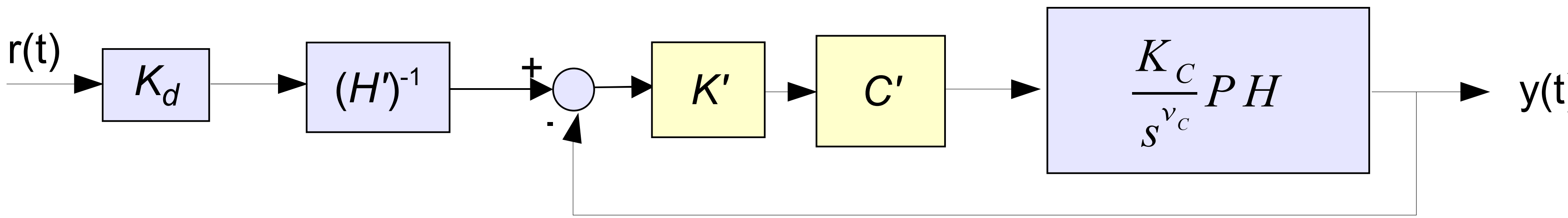
Sintesi mediante luogo delle radici

- Specifiche del transitorio nel piano complesso
- Criteri generali per la sintesi
- Esempi di sintesi

Specifiche del transitorio nel piano complesso



Il blocco K' può assumere valori inferiori a 1 solo e soltanto se la presenza di un adeguato numero di poli nell'origine nel controllore e/o nel processo garantisce il rispetto delle specifiche sul comportamento a regime a prescindere dal valore del guadagno

$$B_3 \approx \bar{B}_3$$

$$s \leq \bar{s}$$

$$M_r \leq \bar{M}_r$$

$$t_s \leq \bar{t}_s$$

$$t_s \leq \bar{t}_s \Leftrightarrow B_3 \geq \bar{B}_3$$

$$t_a \leq \bar{t}_a$$

$$t_{a_{\varepsilon\%}} \leq \bar{t}_a \Leftrightarrow B_3 \geq \bar{B}_3$$

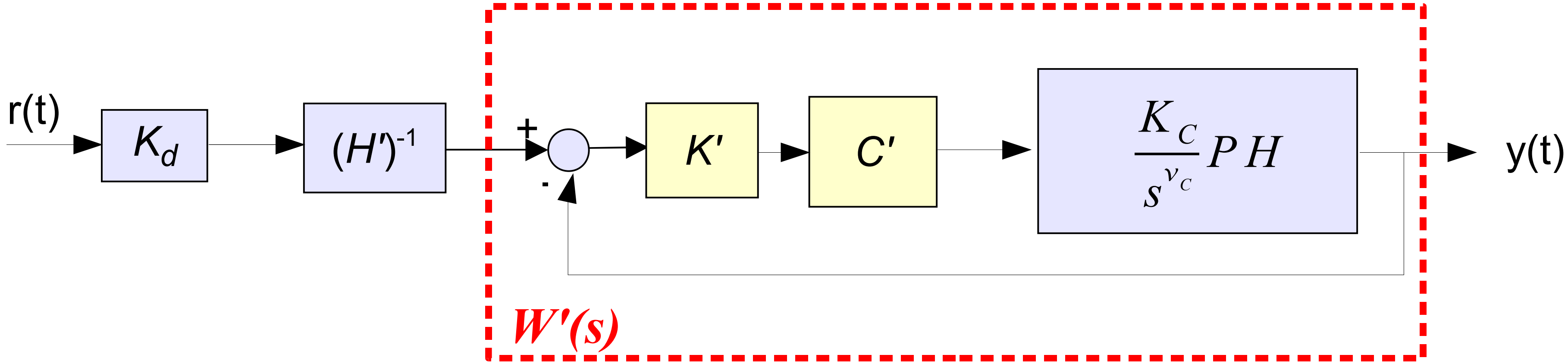
$$B_3 \approx \bar{B}_3 \quad \Leftrightarrow \quad \omega_n \approx \bar{\omega}_n$$

$$M_r \leq \bar{M}_r; \quad s \leq \bar{s} \quad \Leftrightarrow \quad \xi \geq \bar{\xi}$$

$$t_s \leq \bar{t}_s \Rightarrow \omega_n \geq \bar{\omega}_n$$

$$t_{a_{\varepsilon\%}} \leq \bar{t}_a \quad \Rightarrow \quad \tau_{eq} = \frac{1}{\xi \omega_n} \leq \bar{\tau}$$

Specifiche del transitorio nel piano complesso



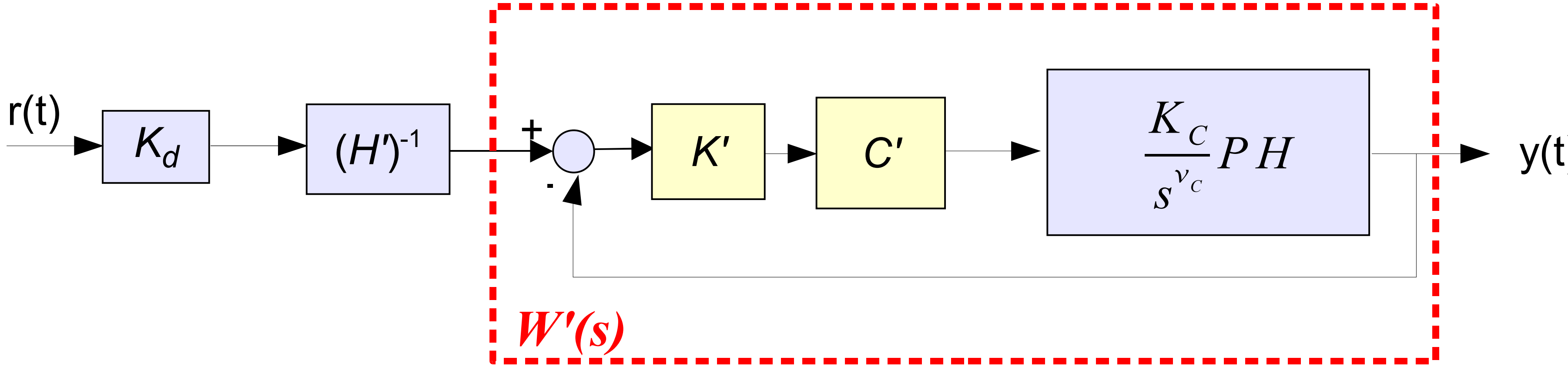
$$W'(s) = \frac{F'(s)}{1+F'(s)} = \frac{N_{F'}(s)}{D_{F'}(s) + K' N_{F'}(s)}$$

$$F'(s) = K' C'(s) \frac{K_C}{s^{\nu_C}} P(s) H(s)$$

I poli di W' possono essere valutati con il metodo del luogo delle radici

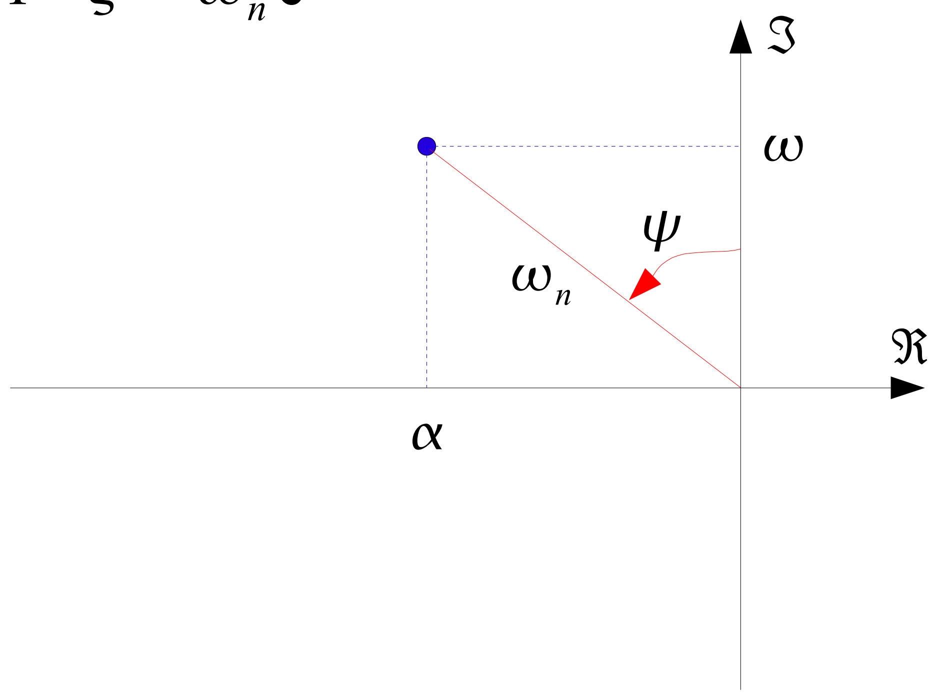
Inserendo poli e zeri in C' il luogo delle radici può essere modificato in modo che sia possibile definire valori di K' per cui i poli a ciclo chiuso giacciano in una specifica zona del piano complesso

Specifiche del transitorio nel piano complesso



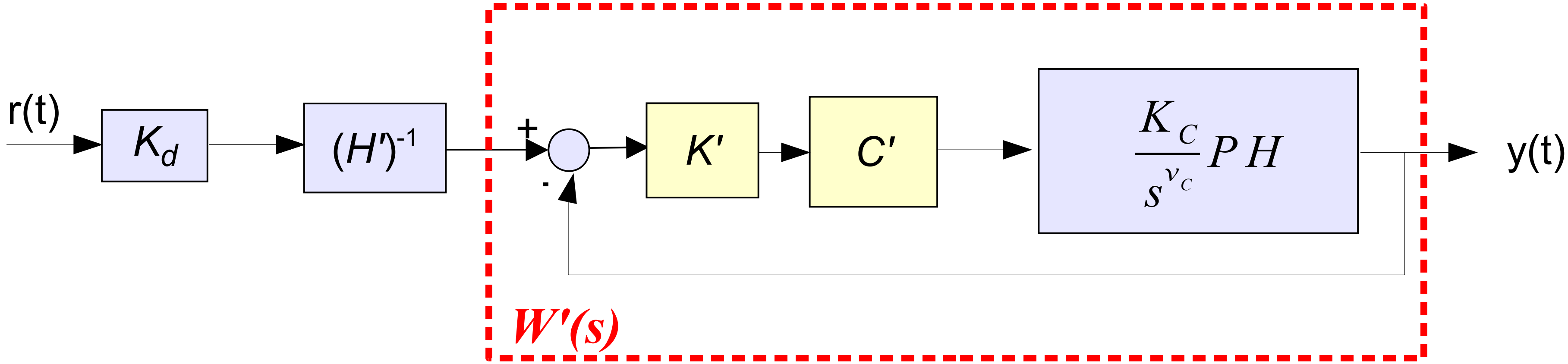
$$p_{i,i+1}^{W'} = \alpha \pm j\omega = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = \omega_n e^{\pm j\left(\frac{\pi}{2}+\psi\right)}$$

$$\xi = \sin(\psi)$$



I poli di W' possono essere valutati con il metodo del luogo delle radici

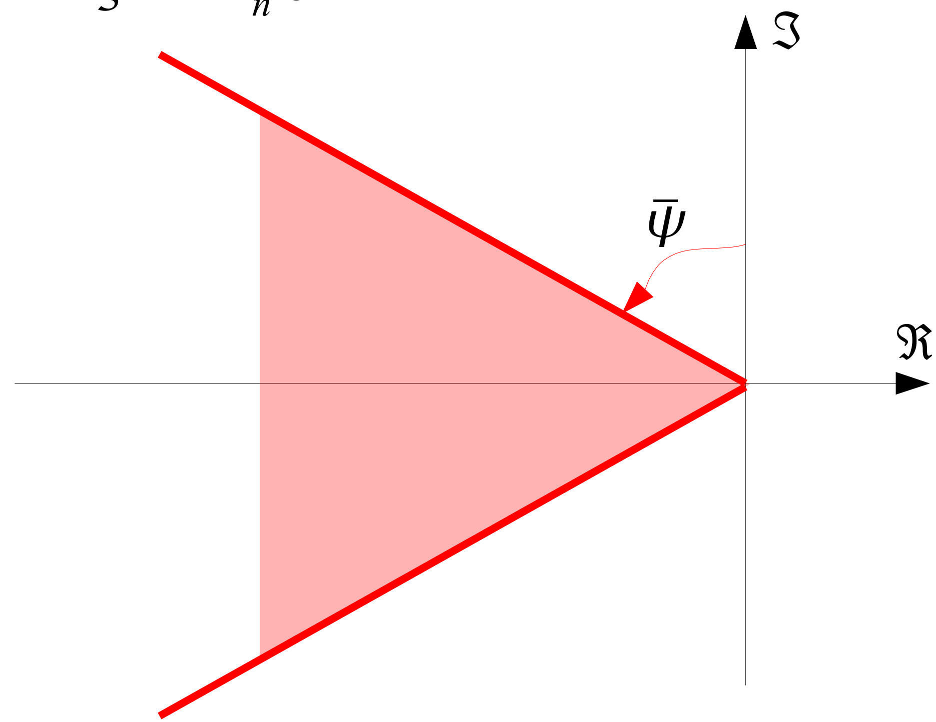
Specifiche del transitorio nel piano complesso



$$p_{i,i+1}^{W'} = \alpha \pm j\omega = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = \omega_n e^{\pm j\left(\frac{\pi}{2}+\psi\right)}$$

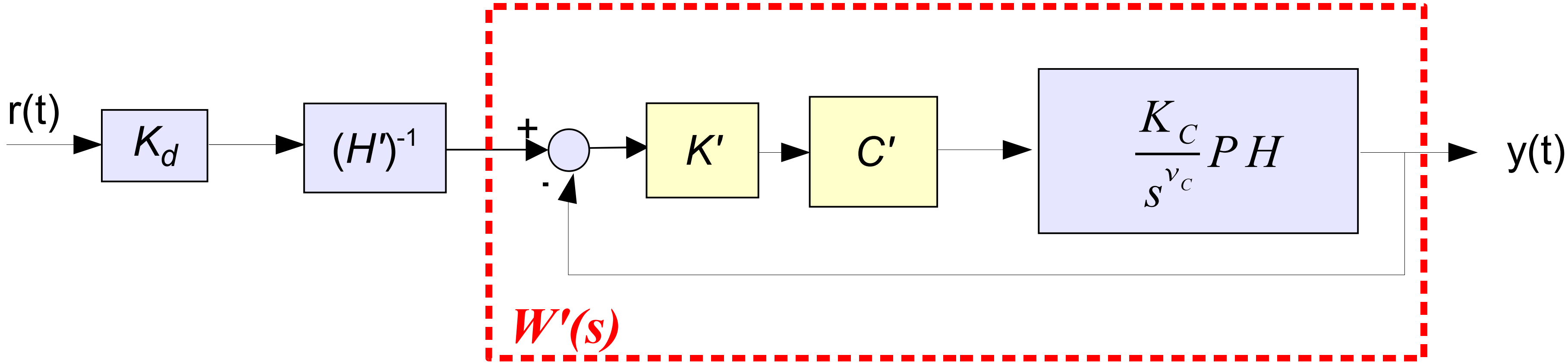
$$\xi = \sin(\psi)$$

$$\boxed{\xi \geq \bar{\xi} \Leftrightarrow \psi \geq \bar{\psi}}$$



I poli di W' possono essere valutati con il metodo del luogo delle radici

Specifiche del transitorio nel piano complesso

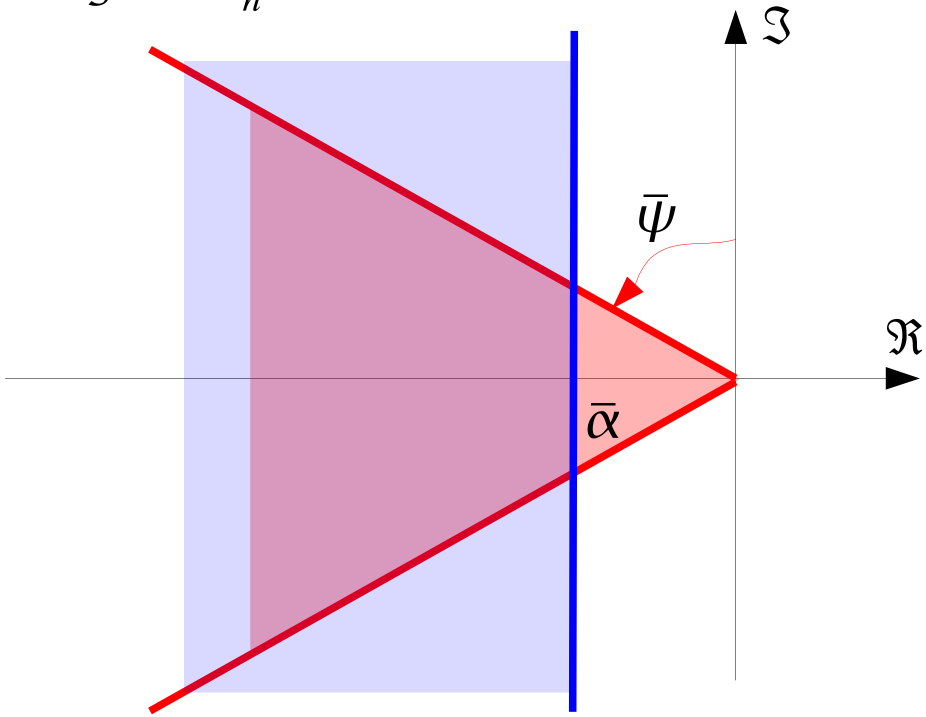


$$p_{i,i+1}^{W'} = \alpha \pm j\omega = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = \omega_n e^{\pm j\left(\frac{\pi}{2}+\psi\right)}$$

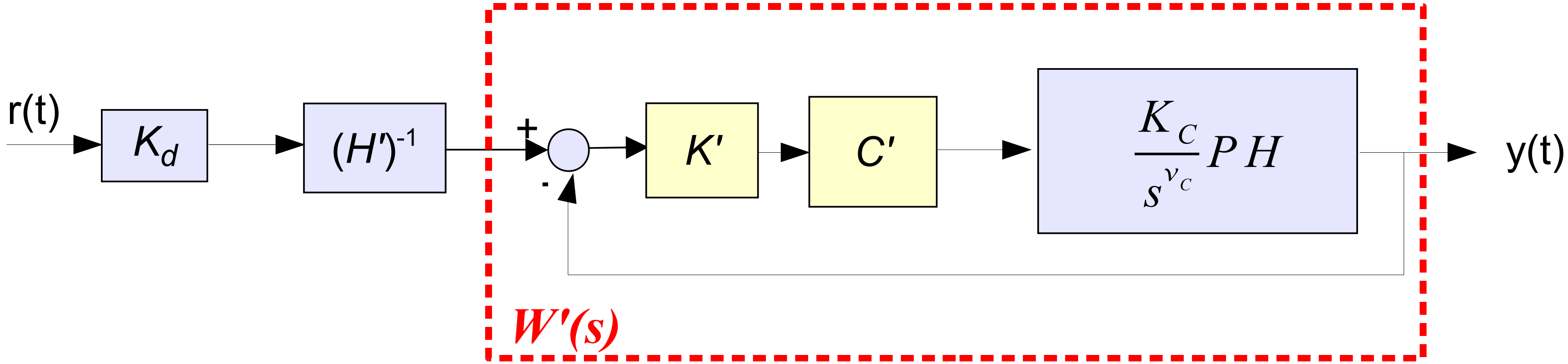
$$\xi = \sin(\psi)$$

$$\xi \geq \bar{\xi} \Leftrightarrow \psi \geq \bar{\psi}$$

$$\xi\omega_n \leq \frac{1}{\bar{\tau}} = \bar{\alpha} \Leftrightarrow \alpha \leq \bar{\alpha}$$



Specifiche del transitorio nel piano complesso



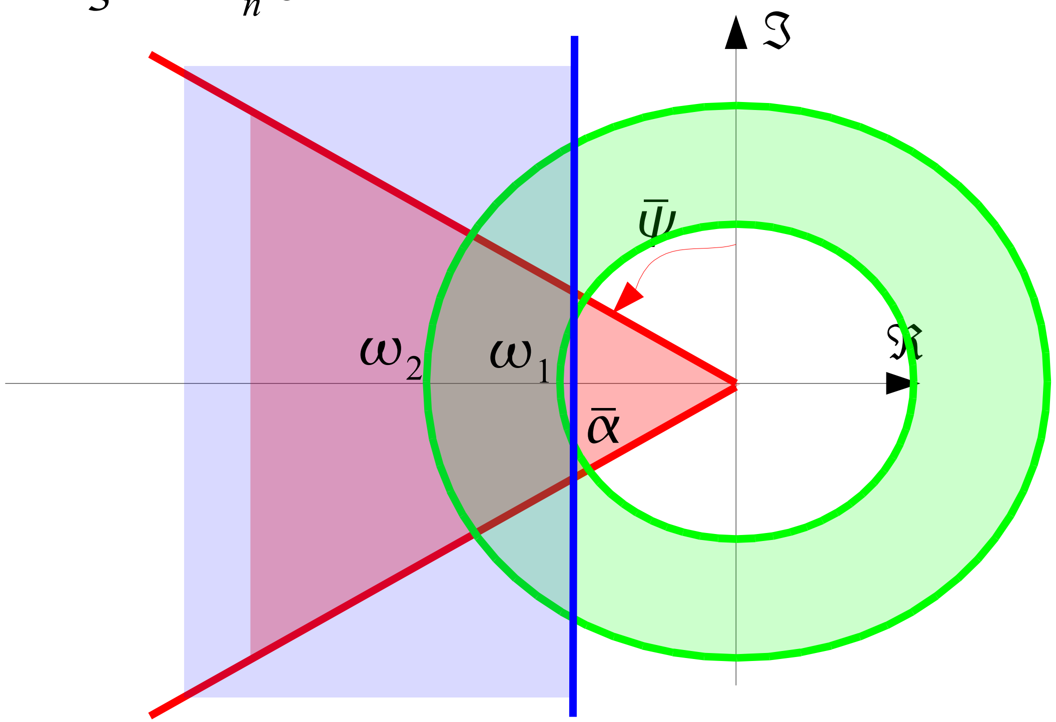
$$p_{i,i+1}^{W'} = \alpha \pm j\omega = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = \omega_n e^{\pm j\left(\frac{\pi}{2}+\psi\right)}$$

$$\xi = \sin(\psi)$$

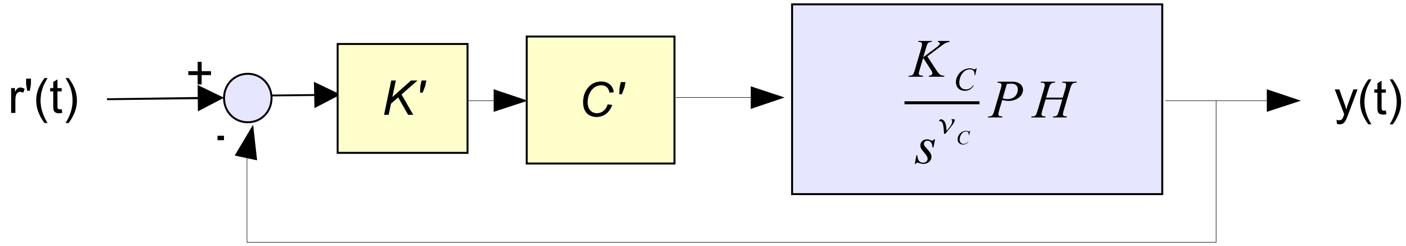
$$\xi \geq \bar{\xi} \Leftrightarrow \psi \geq \bar{\psi}$$

$$\xi\omega_n \leq \frac{1}{\bar{\tau}} = \bar{\alpha} \Leftrightarrow \alpha \leq \bar{\alpha}$$

$$\omega_n \approx \bar{\omega}_n \Leftrightarrow \omega_1 \leq \omega_n \leq \omega_2$$



