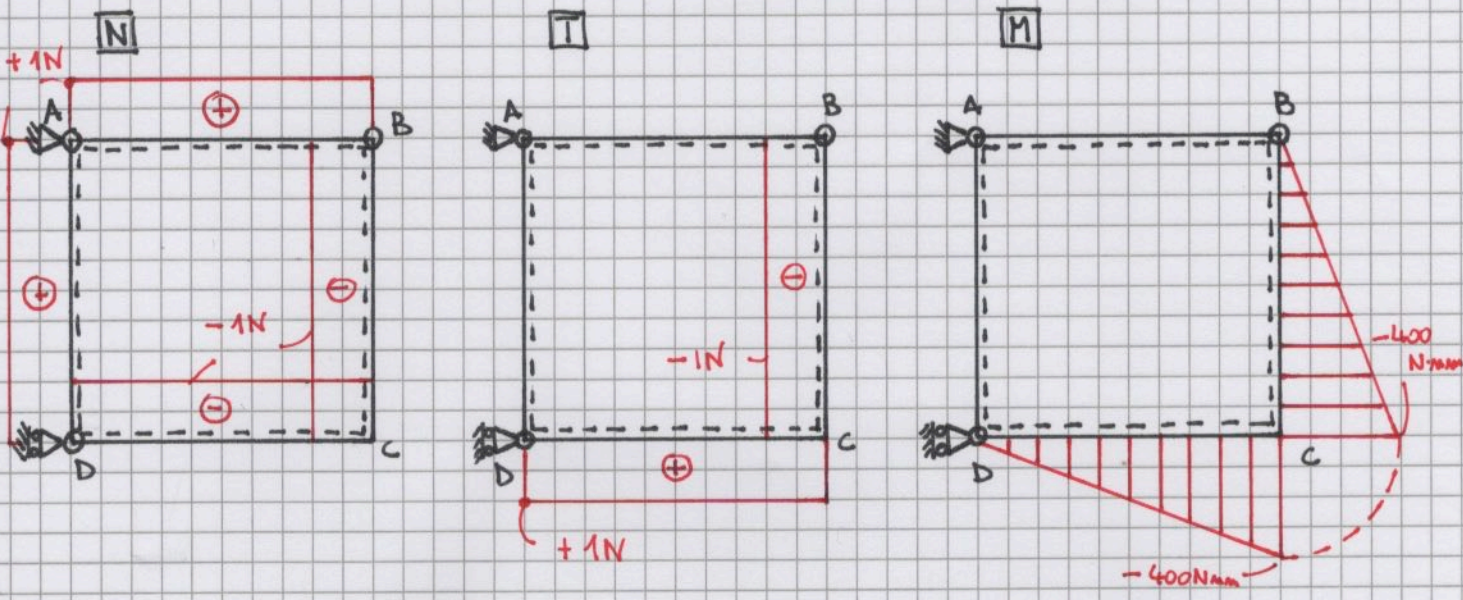
$$N_{CB}^{(v)} = -V_D' = -1N$$

$$T_{CB}^{(v)} = -H_D' = -1N$$

$$M_{CB}^{(v)} = -V_D'(400) + H_D' \eta = -400 + \eta$$

$$M(\phi) = -400 \text{ N}\cdot\text{mm}$$

$$M(400) = \phi$$



CALCOLO SPOLAMENTO

$$\begin{aligned}
 \mathcal{L}_{INT} &= \int_0^{400} \left[ \frac{N_{AB}^{(N)} N_{AB}^{(R)}}{EA} \right] dx + \int_0^{400} \left[ \frac{N_{BC}^{(V)} N_{BC}^{(R)}}{EA} + \chi \frac{T_{BC}^{(V)} T_{BC}^{(R)}}{GA} + \frac{M_{BC}^{(V)} M_{BC}^{(R)}}{EI} \right] dx \\
 &+ \int_0^{400} \left[ \frac{N_{CD}^{(V)} N_{CD}^{(R)}}{EA} + \chi \frac{T_{CD}^{(V)} T_{CD}^{(R)}}{GA} + \frac{M_{CD}^{(V)} M_{CD}^{(R)}}{EI} \right] dx + \int_0^{400} \frac{N_{AD}^{(V)} N_{AD}^{(R)}}{EA} dy = \\
 &= 1.1905 E-04 + 0.0123 + 0.0154 + 2.3810 E-04 = \\
 &= 0.0280
 \end{aligned}$$

$\mathcal{L}_{EST} = 1 \cdot \delta_B$

$\mathcal{L}_{EST} = \mathcal{L}_{INT}$

$\delta_B = 0.0280 \text{ mm} \rightarrow$  positivo: significa che lo spostamento avviene nello stesso verso in cui è posizionata la forza applicata unitaria.