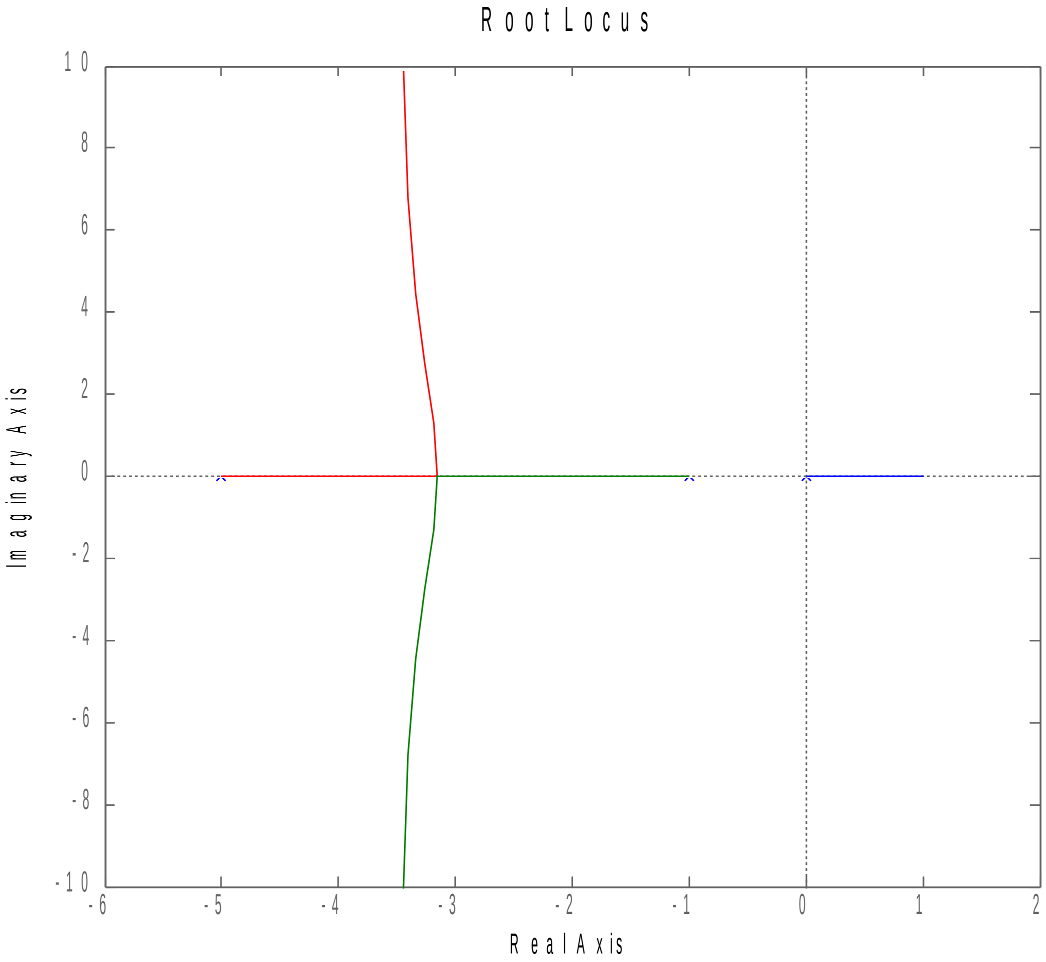
Criteri generali per la sintesi



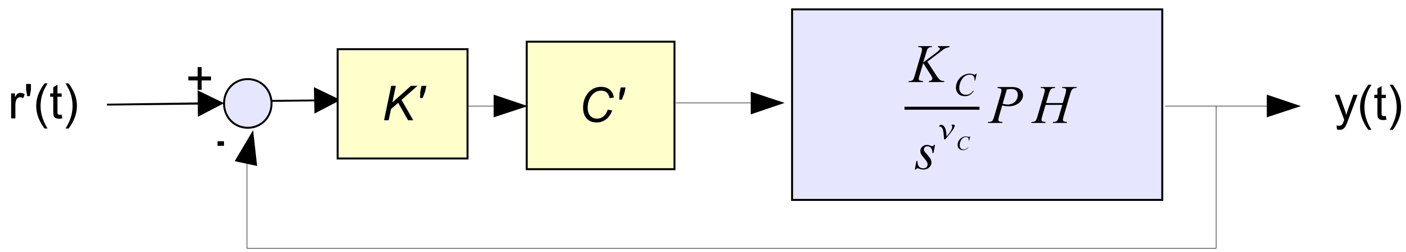
NON cancellare poli e zeri a parte reale positiva

$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{s-1}{(s+1)(s+5)}$$

La parte del semiasse reale a destra del polo nell'origine appartiene al luogo



Criteri generali per la sintesi

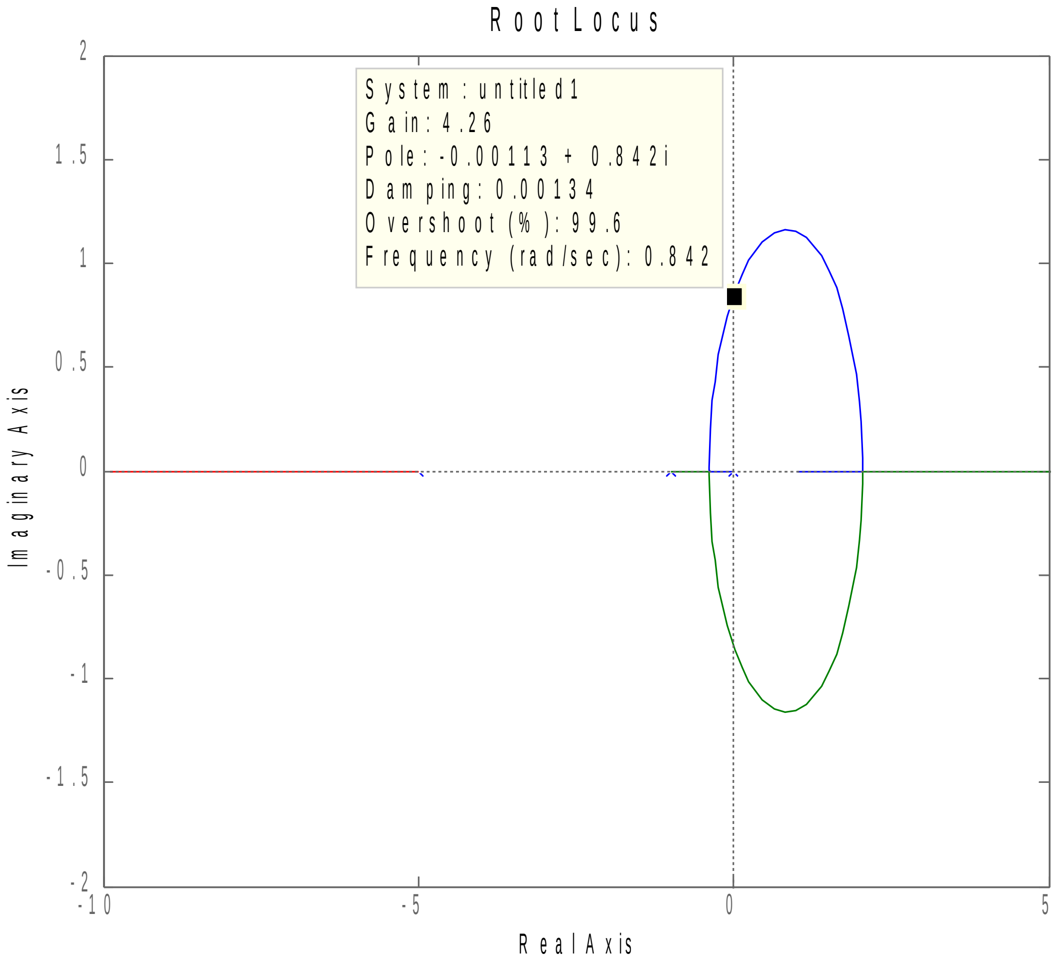


NON cancellare poli e zeri a parte reale positiva

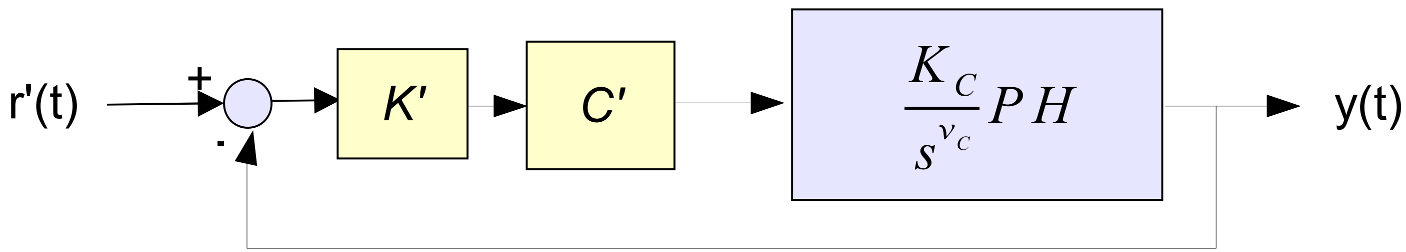
$$H = -1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{s-1}{(s+1)(s+5)}$$

Realizzare un sistema a retroazione positiva

$$K' \leq 4{,}26$$

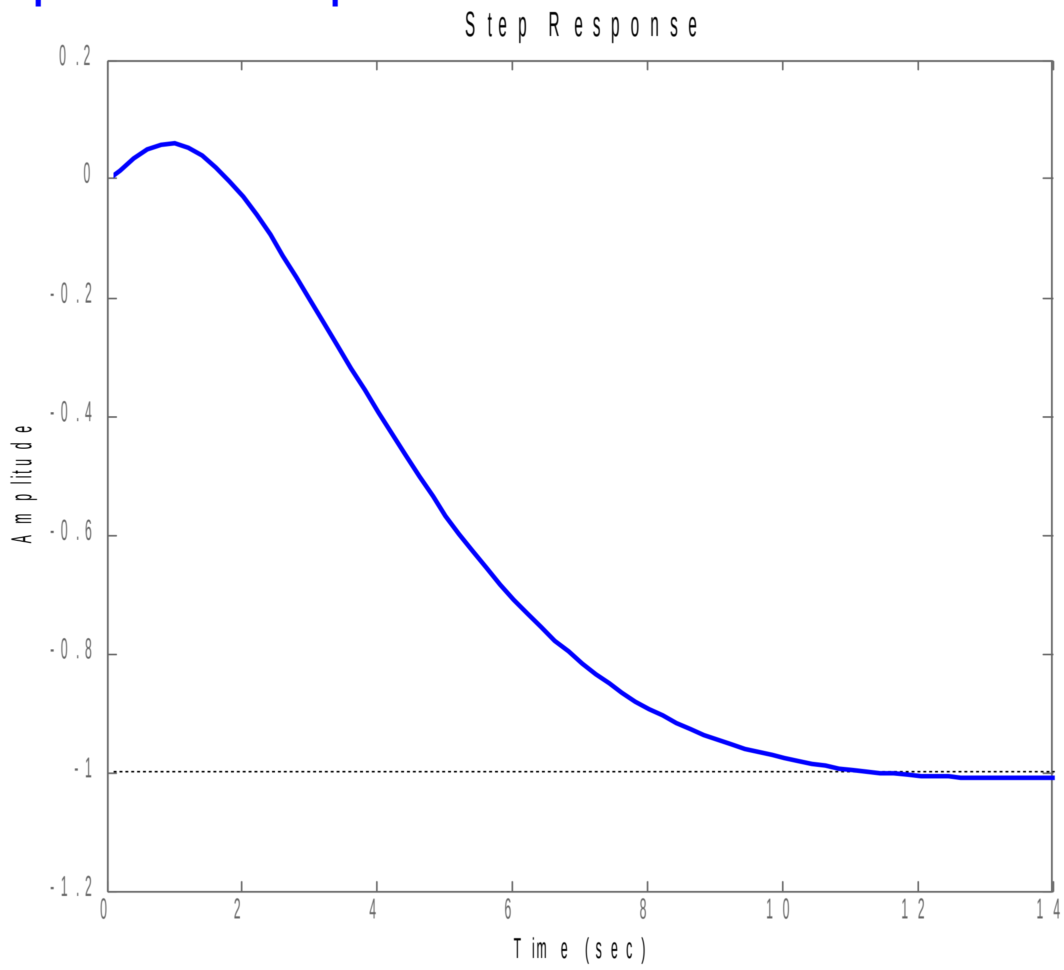


Criteri generali per la sintesi



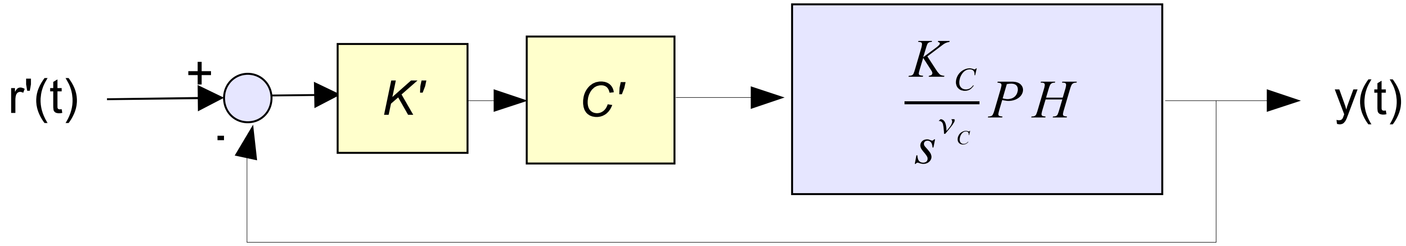
NON cancellare poli e zeri a parte reale positiva

$$H = -1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{s-1}{(s+1)(s+5)}$$



A causa del guadagno negativo di P si ha guadagno negativo a ciclo chiuso

Criteri generali per la sintesi

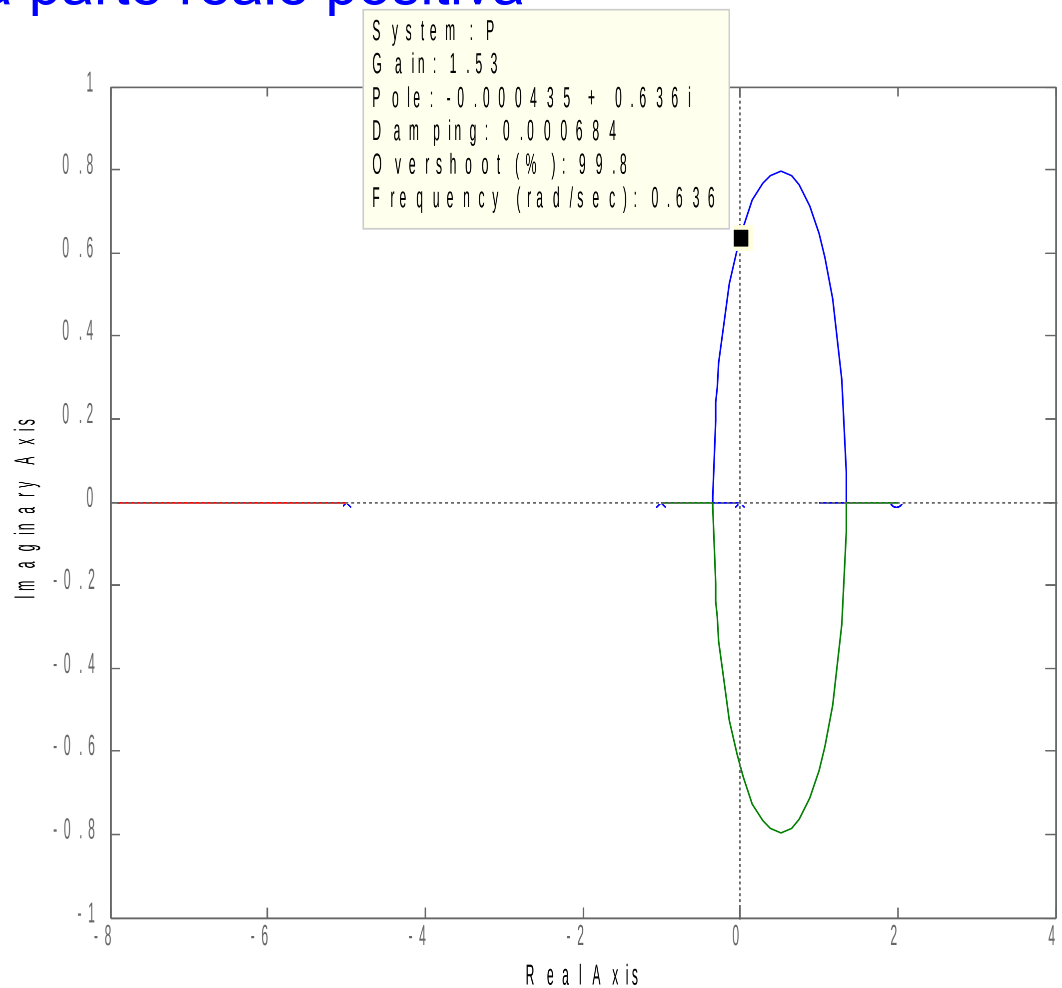


NON cancellare poli e zeri a parte reale positiva

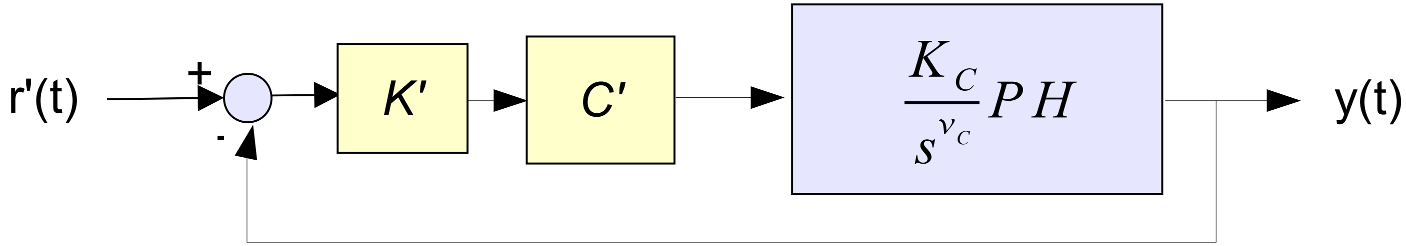
$$H=1$$
$$K_C=1$$
$$\nu_C=1$$
$$P(s)=\frac{s-1}{(s+1)(s+5)}$$

Inserire uno zero a parte reale positiva nel controllore

$$C(s)=K'\frac{s-2}{s}$$
$$K'\leq 1{,}53$$



Criteri generali per la sintesi

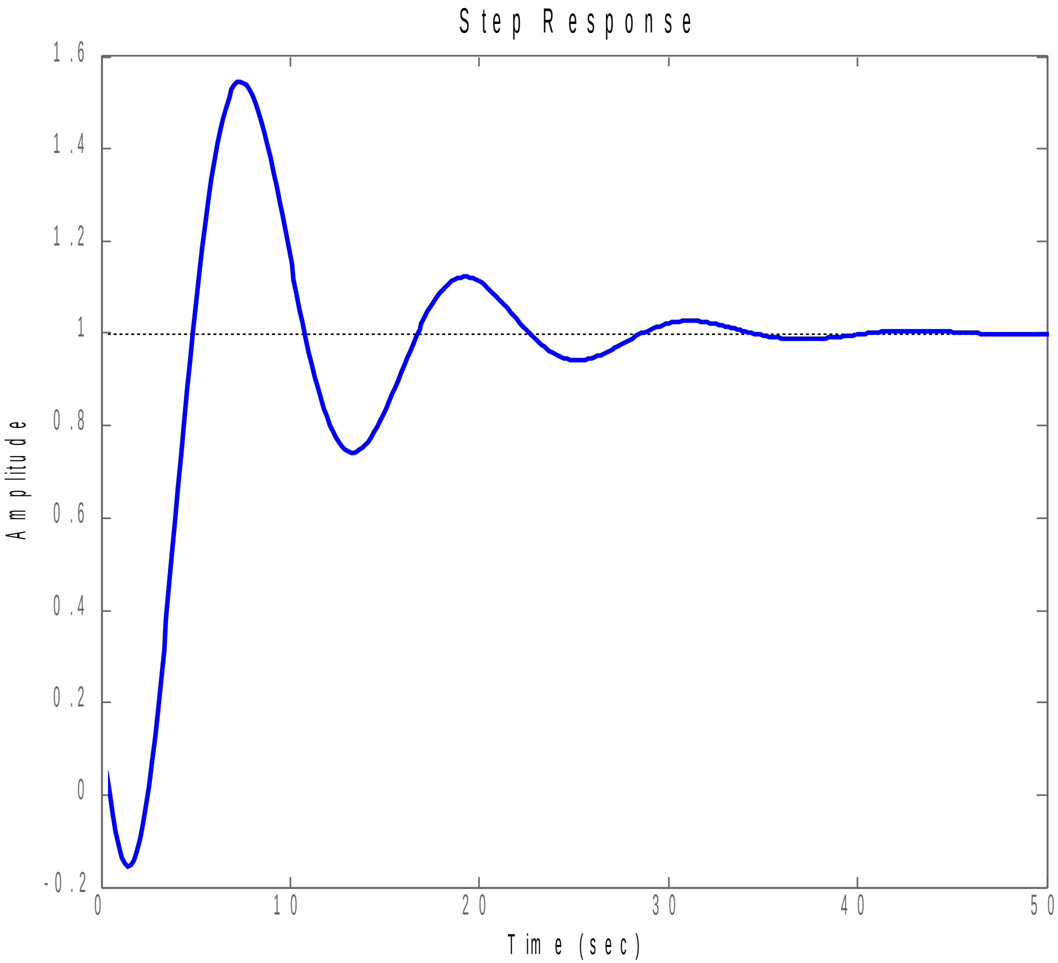


NON cancellare poli e zeri a parte reale positiva

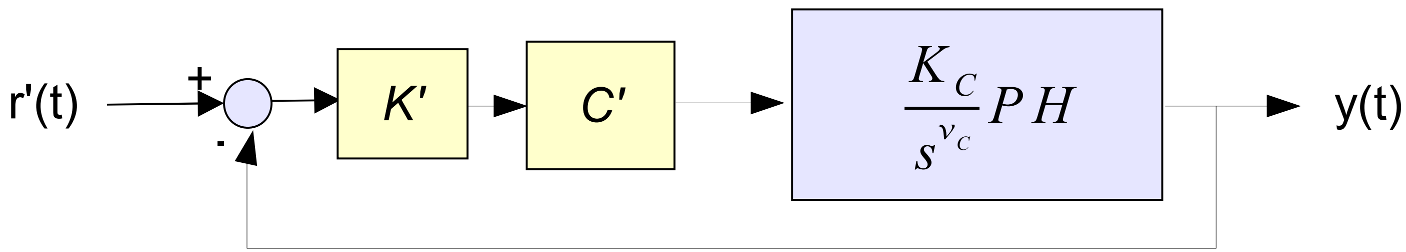
$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{s-1}{(s+1)(s+5)}$$

Inserire uno zero a parte reale positiva nel controllore

$$C(s) = K' \frac{s-2}{s}$$
$$K' = 1 \le 1{,}53$$

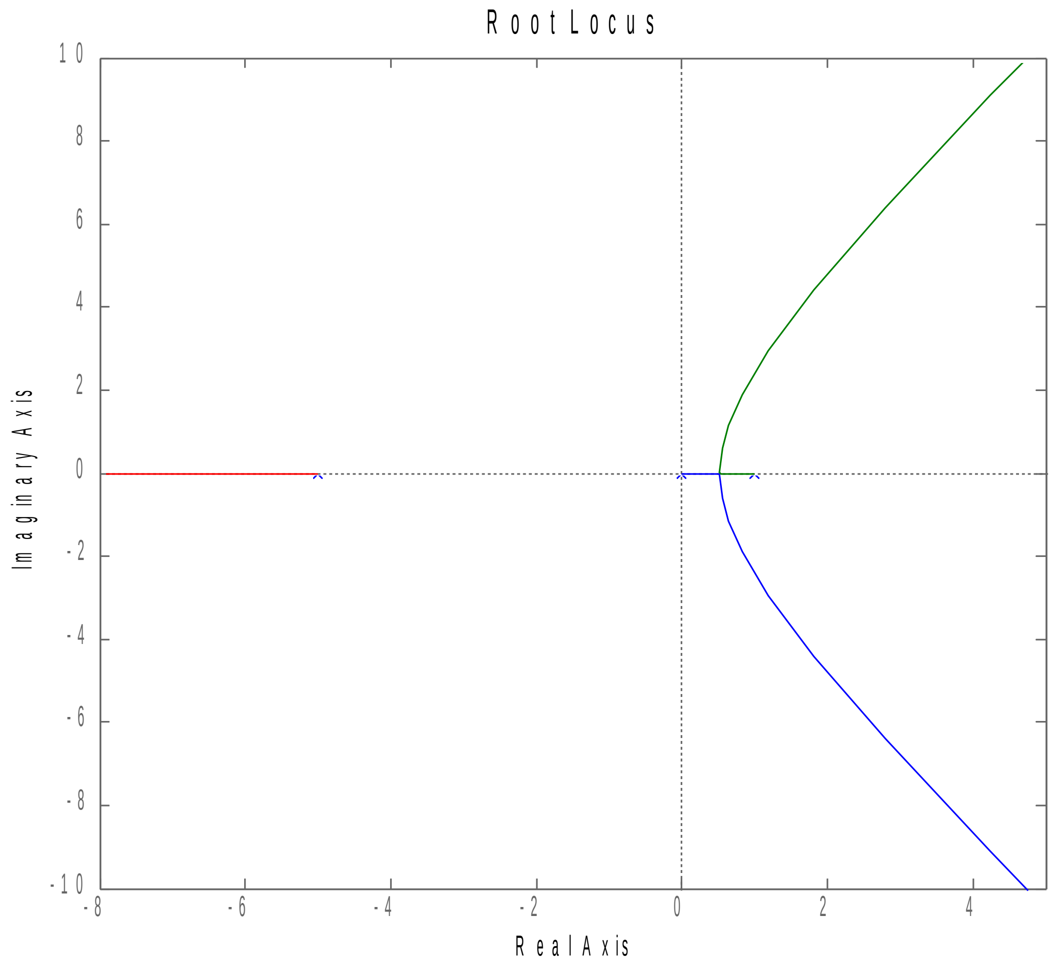


Criteri generali per la sintesi

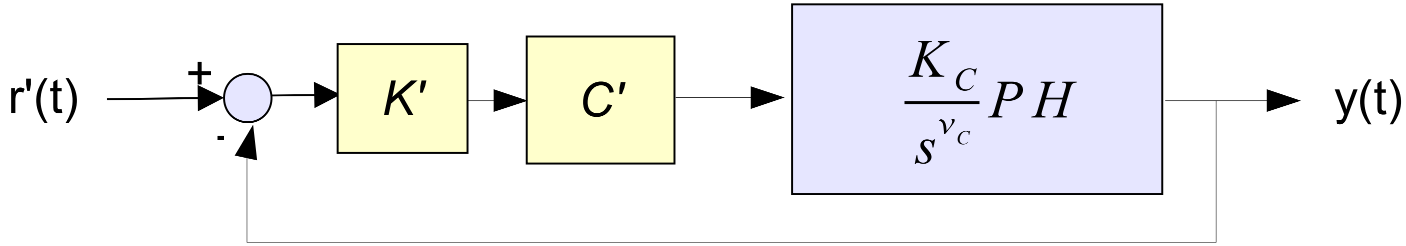


NON cancellare poli e zeri a parte reale positiva

$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{1}{(s-1)(s+5)}$$



Criteri generali per la sintesi

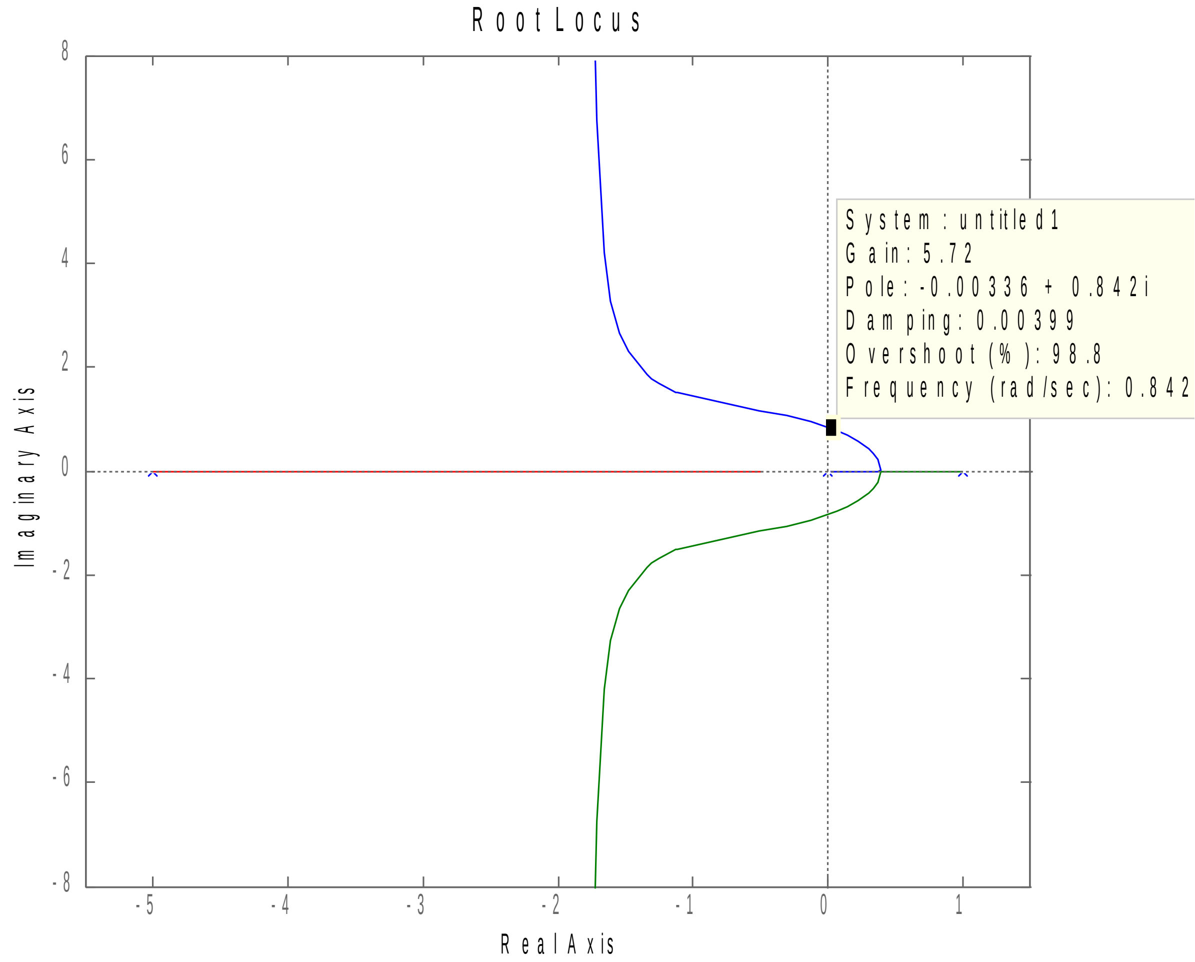


NON cancellare poli e zeri a parte reale positiva

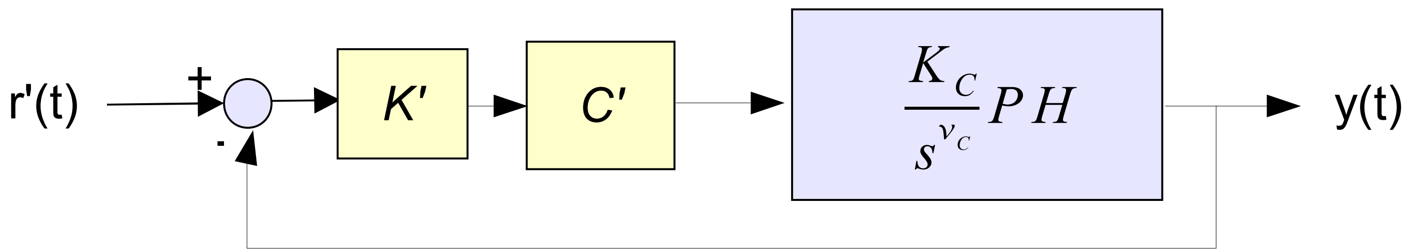
$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{1}{(s-1)(s+5)}$$

Inserire uno zero a parte reale negativa nel controllore

$$C(s) = K' \frac{s+0.5}{s}$$
$$K' \geq 5{,}72$$



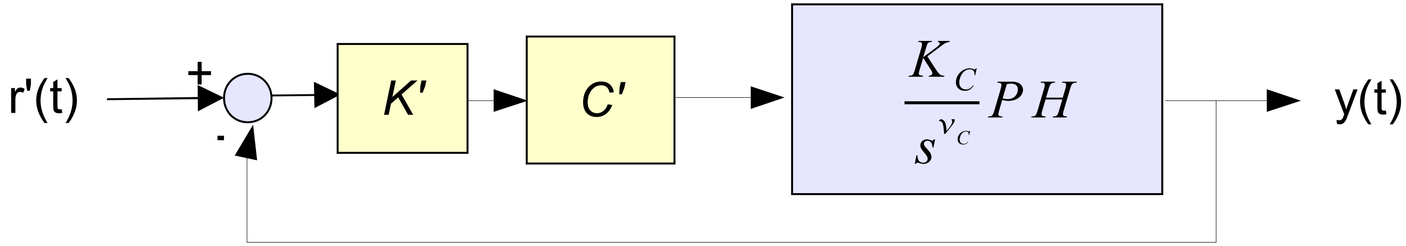
Criteri generali per la sintesi



NON cancellare poli a parte reale nulla

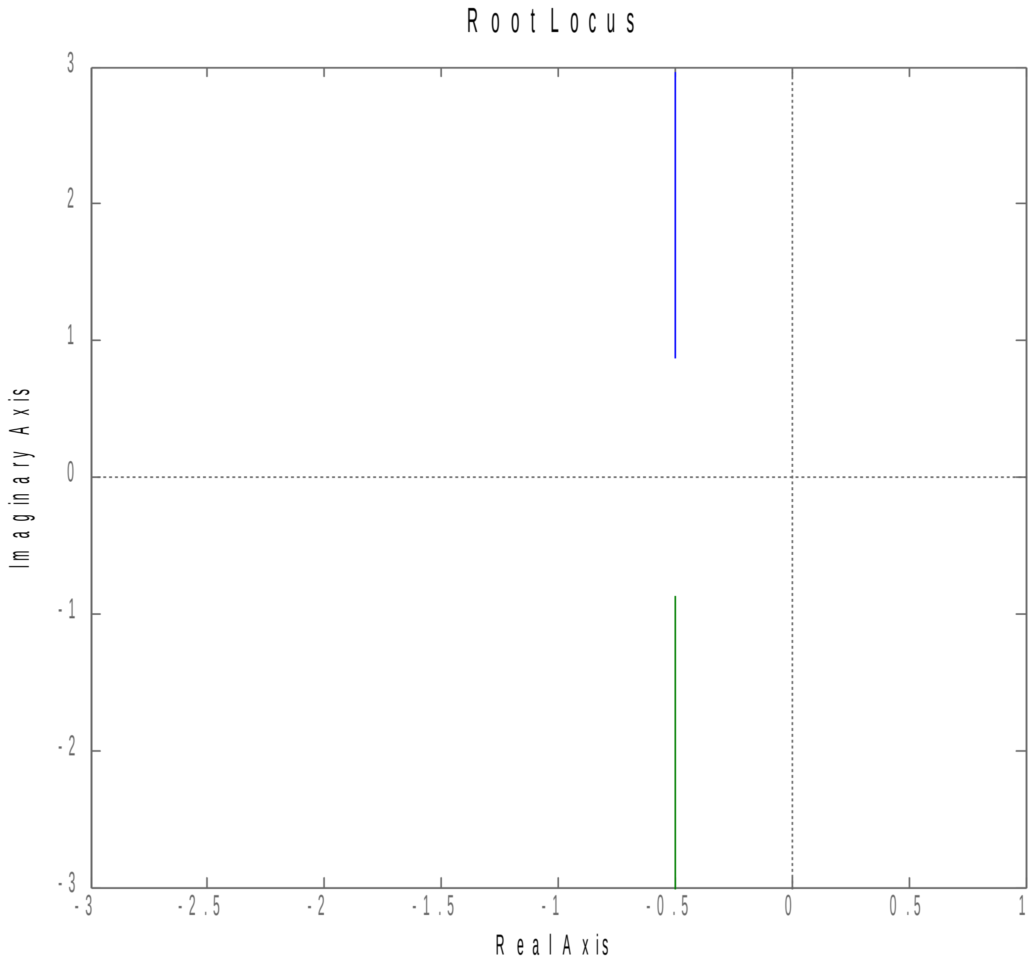
- La cancellazione di un polo nell'origine può causare il NON soddisfacimento delle specifiche sul comportamento a regime
- A causa di variazioni parametriche, il polo o lo zero a parte reale nulla introdotto o del sistema possono spostarsi nel semipiano positivo e originare un modo instabile
- L'introduzione del controllore deve anche garantire sufficienti margini di stabilità al sistema a ciclo chiuso: i suoi poli devono essere sufficientemente lontani dall'asse immaginario

Criteri generali per la sintesi

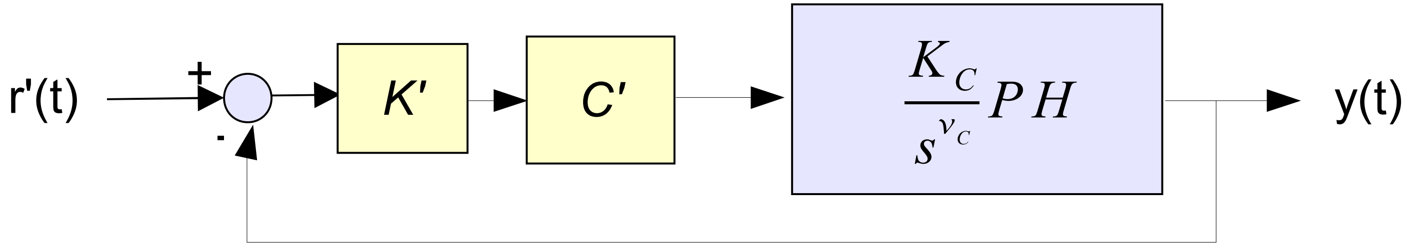


L'introduzione di un polo sposta il luogo delle radici verso il semipiano positivo

$$H=1$$
$$K_C=1$$
$$\nu_C=0$$
$$P(s)=\frac{1}{s^2+s+1}$$



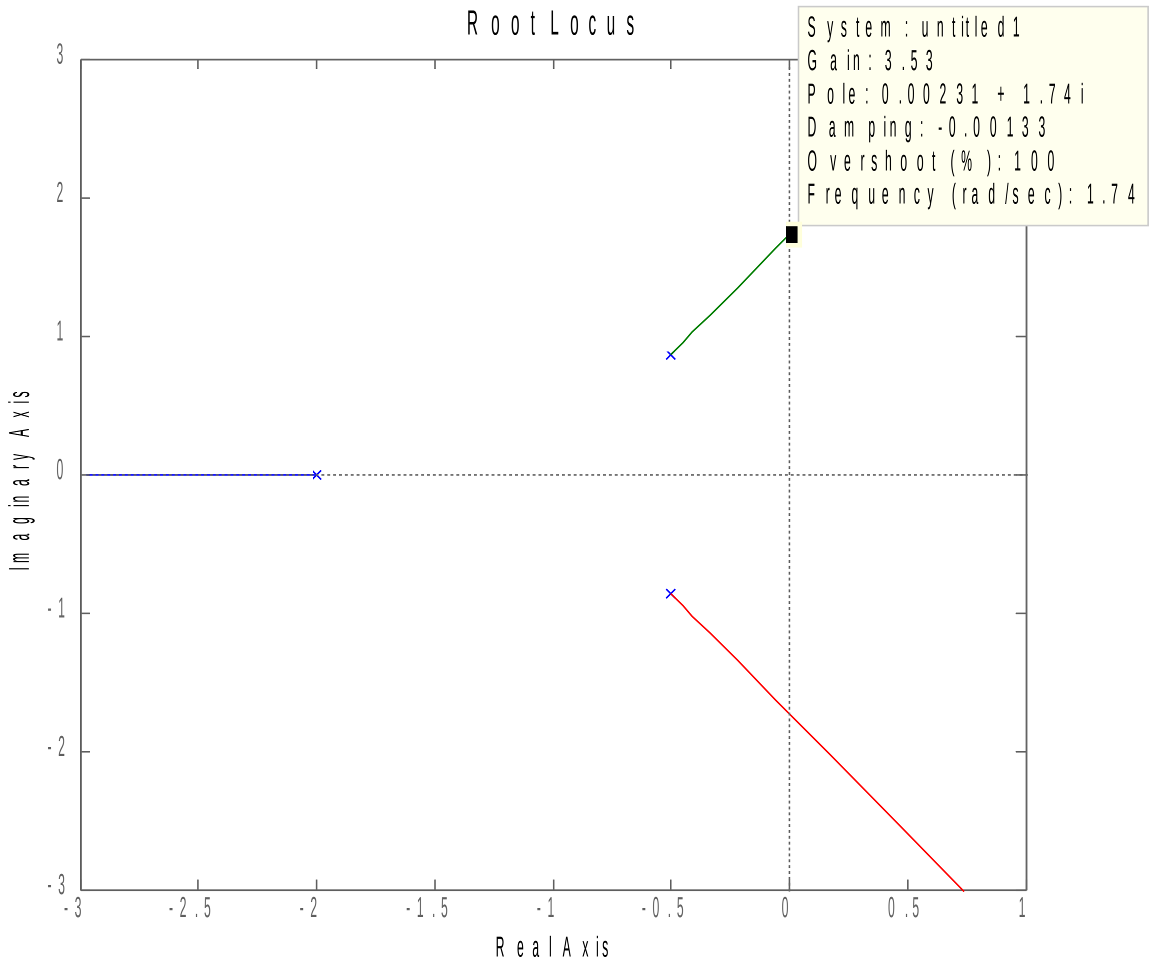
Criteri generali per la sintesi



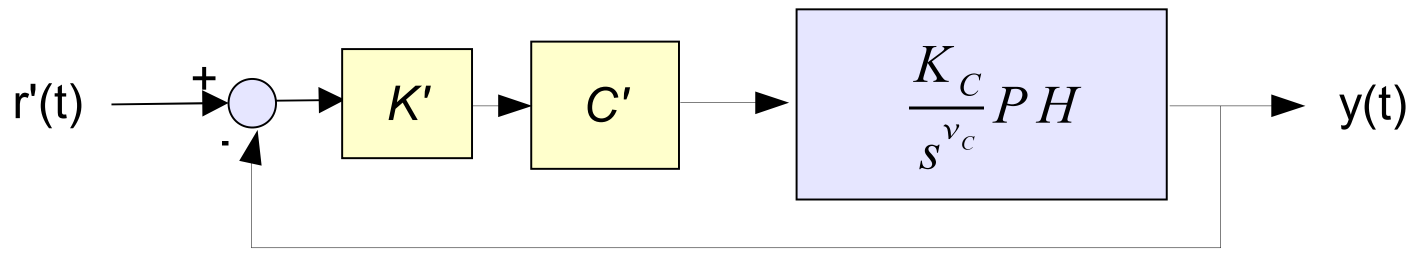
L'introduzione di un polo sposta il luogo delle radici verso il semipiano positivo

$$H = 1$$
$$K_C = 1$$
$$\nu_C = 0$$
$$P(s) = \frac{1}{s^2 + s + 1}$$

$$C(s) = K' \frac{1}{0.5s + 1}$$
$$K' = 1 \leq 3{,}53$$



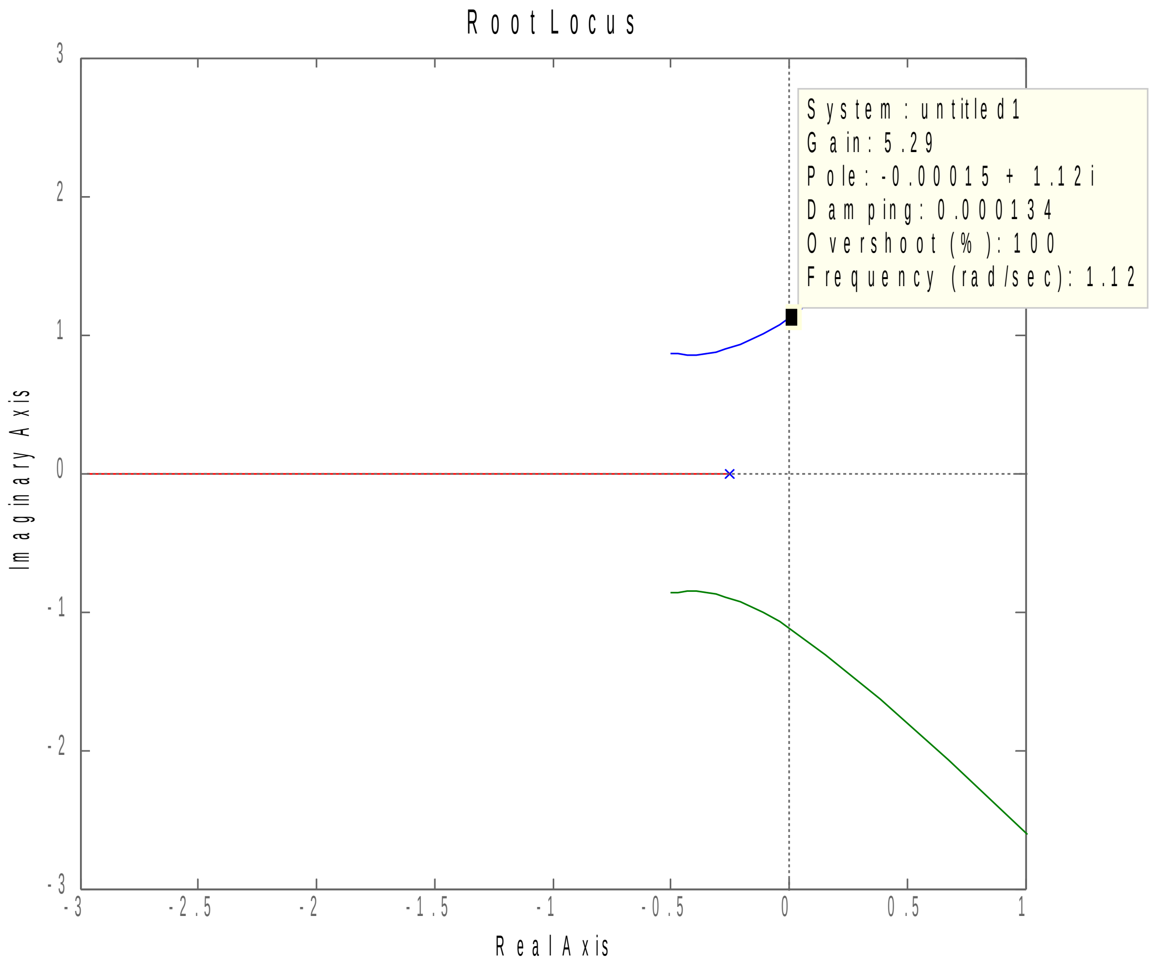
Criteri generali per la sintesi



L'introduzione di un polo sposta il luogo delle radici verso il semipiano positivo

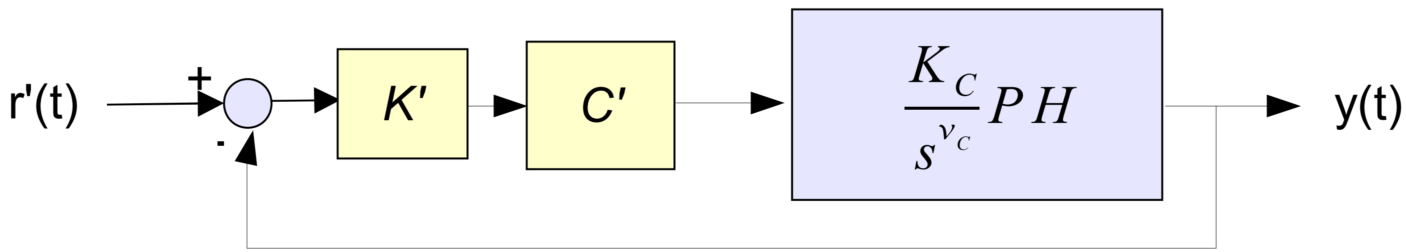
$$H = 1$$
$$K_C = 1$$
$$\nu_C = 0$$
$$P(s) = \frac{1}{s^2 + s + 1}$$

$$C(s) = K' \frac{1}{4s + 1}$$
$$K' = 1 \leq 3{,}29$$



Lo spostamento è tanto maggiore quanto il polo è vicino all'origine

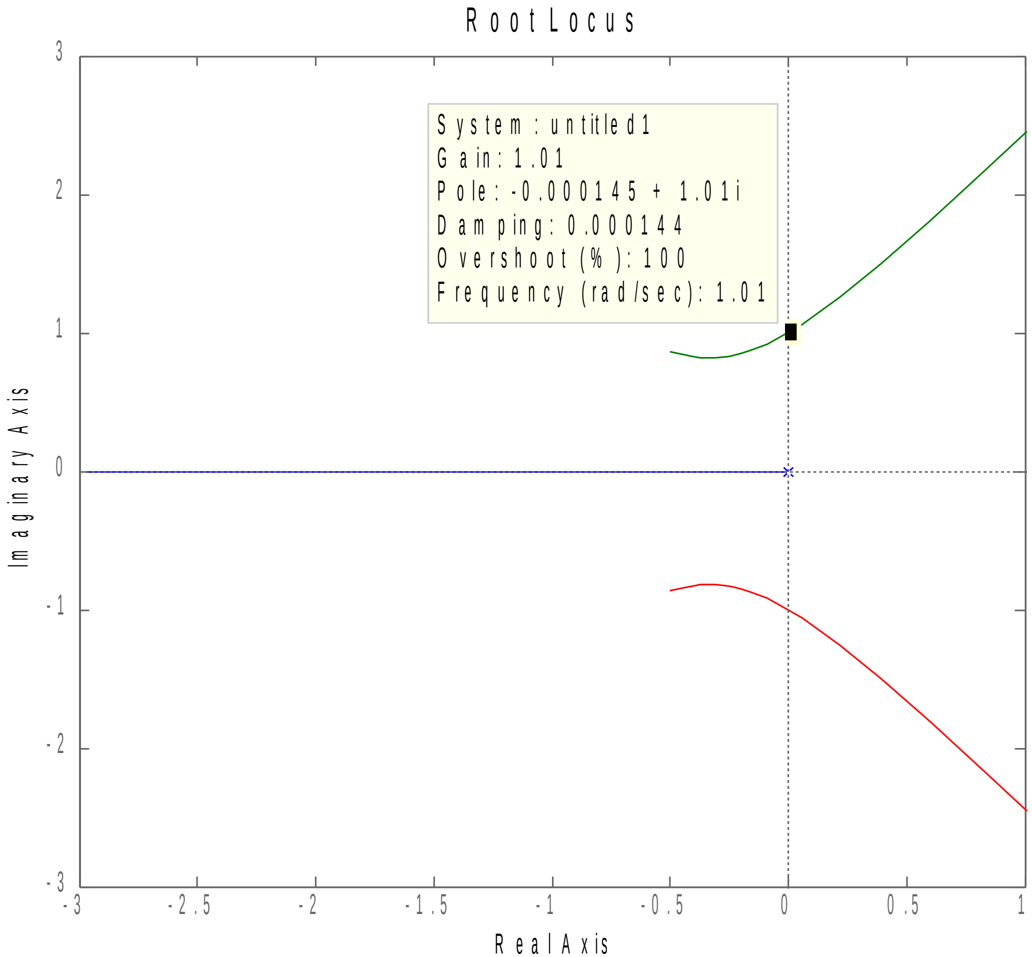
Criteri generali per la sintesi



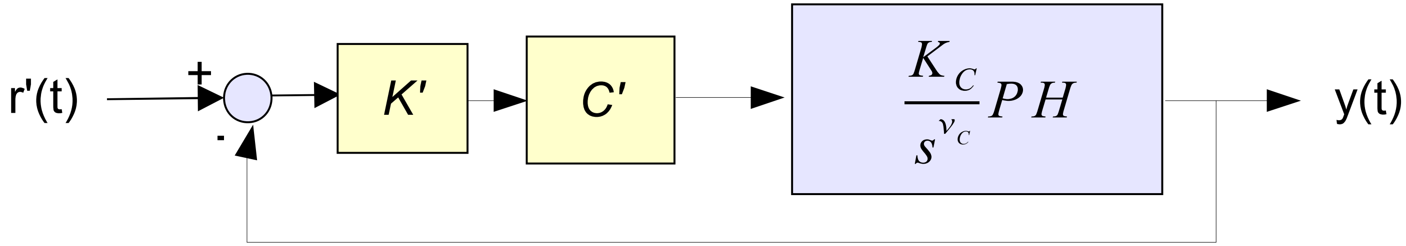
L'introduzione di uno zero sposta il luogo delle radici verso il semipiano negativo

$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{1}{s^2 + s + 1}$$

$$K' \leq 1$$



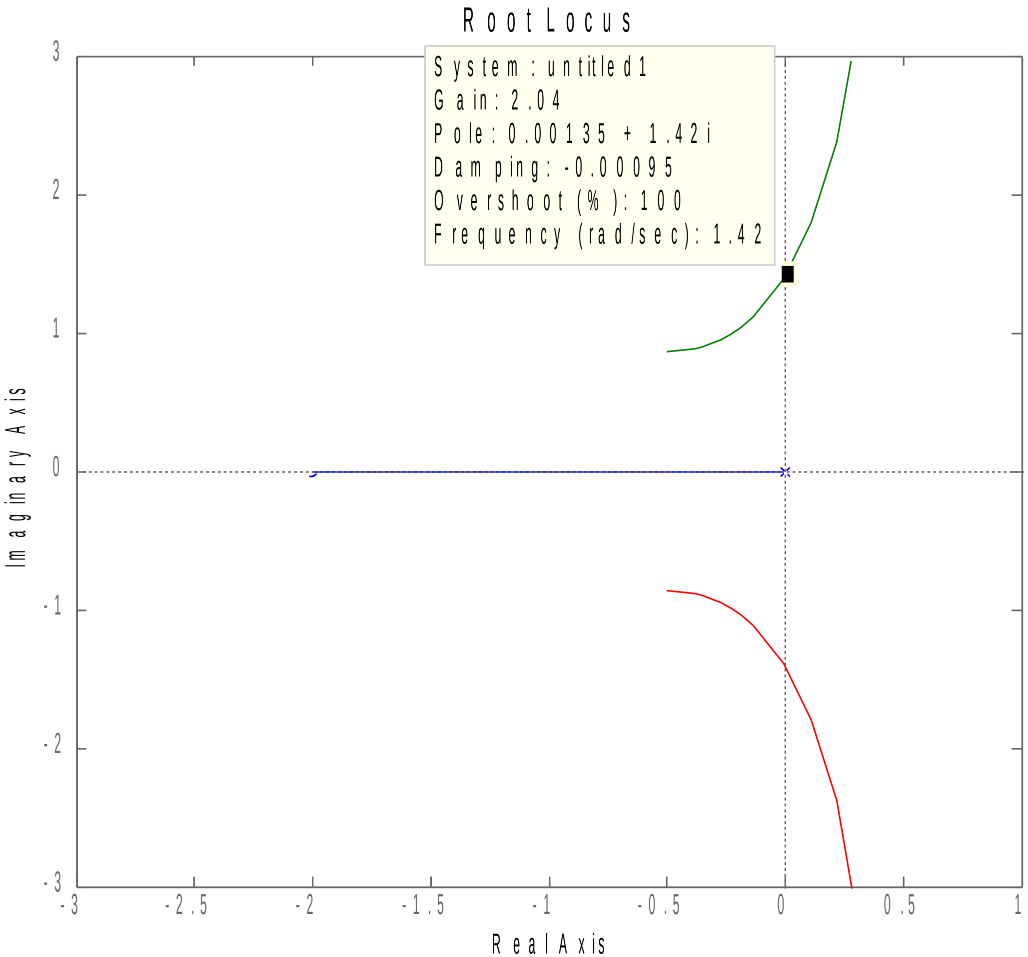
Criteri generali per la sintesi



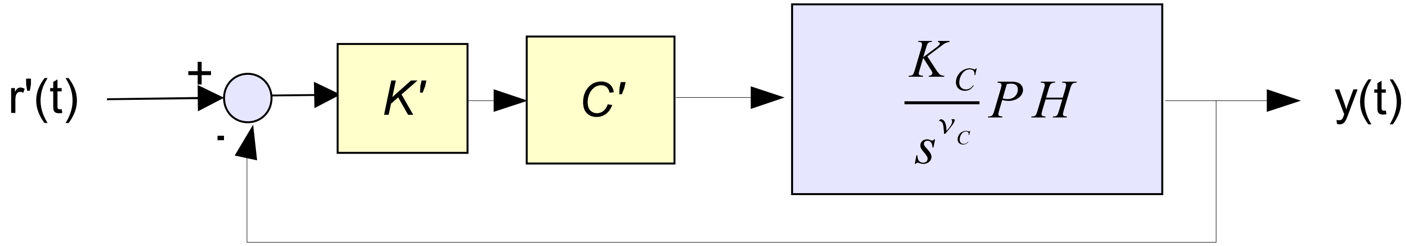
L'introduzione di uno zero sposta il luogo delle radici verso il semipiano negativo

$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{1}{s^2 + s + 1}$$

$$C(s) = K' \frac{0.5 s + 1}{s}$$
$$K' = 1 \leq 2$$



Criteri generali per la sintesi



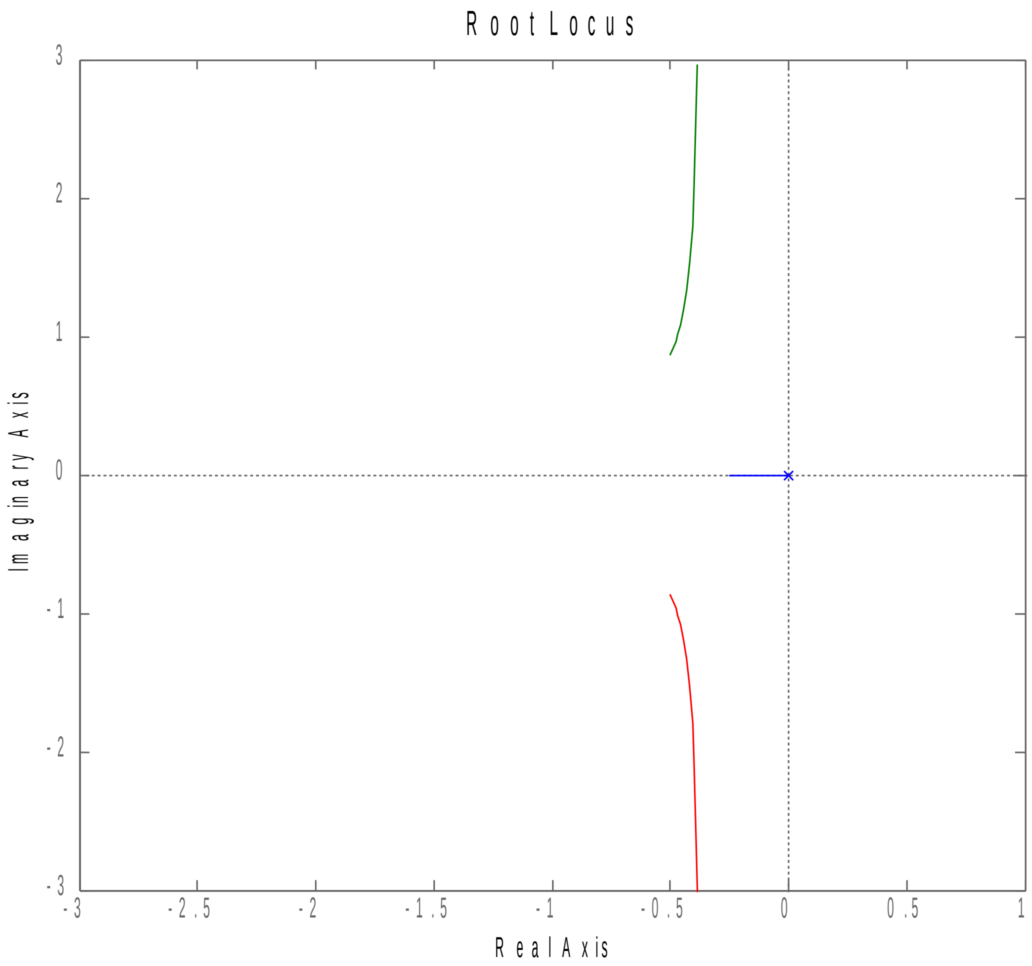
L'introduzione di uno zero sposta il luogo delle radici verso il semipiano negativo

$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{1}{s^2 + s + 1}$$

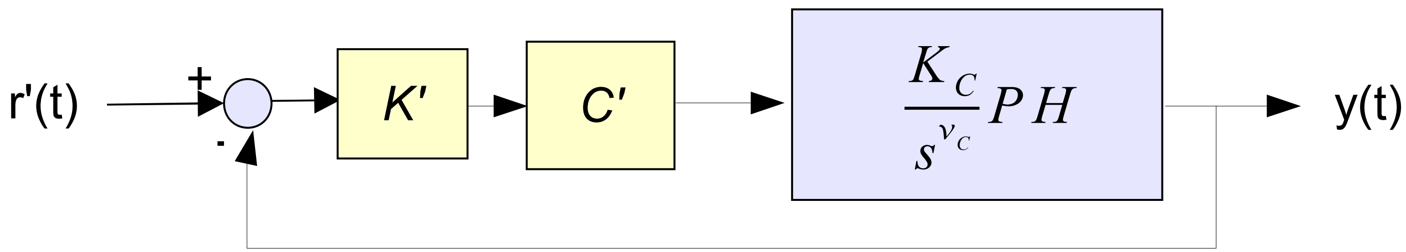
$$C(s) = K' \frac{4s+1}{s}$$

K' *non limitato*

Lo spostamento è tanto maggiore quanto lo zero è vicino all'origine



Criteri generali per la sintesi



$$\frac{K_C}{s^{\nu_C}} P H$$

L'introduzione di una coppia polo - zero vicino all'origine tende a modificare la taratura del luogo in modo da avere un guadagno critico inferiore

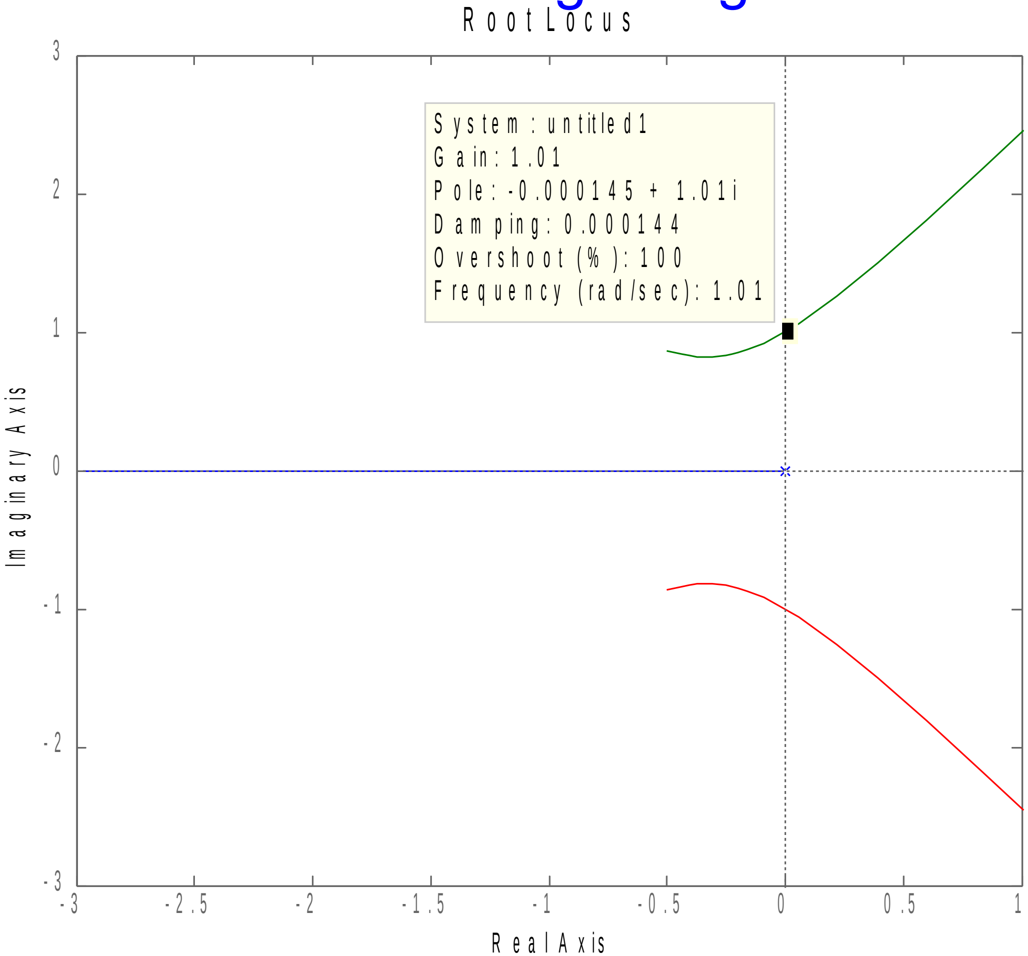
$$H = 1$$

$$K_C = 1$$

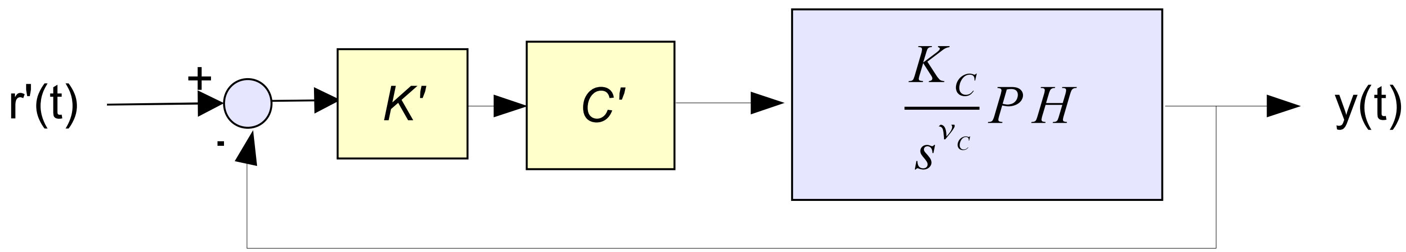
$$\nu_C = 1$$

$$P(s) = \frac{1}{s^2 + s + 1}$$

$$K' \leq 1$$



Criteri generali per la sintesi

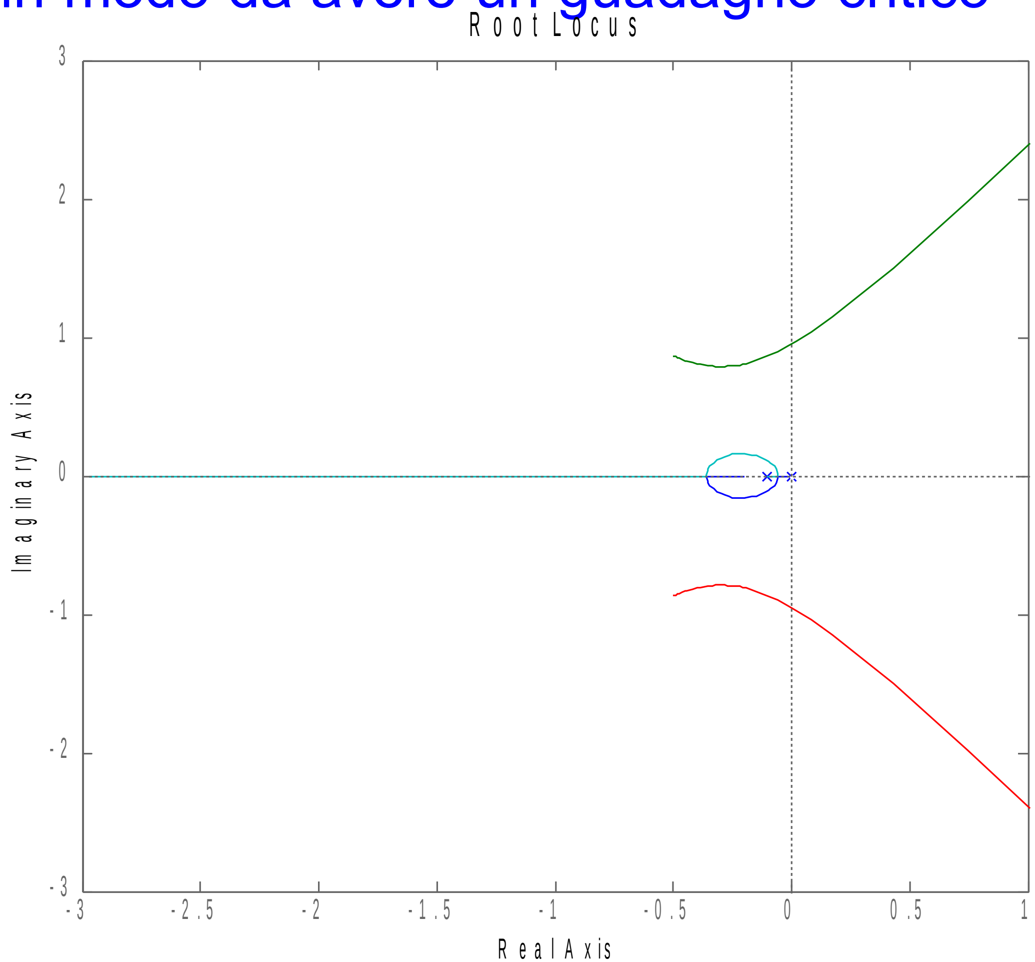


L'introduzione di una coppia polo - zero vicino all'origine tende a modificare la taratura del luogo in modo da avere un guadagno critico inferiore

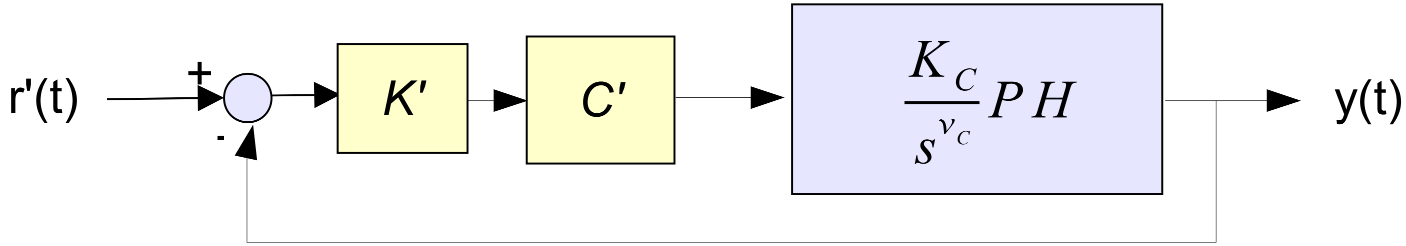
$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{1}{s^2 + s + 1}$$

$$K' \leq 1{,}79$$

$$C(s) = K' \frac{5s + 1}{s(10s + 1)}$$



Criteri generali per la sintesi



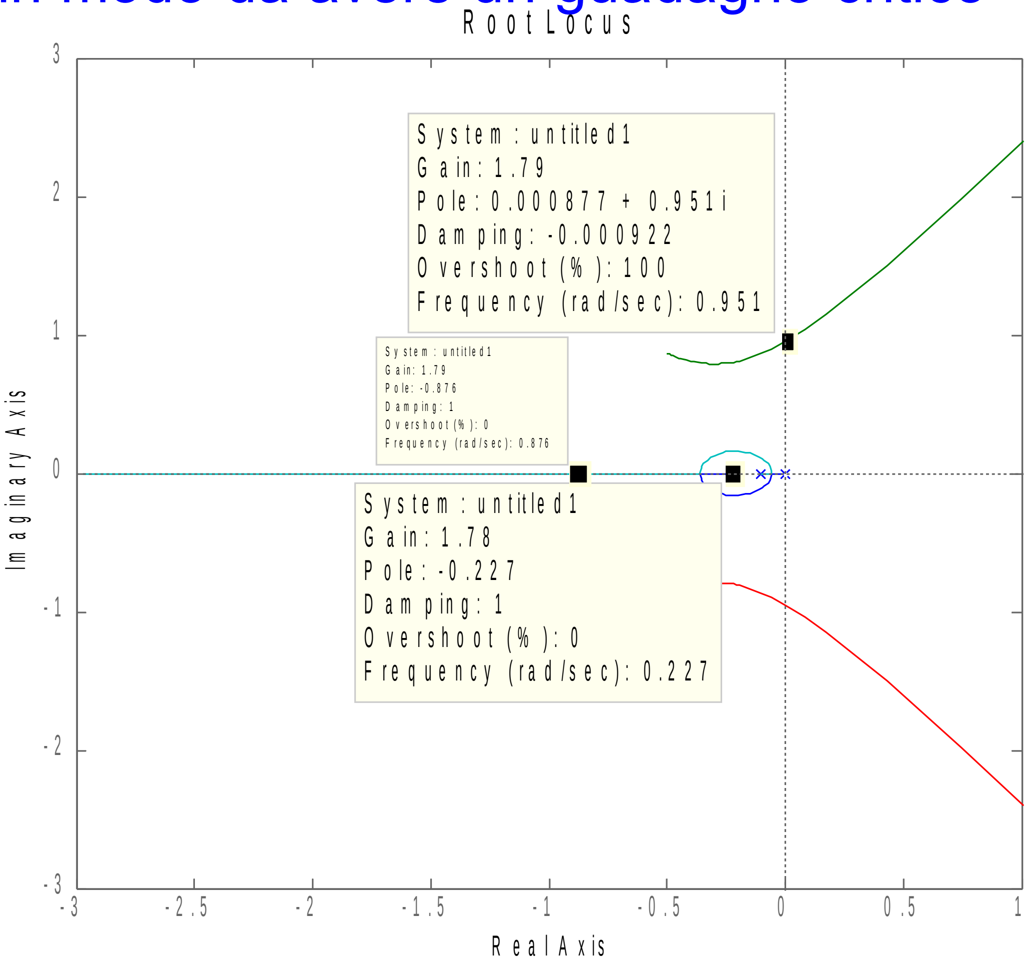
$$\frac{K_C}{s^{\nu_C}} P H$$

L'introduzione di una coppia polo - zero vicino all'origine tende a modificare la taratura del luogo in modo da avere un guadagno critico inferiore

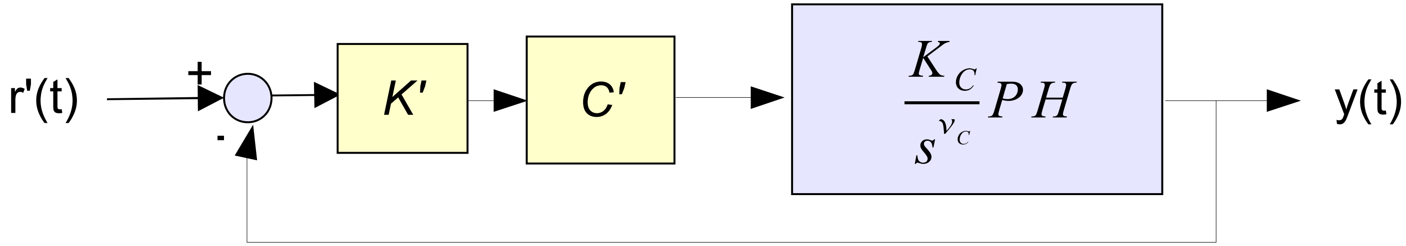
$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{1}{s^2 + s + 1}$$

$$K' \leq 1{,}79$$

$$C(s) = K' \frac{5s+1}{s(10s+1)}$$



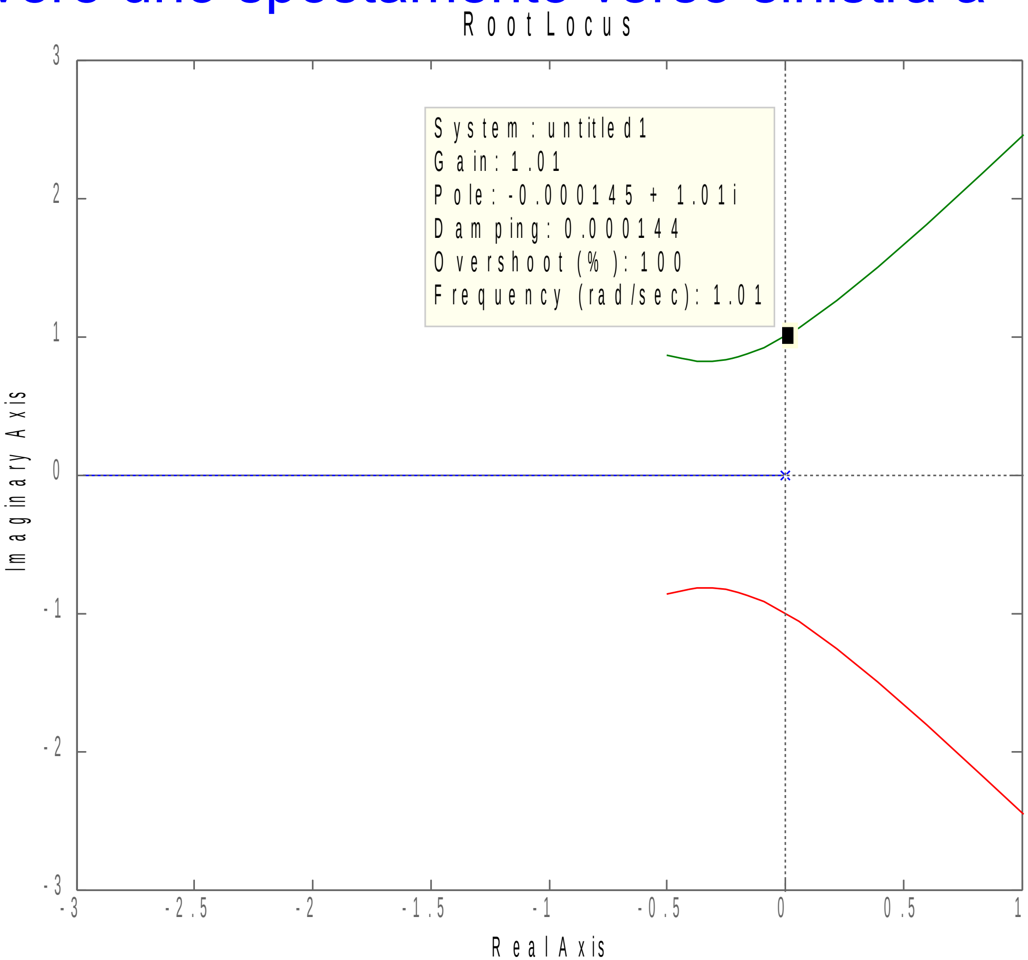
Criteri generali per la sintesi



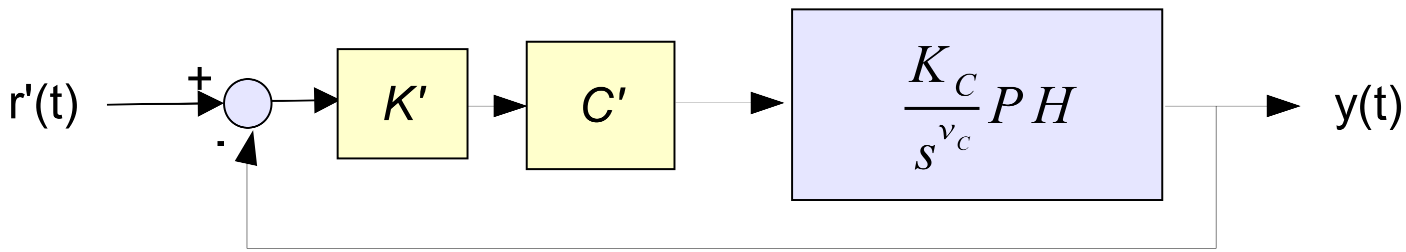
L'introduzione di una coppia zero - polo vicino all'origine modifica la taratura del luogo in modo da avere uno spostamento verso sinistra a parità di guadagno critico

$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{1}{s^2 + s + 1}$$

$$K' \leq 1$$



Criteri generali per la sintesi

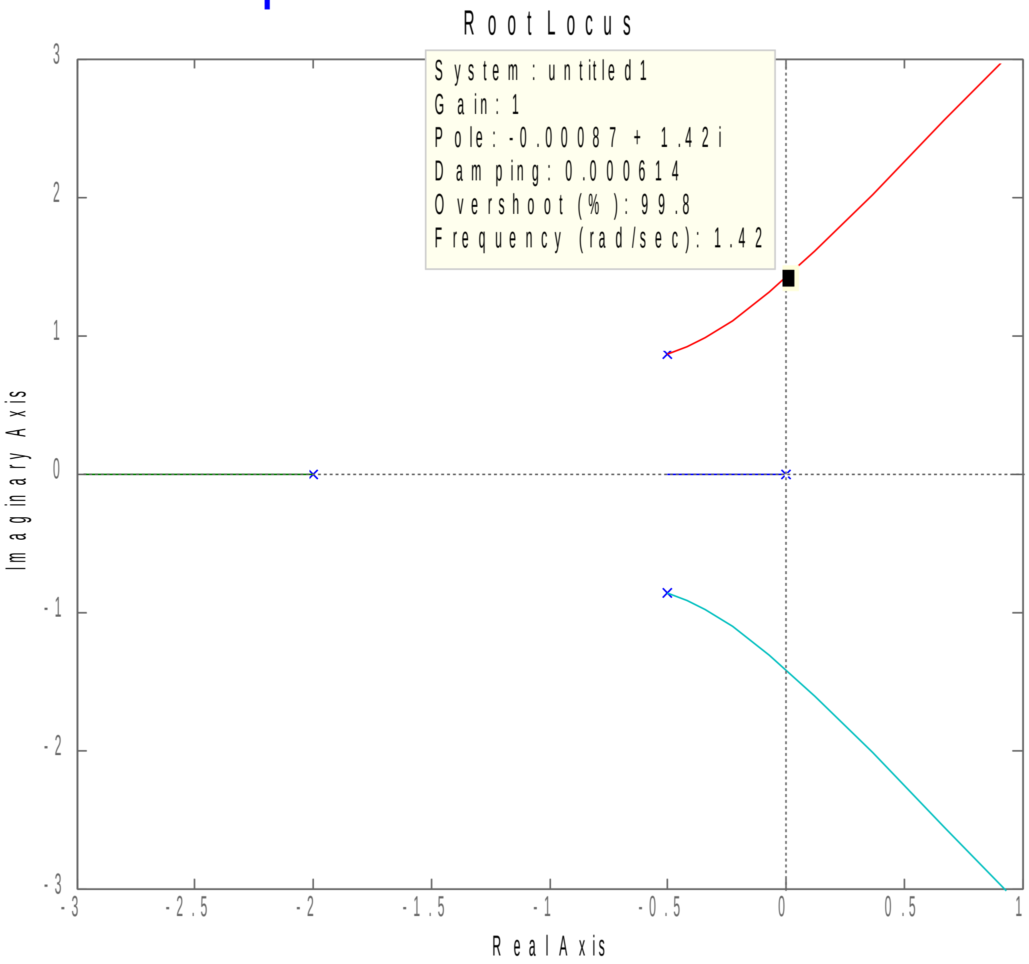


L'introduzione di una coppia zero - polo vicino all'origine modifica la taratura del luogo in modo da avere uno spostamento verso sinistra a parità di guadagno critico

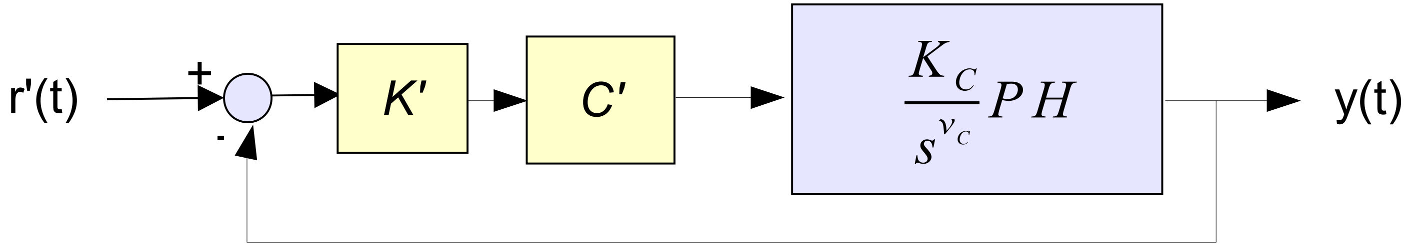
$$H=1$$
$$K_C=1$$
$$\nu_C=1$$
$$P(s)=\frac{1}{s^2+s+1}$$

$$K'\leq 1$$

$$C(s)=K'\frac{5s+1}{s(0,5s+1)}$$



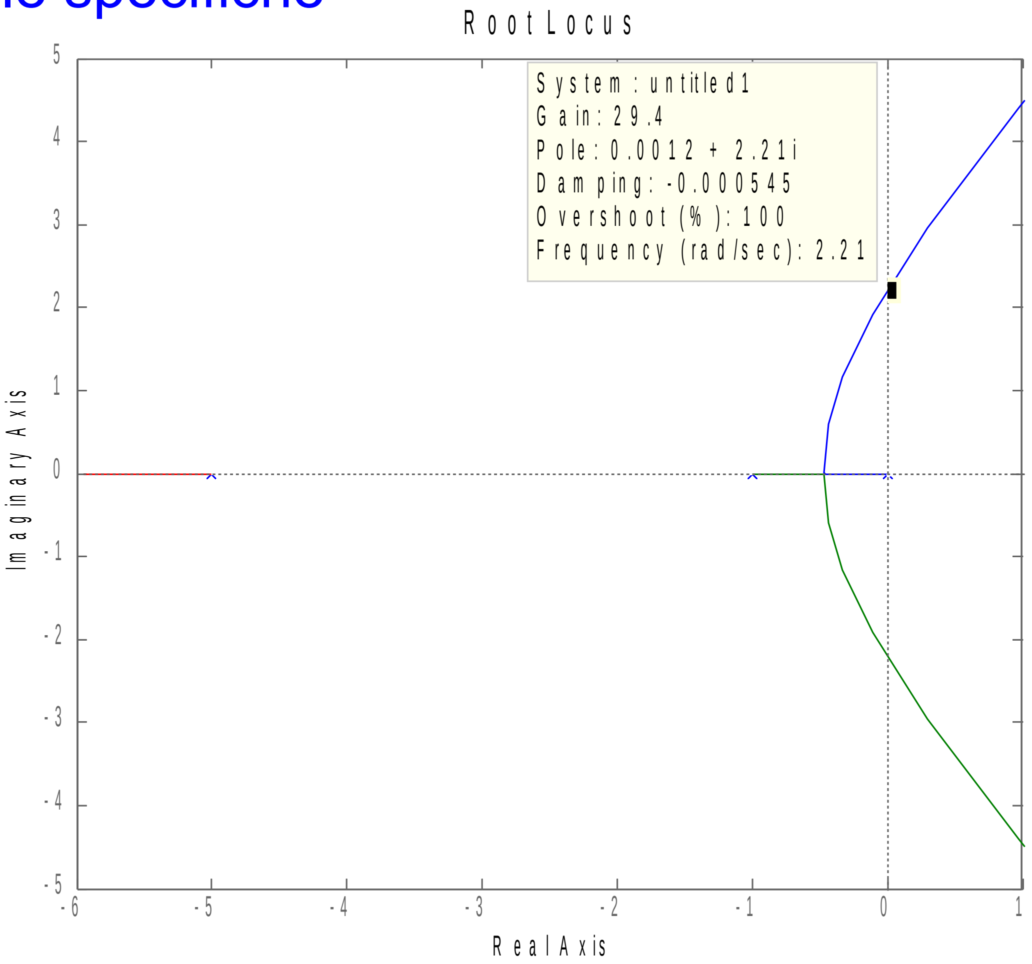
Criteri generali per la sintesi



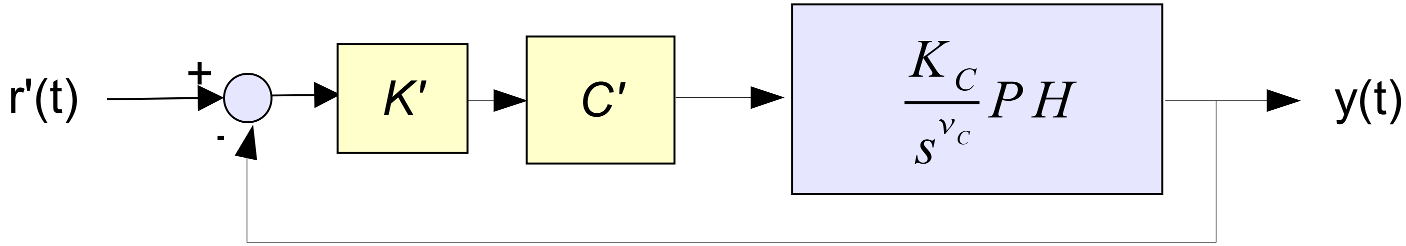
La cancellazione di parte del processo può consentire la modifica del luogo delle radici per rispettare le specifiche

$$H = 1$$
$$K_C = 1$$
$$\nu_C = 1$$
$$P(s) = \frac{1}{(s+1)(s+5)}$$

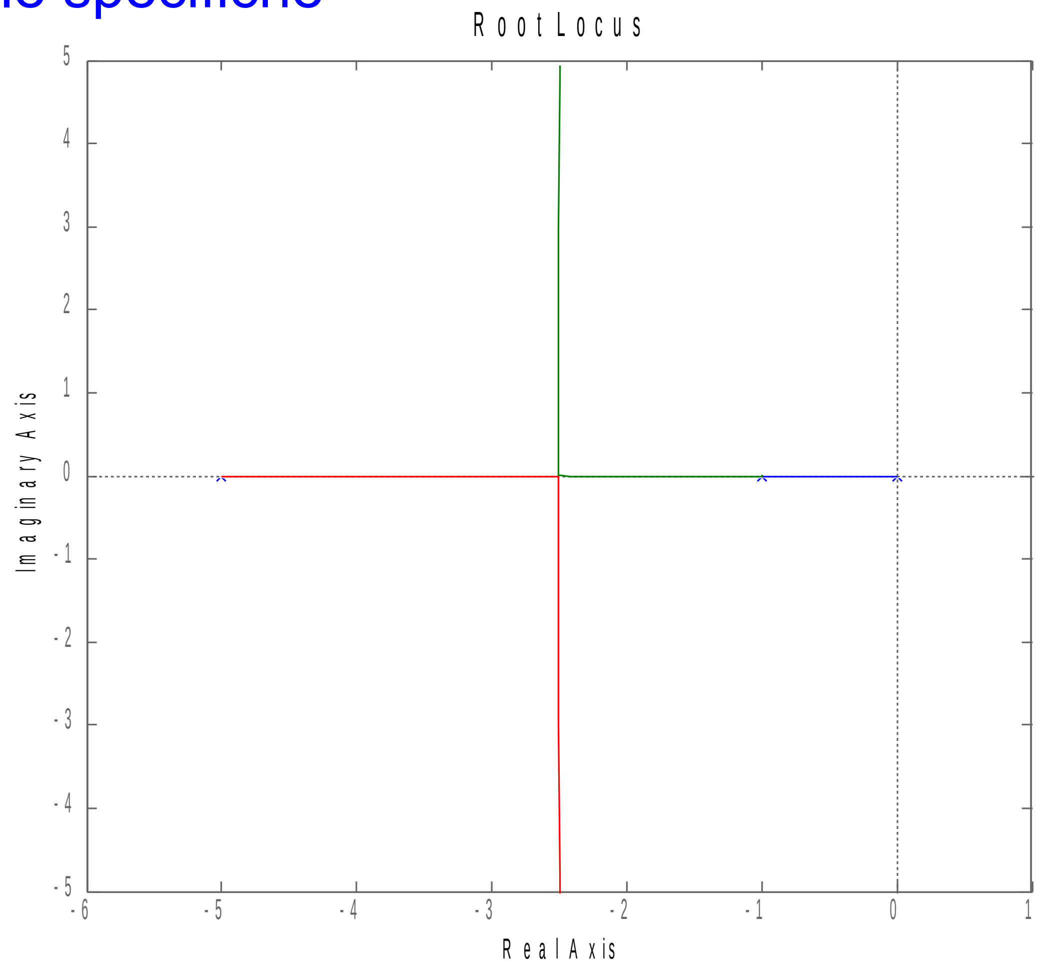
$$K' \leq 29{,}4$$



Criteri generali per la sintesi



La cancellazione di parte del processo può consentire la modifica del luogo delle radici per rispettare le specifiche



$$H = 1$$

$$K_C = 1$$

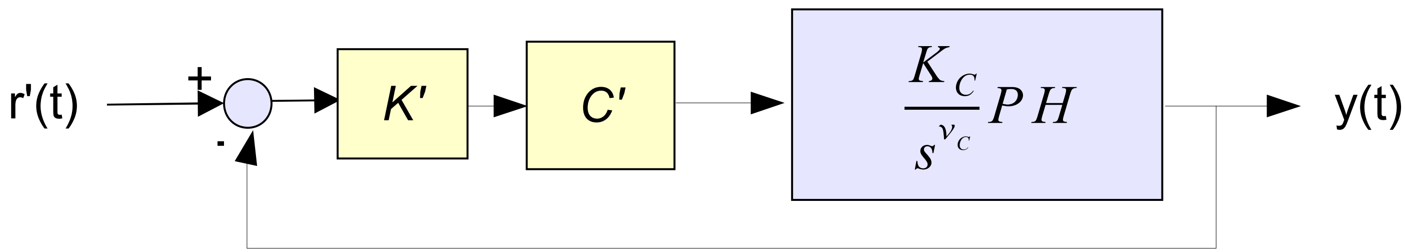
$$\nu_C = 1$$

$$P(s) = \frac{1}{(s+1)(s+5)}$$

$$K' \, non \; limitato$$

$$C(s) = K' \frac{s+1}{s}$$

Esempio



$$\frac{K_C}{s^{\nu_C}} P H$$

$$P(s)=\frac{25}{(s+1)(s+5)(s+10)}$$

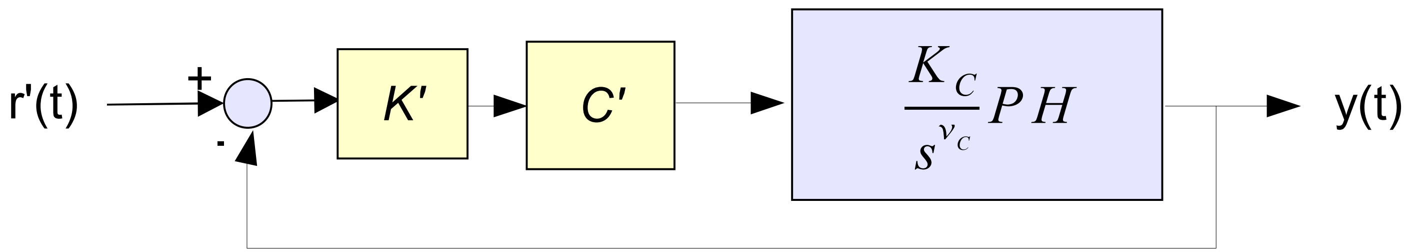
$$K_{W_r}=0,5$$

$$|e_\infty(t)|=|e_{r_\infty}(t)|_{\delta_{-2}}+|e_{d_\infty}(t)|_{\delta_{-1}}\le 0,1$$

$$t_{a_{1\%}}\le 2$$

$$s_{\%}\le 5\%$$

Esempio



$$P(s) = \frac{25}{(s+1)(s+5)(s+10)}$$

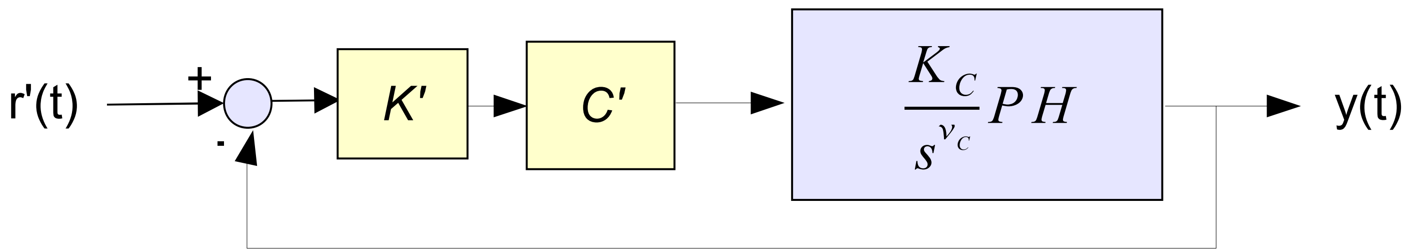
$$K_{W_r} = 0{,}5 \Rightarrow K_d = 0{,}5 \Rightarrow H = 2$$

$$|e_\infty(t)| = |e_{r_\infty}(t)|_{\delta_{-2}} + |e_{d_\infty}(t)|_{\delta_{-1}} \le 0{,}1 \Rightarrow \begin{matrix} \nu_C = 1 \\ K_C \ge 5 \end{matrix}$$

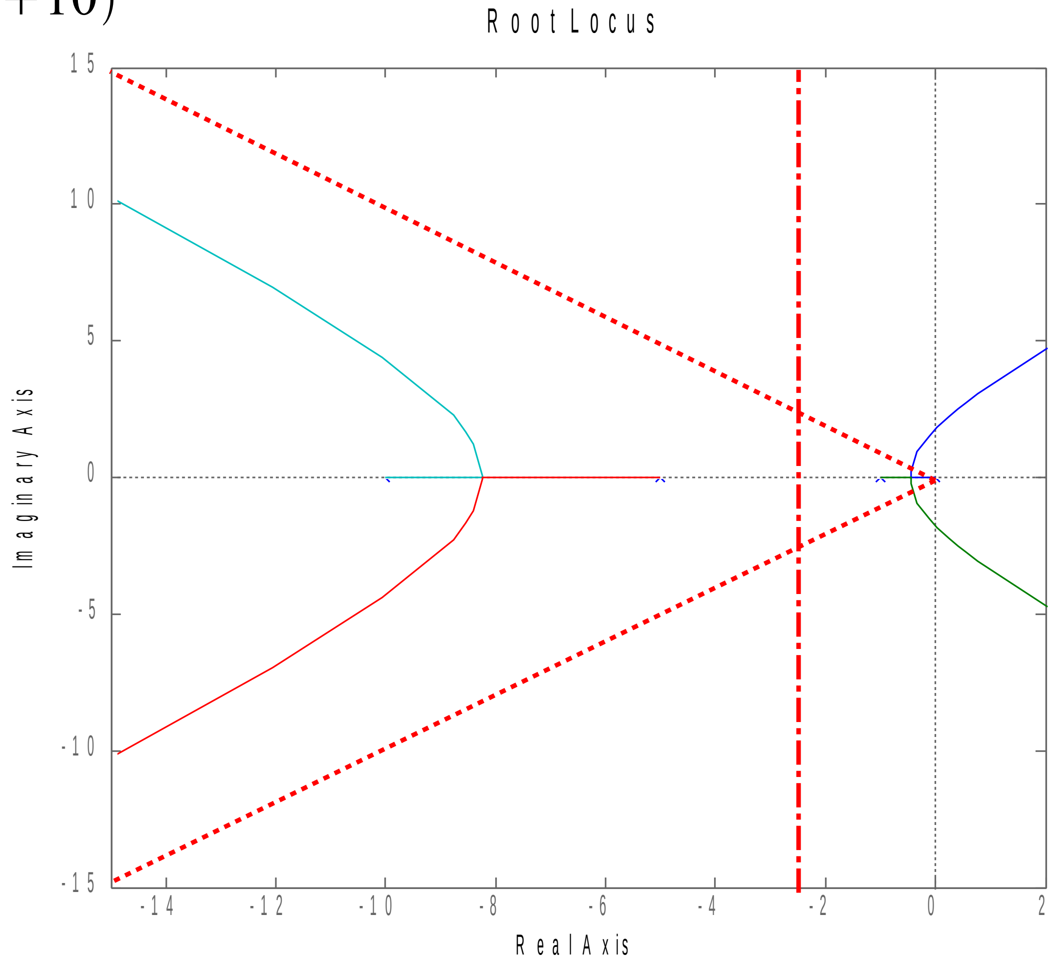
$$t_{a_{1\%}} \le 2 \Rightarrow \tau_{eq} \le \frac{2}{5} \Rightarrow \bar{\alpha} = 2{,}5$$

$$s_{\%} \le 5\% \Rightarrow \xi \ge 0{,}7 \Rightarrow \bar{\psi} = \frac{\pi}{4}$$

Esempio



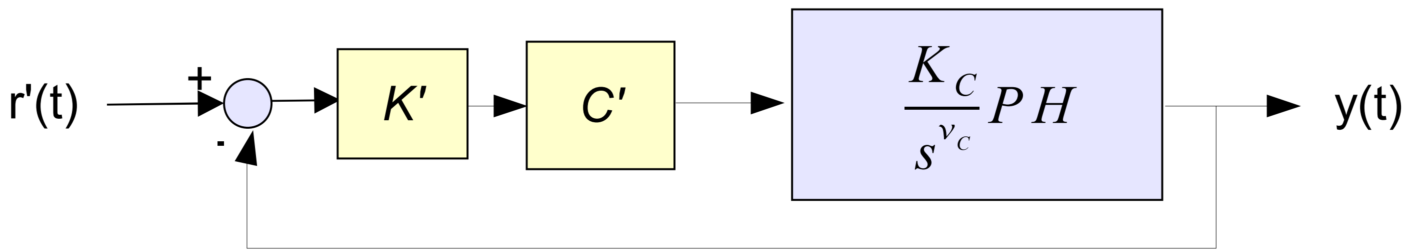
$$P(s)=\frac{25}{(s+1)(s+5)(s+10)};\quad H=2;\quad K_C=5;\quad \nu_C=1$$



$$t_{a_{1\%}}\leq 2\Rightarrow \tau_{eq}\leq\frac{2}{5}\Rightarrow\bar{\alpha}=2,5$$

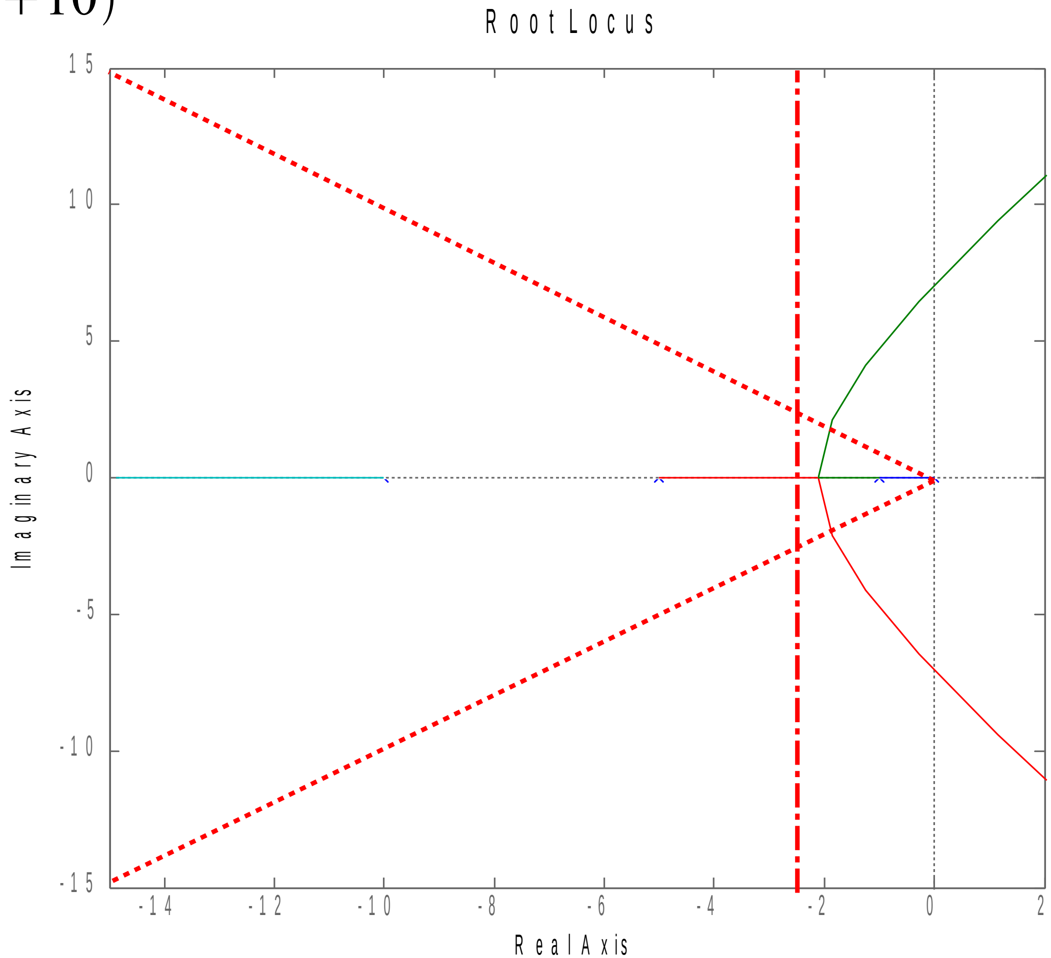
$$s_{\%}\leq 5\%\Rightarrow\xi\geq 0,7\Rightarrow\bar{\psi}=\frac{\pi}{4}$$

Esempio

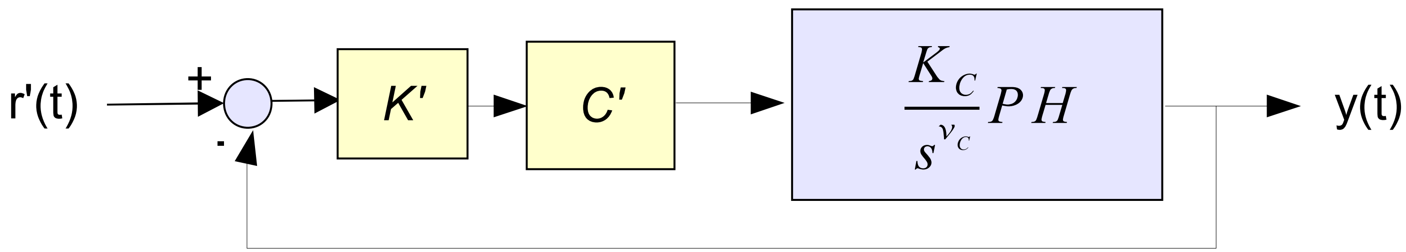


$$P(s) = \frac{25}{(s+1)(s+5)(s+10)}; \quad H = 2; \quad K_C = 5; \quad \nu_C = 1$$

$$C(s) = 5\,\frac{s+1}{s}$$

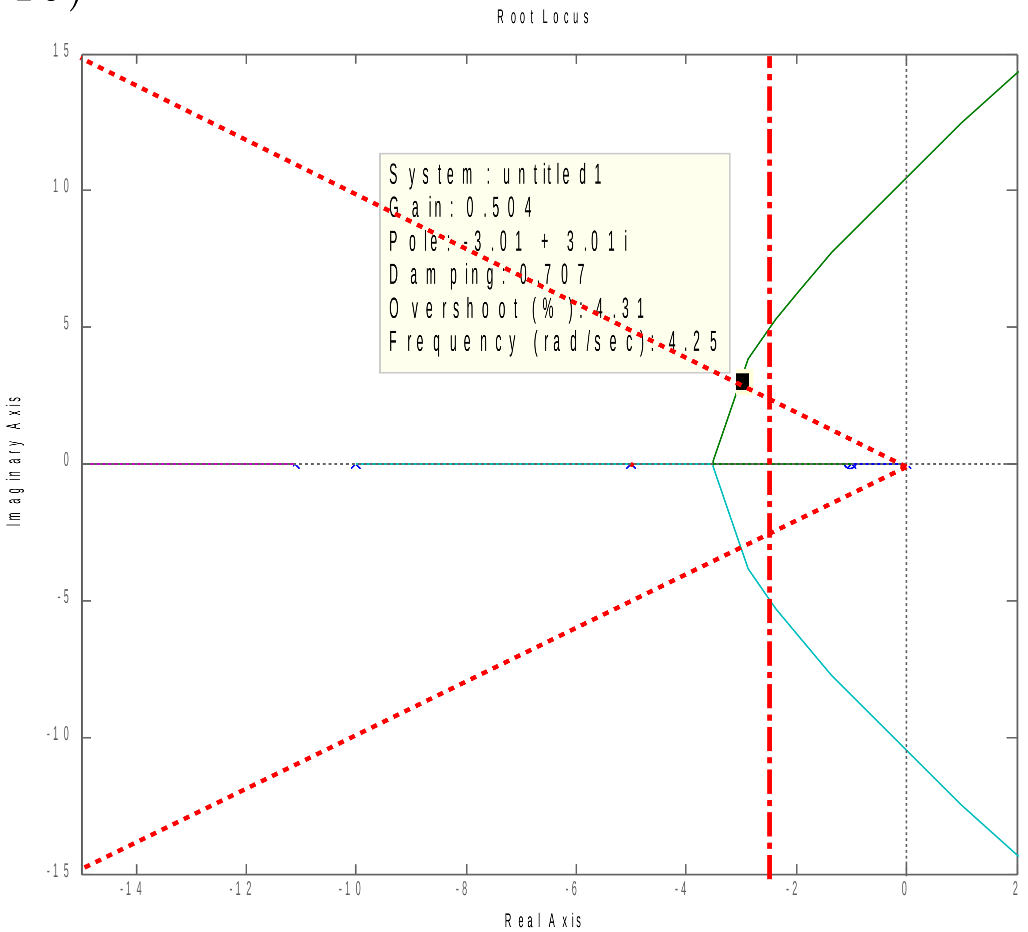


Esempio

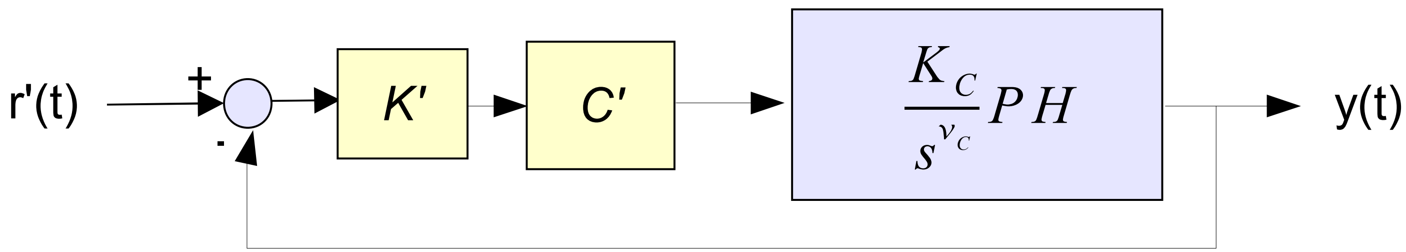


$$P(s) = \frac{25}{(s+1)(s+5)(s+10)}; \quad H = 2; \quad K_C = 5; \quad \nu_C = 1$$

$$C(s) = 5\,\frac{(s+1)(0.2s+1)}{s(0.09s+1)}$$

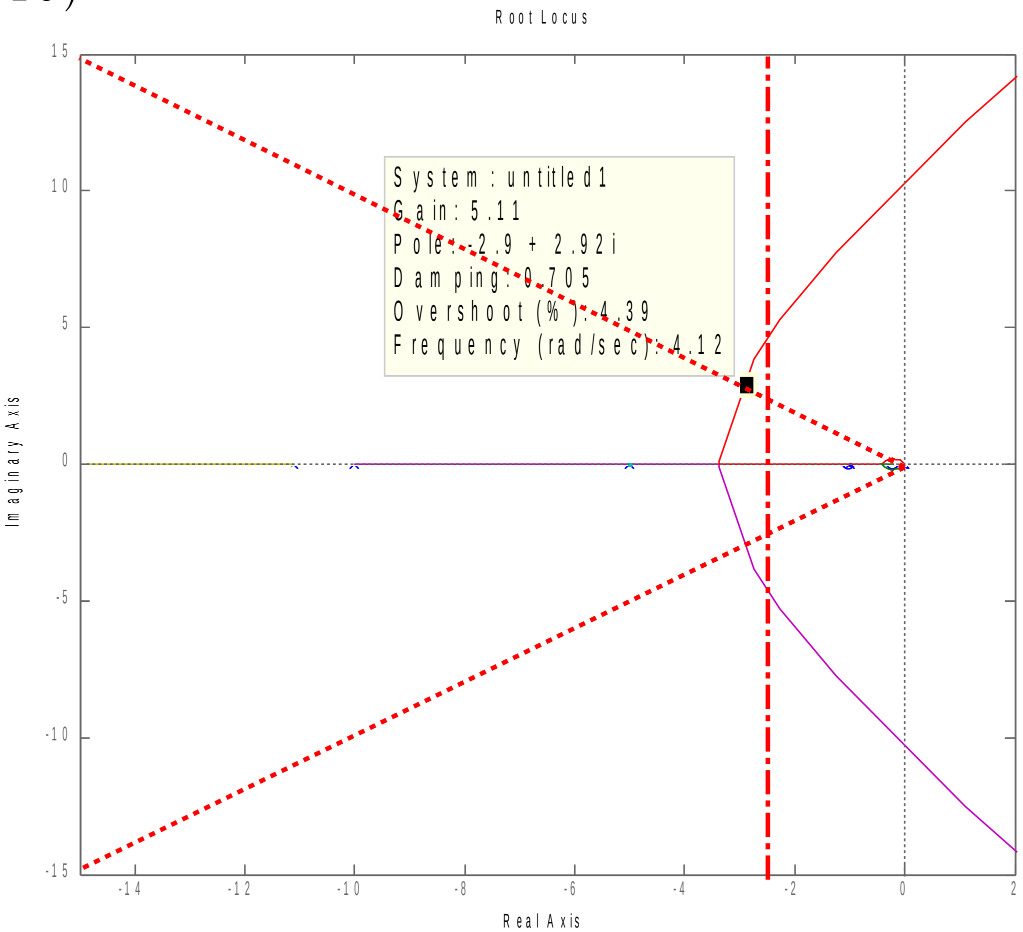


Esempio

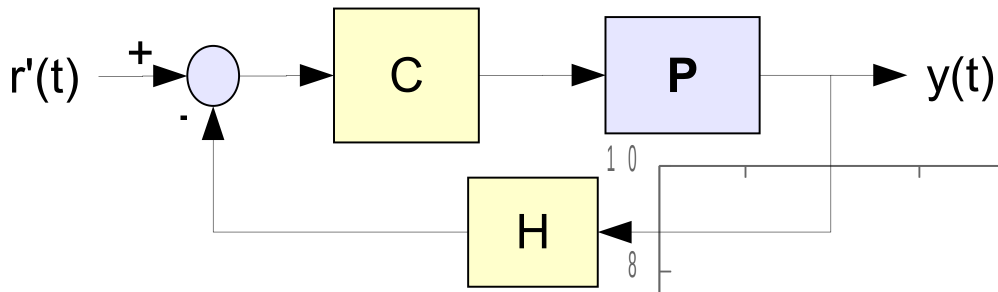


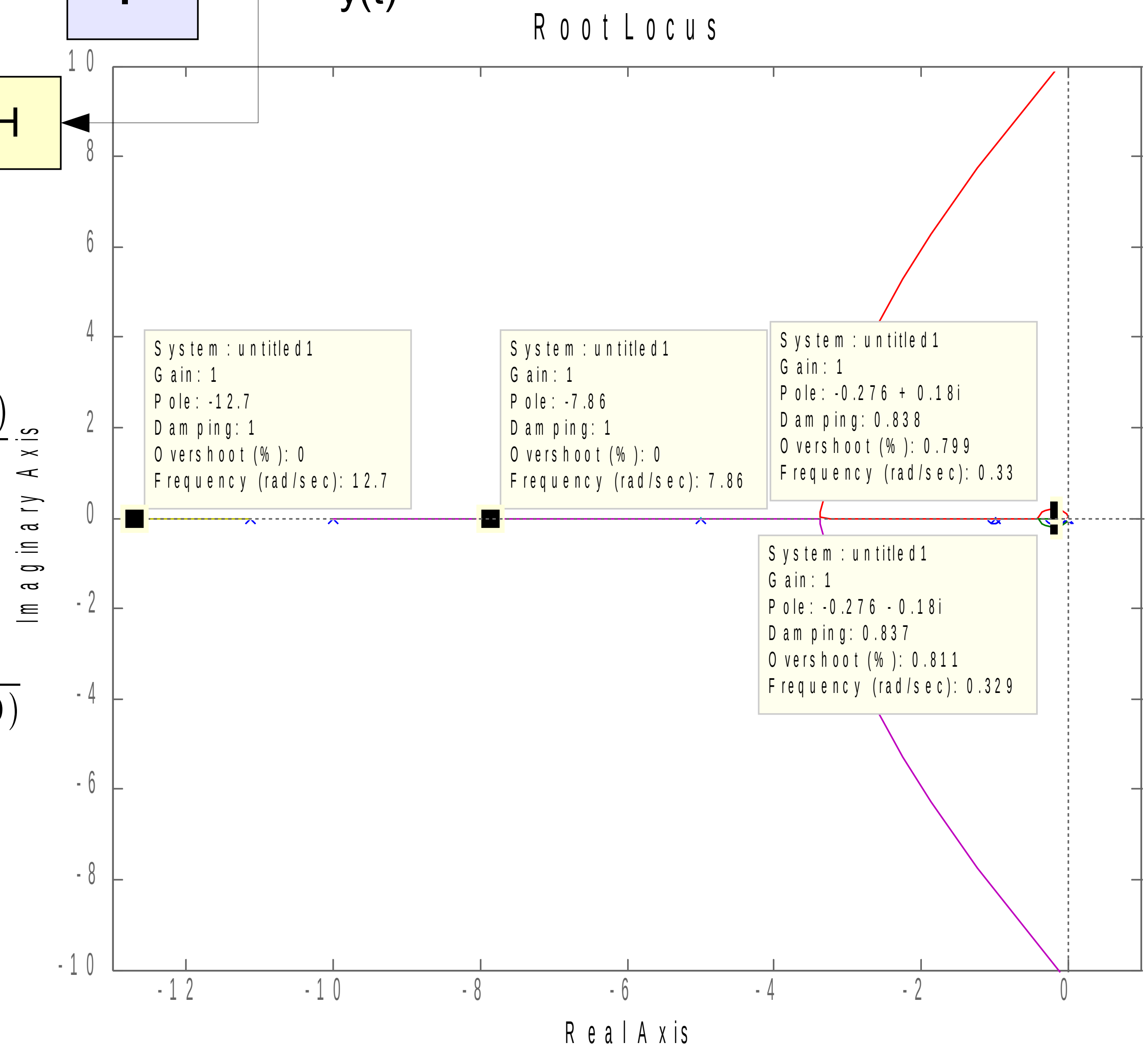
$$P(s) = \frac{25}{(s+1)(s+5)(s+10)}; \quad H = 2; \quad K_C = 5; \quad \nu_C = 1$$

$$C(s) = 5\frac{(5s+1)(s+1)(0.2s+1)}{s(50s+1)(0.09s+1)}$$



Esempio





$$P(s)=\frac{25}{(s+1)(s+5)(s+10)}$$

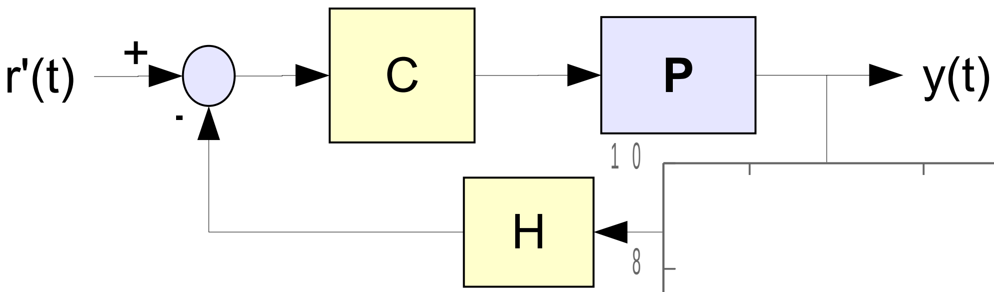
$$H=2$$

$$C(s)=5\frac{(5s+1)(s+1)(0.2s+1)}{s(50s+1)(0.09s+1)}$$

$$F(s)=\frac{50(5s+1)}{s(50s+1)(0.09s+1)(s+10)}$$

Il guadagno non è sufficiente ed il modo dominante è troppo in bassa frequenza

Esempio

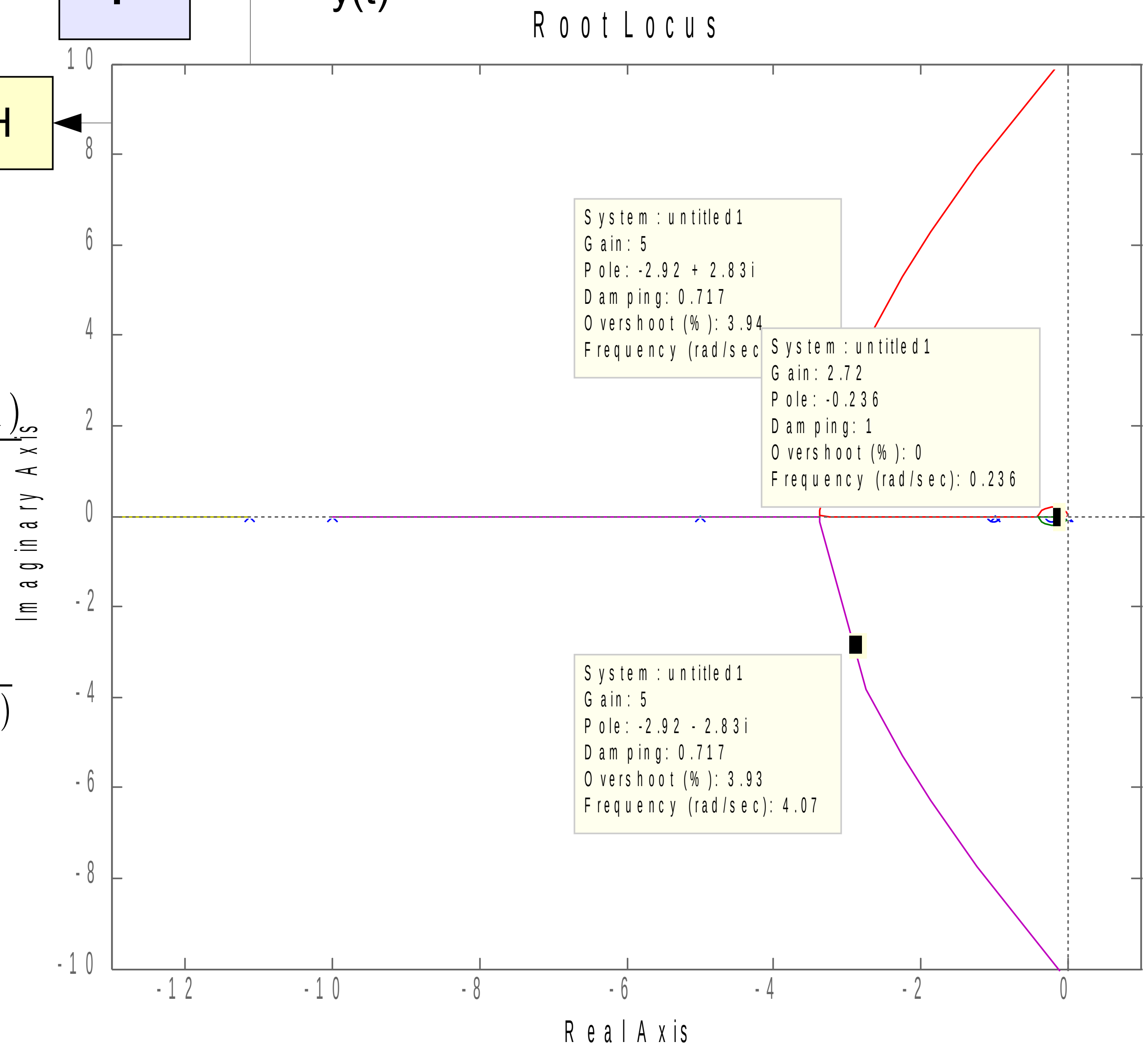


$$P(s)=\frac{25}{(s+1)(s+5)(s+10)}$$

$$H=2$$

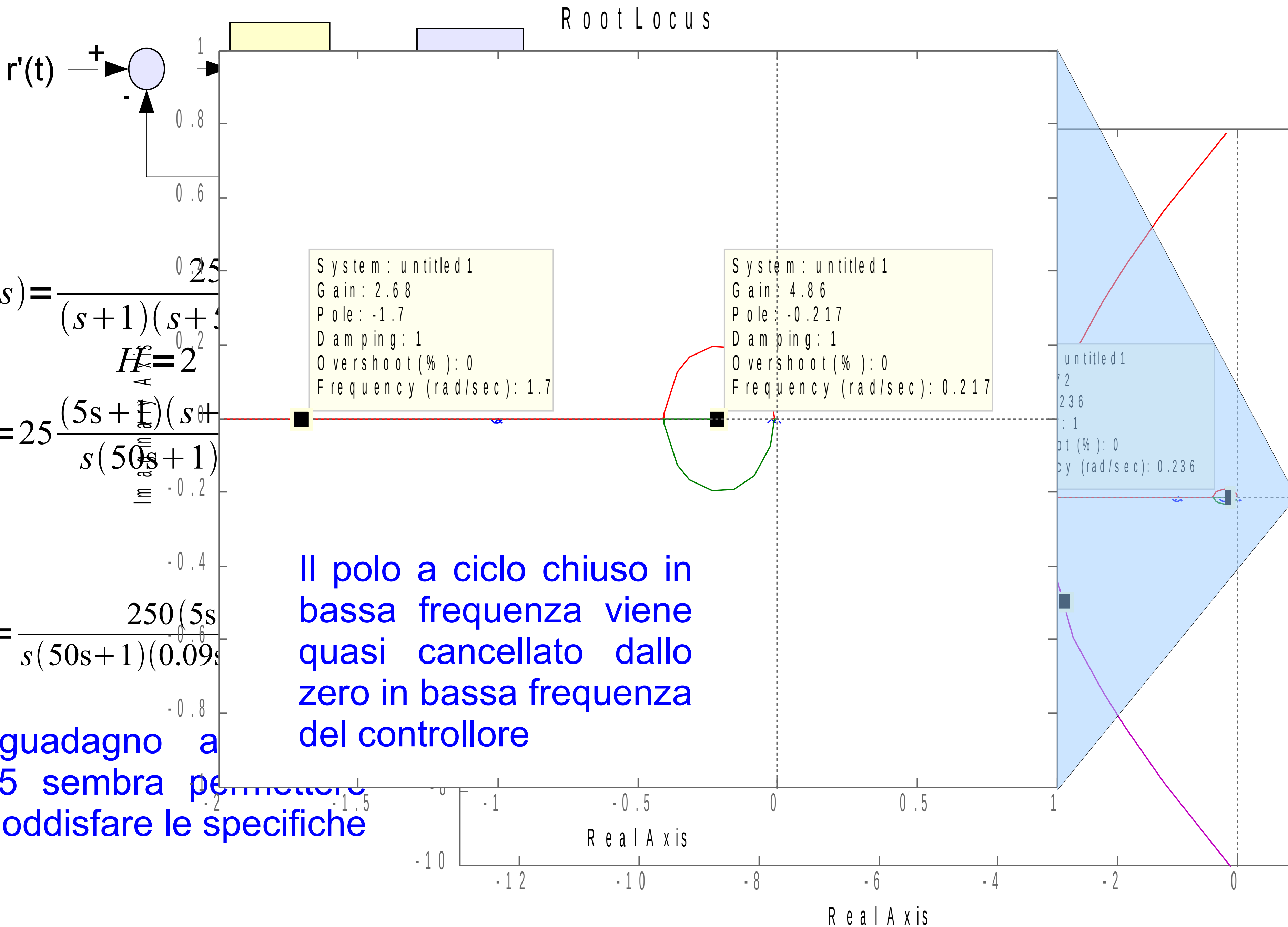
$$C(s)=25\frac{(5s+1)(s+1)(0.2s+1)}{s(50s+1)(0.09s+1)}$$

$$F(s)=\frac{250(5s+1)}{s(50s+1)(0.09s+1)(s+10)}$$



Il guadagno aggiuntivo K'=5 sembra permettere di soddisfare le specifiche

Esempio



$$P(s)=\frac{25}{(s+1)(s+\ldots}$$

$$H=2$$

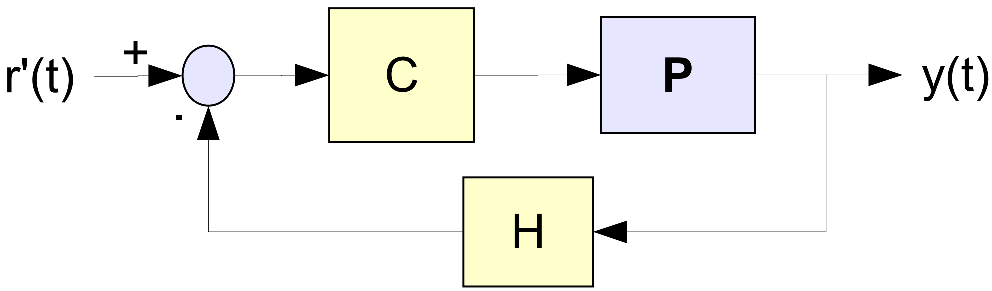
$$C(s)=25\frac{(5s+1)(s+\ldots}{s(50s+1)\ldots}$$

$$F(s)=\frac{250(5s\ldots}{s(50s+1)(0.09s\ldots}$$

Il polo a ciclo chiuso in bassa frequenza viene quasi cancellato dallo zero in bassa frequenza del controllore

Il guadagno a K'=5 sembra permettere di soddisfare le specifiche

Esempio



$$P(s)=\frac{25}{(s+1)(s+5)(s+10)}$$

$$H=2$$

$$C(s)=25\frac{(5s+1)(s+1)(0.2s+1)}{s(50s+1)(0.09s+1)}$$

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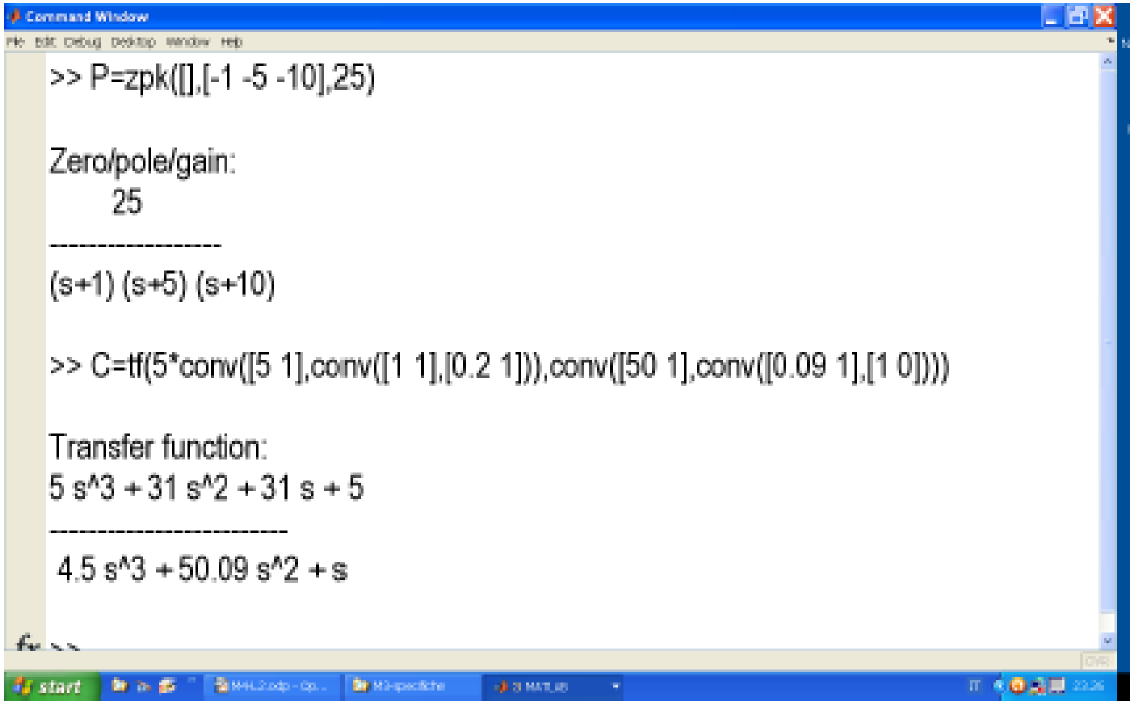
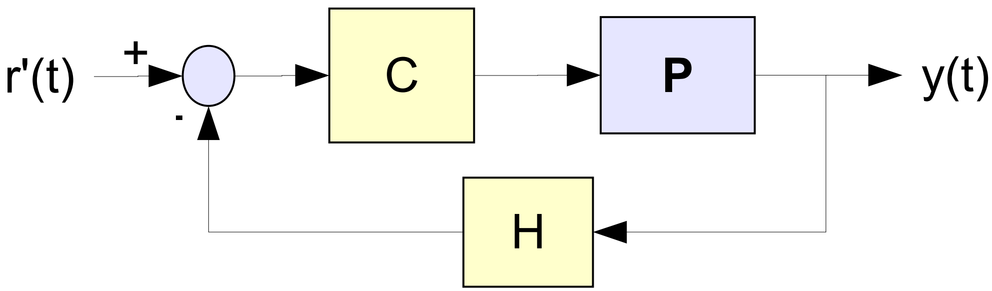


Esempio



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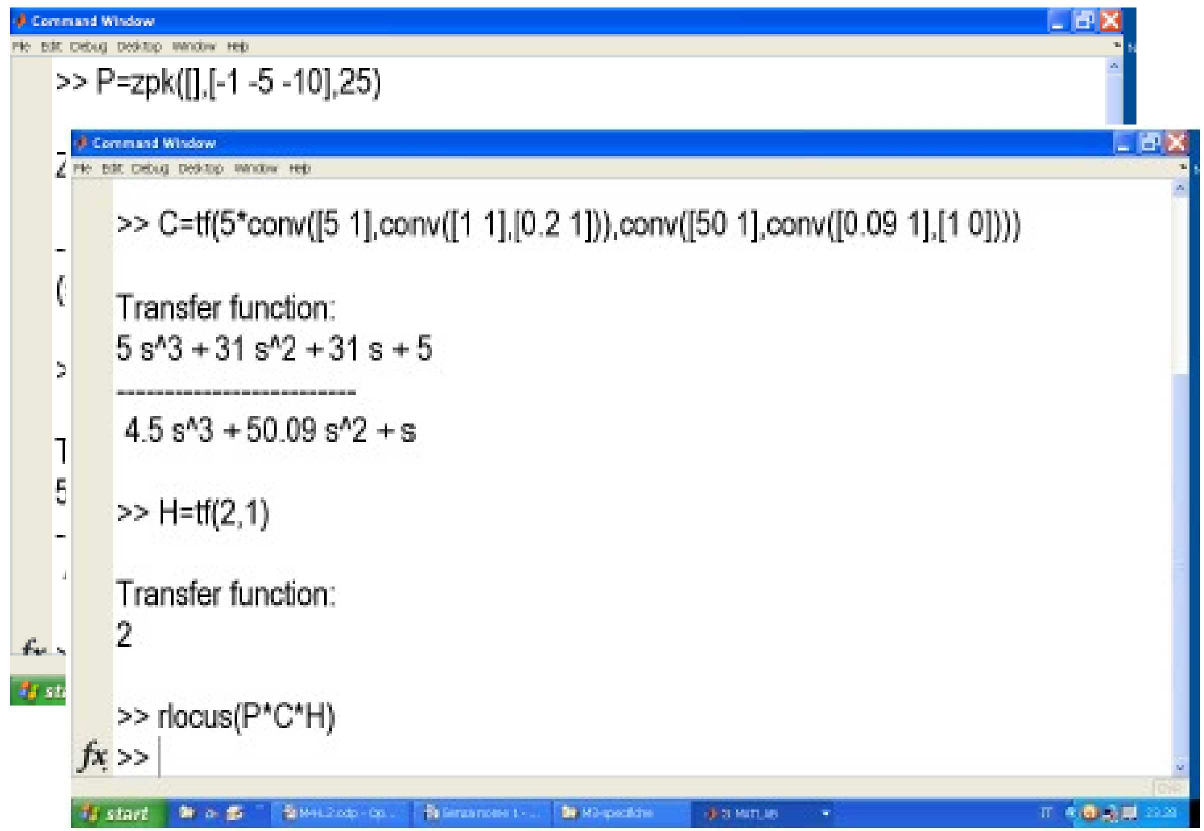
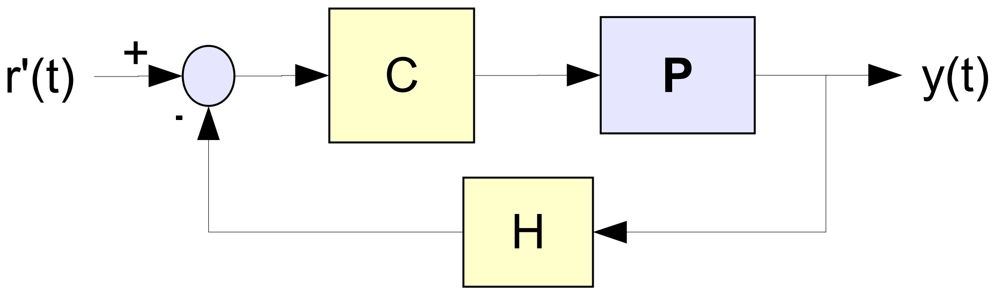


Esempio



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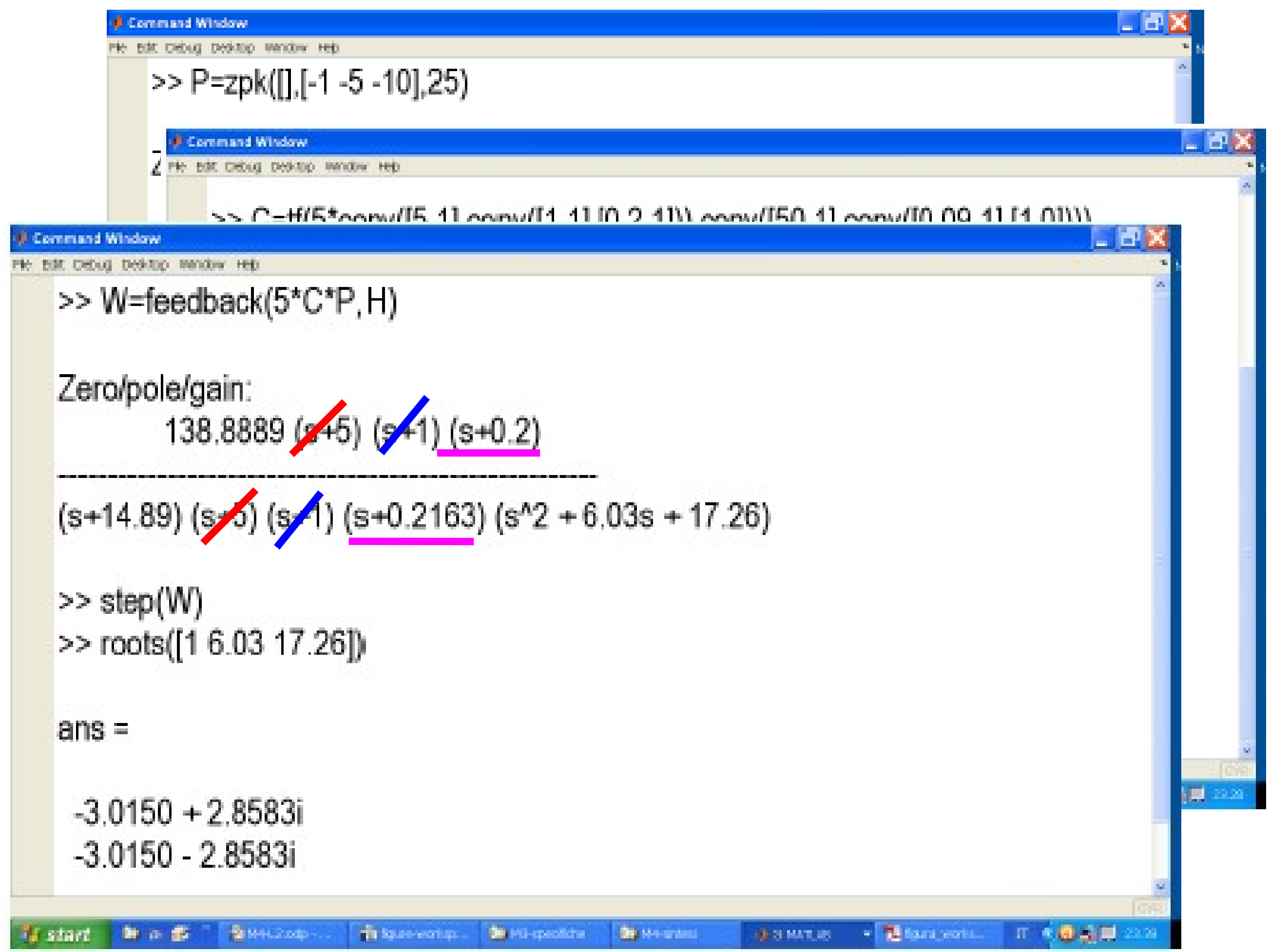
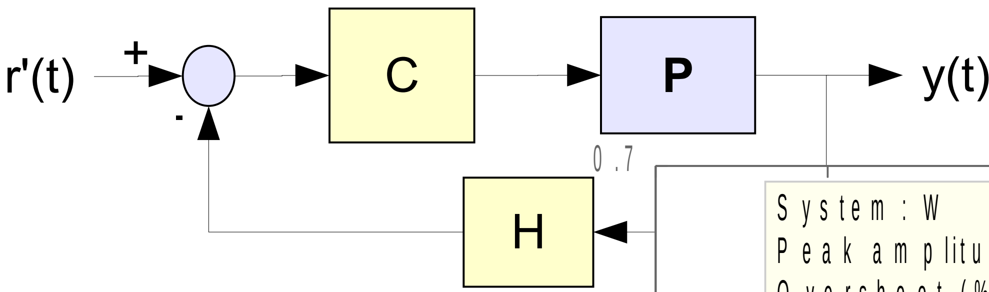
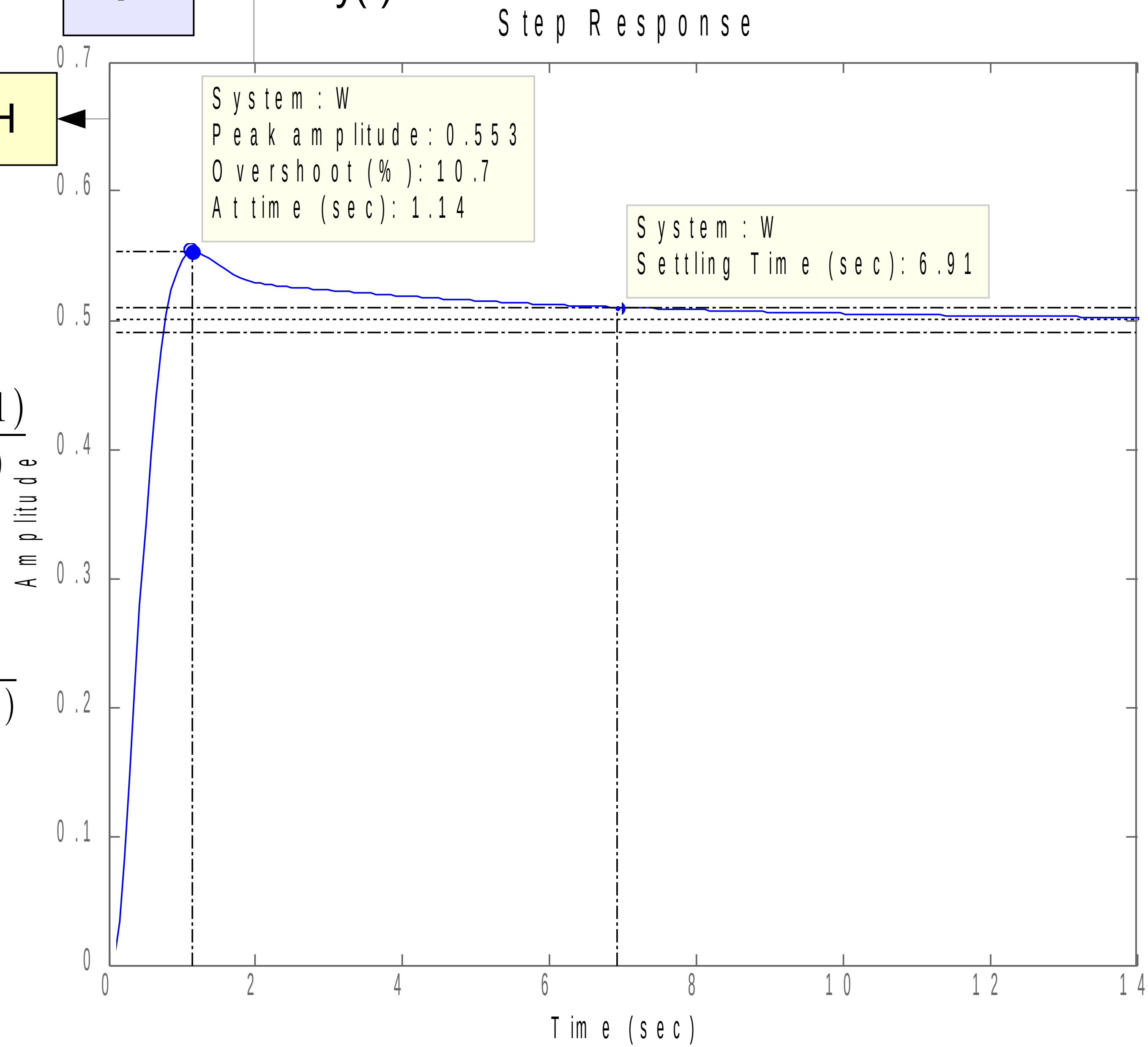


Esempio





$$P(s)=\frac{25}{(s+1)(s+5)(s+10)}$$

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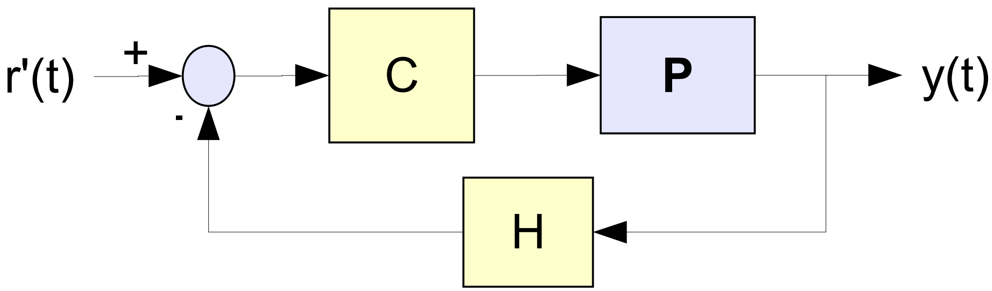
Esempio

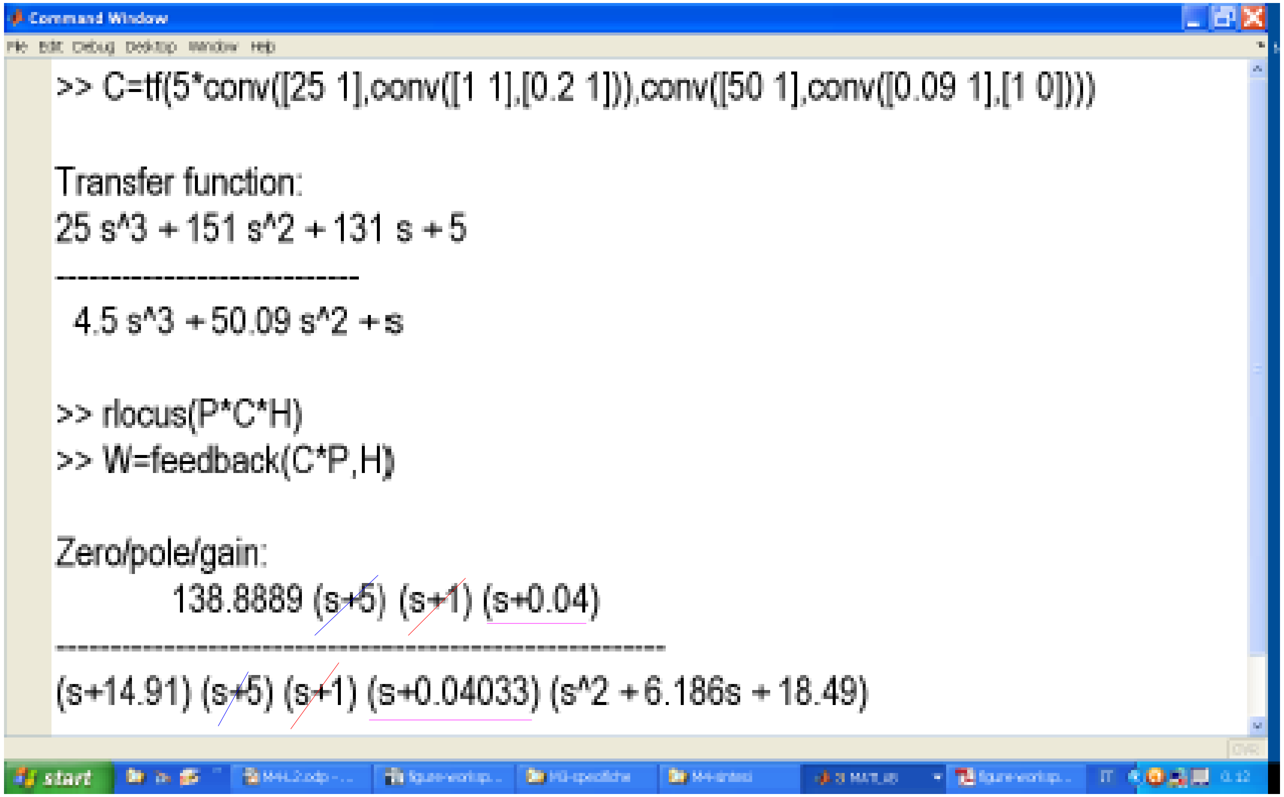


$$P(s)=\frac{25}{(s+1)(s+5)(s+10)}$$

$$H=2$$

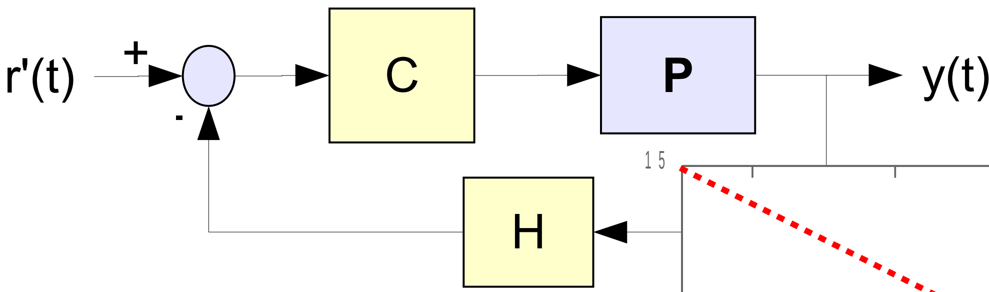
$$C(s)=5\,\frac{(25s+1)(s+1)(0.2s+1)}{s(50s+1)(0.09s+1)}$$

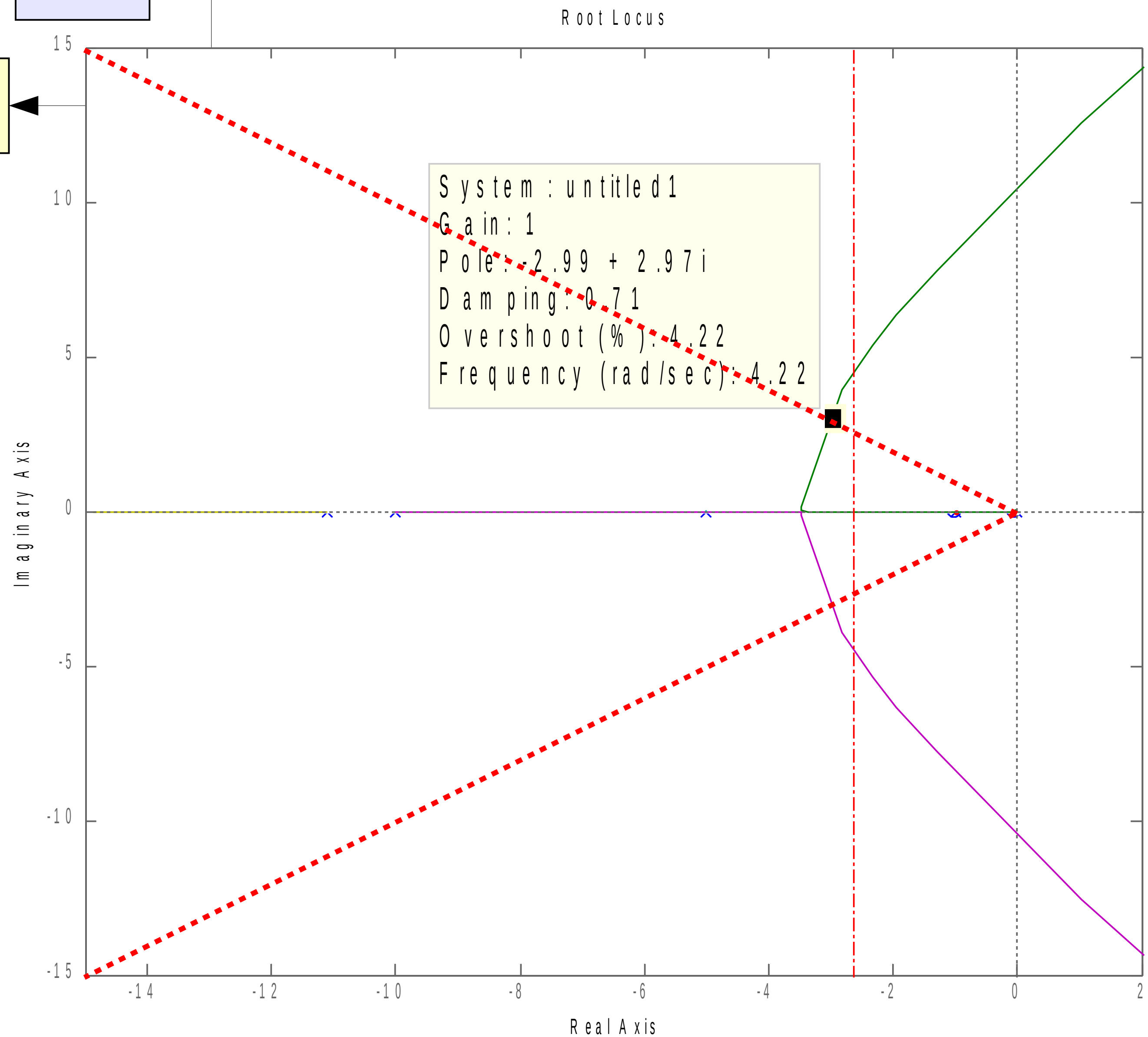
$$F(s)=\frac{50(25s+1)}{s(50s+1)(0.09s+1)(s+10)}$$



Avvicinando lo zero della coppia polo-zero all'origine e non incrementando il guadagno del controllore....

Esempio





$$P(s)=\frac{25}{(s+1)(s+5)(s+10)}$$

$$H=2$$

$$C(s)=5\frac{(25s+1)(s+1)(0.2s+1)}{s(50s+1)(0.09s+1)}$$

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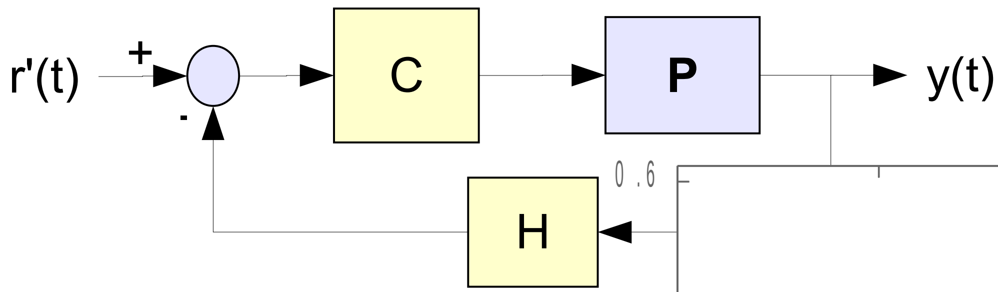
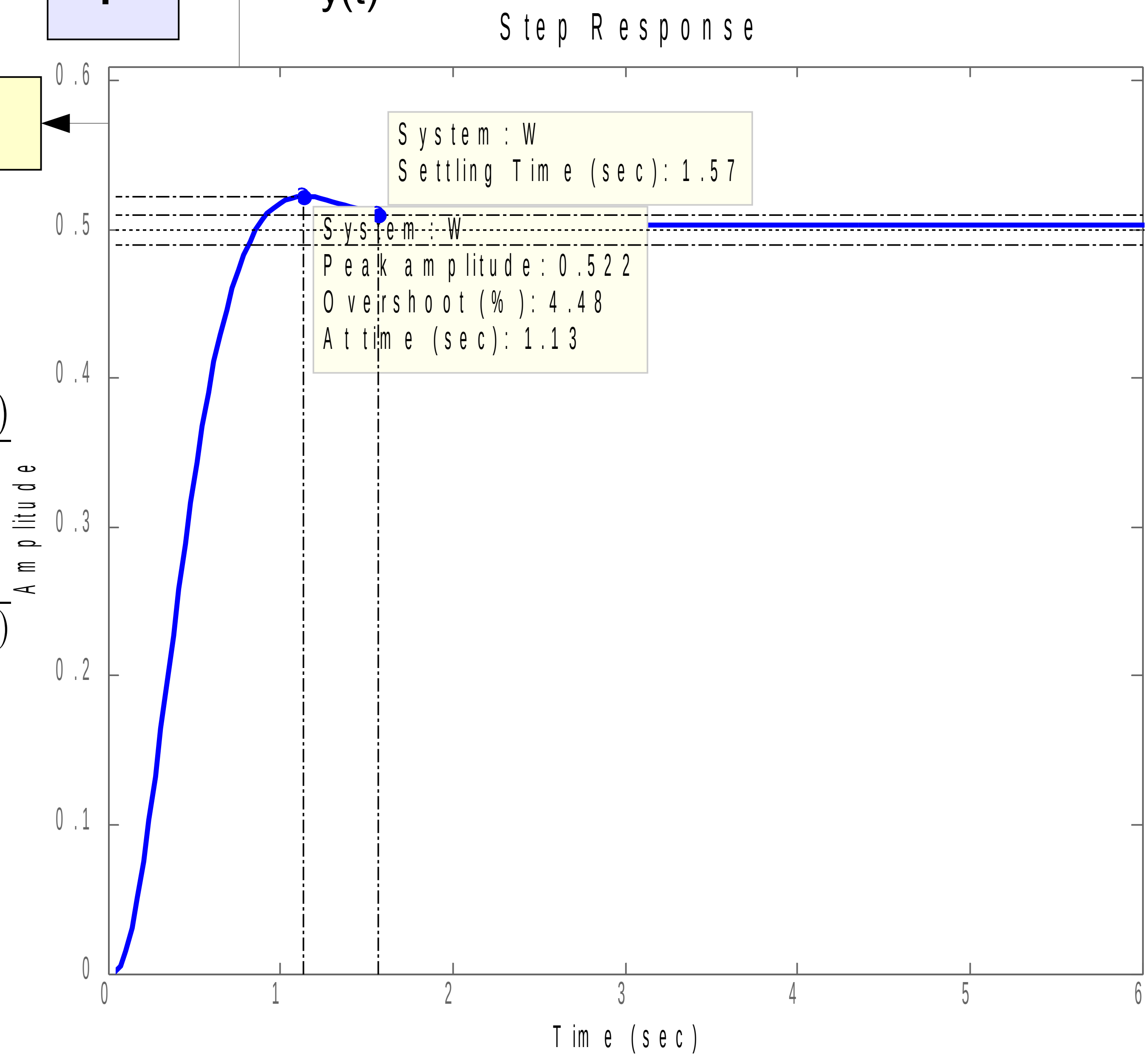
Avvicinando lo zero della coppia polo-zero all'origine e non incrementando il guadagno del controllore....

Esempio





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Avvicinando lo zero della coppia polo-zero all'origine e non incrementando il guadagno del controllore....

Riepilogo

- Le specifiche sul comportamento nel transitorio sono state riportate in forma grafica sul piano complesso (di Gauss)

- Sono state presentate i principali effetti della introduzione di poli e zeri a ciclo aperto sul luogo delle radici del polinomio caratteristico a ciclo chiuso

- E stato presentato un esempio di sintesi con l'ausilio del luogo delle radici