POTENZE A REGIME SINUSOIDALE

mercoledì 3 giugno 2020 09:00

$\dot{S}$ numero complesso: POTENZA COMPLESSA

$\dot{S} = \frac{1}{2} \bar{V} \bar{I}^*$ valori massimi

$\dot{S} = \bar{V}_{eff} \bar{I}_{eff}$ " efficaci $\begin{cases} \dot{Y} = G + jB \\ \dot{Z} = R + jX \end{cases}$

Poiché $\bar{V} = \dot{Z} \bar{I}$ o $\bar{I} = \dot{Y} \bar{V}$

$\boxed{\dot{S} = \frac{1}{2} \bar{V} \bar{I}^*} = \frac{1}{2} \dot{Z} \underbrace{\bar{I} \cdot \bar{I}^*}_{I^2} = \frac{1}{2} \dot{Z} I^2 =$

$= \frac{1}{2}(R + jX) I^2 = \underbrace{\frac{1}{2} R I^2}_{P} + j \underbrace{X I^2}_{Q}$ reattanza

P potenza attiva [W]; valore medio della potenza istantanea

Q potenza reattiva (L, C, M)

$\dot{S} = P + jQ$

Dualmente:

$\dot{S} = \frac{1}{2} \bar{V} \bar{I}^* = \frac{1}{2} \bar{V} (\dot{Y} \bar{V})^* = \frac{1}{2} \overbrace{\bar{V} \bar{V}^*}^{V^2} Y^* =$

$= \frac{1}{2} V^2 Y^*$

$\dot{Y} = G + jB \rightarrow$ suscettanza

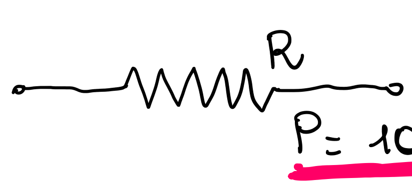
↓ conduttanza

$\dot{Y}^* = G - jB$

$\dot{S} = \frac{1}{2} V^2 (G - jB) = \underbrace{\frac{1}{2} G V^2}_{P} - j \underbrace{\frac{1}{2} B V^2}_{Q}$

Esercizio

Una lampada assorbe 100W alla tensione efficace della rete elettrica italiana. Calcolare il valore efficace della corrente, il valore di picco, la resistenza della lampada.



P = 100 W $\quad V_{eff} = 230\,V$

$V_{eff} = V_M / \sqrt{2} \qquad v(t) = V_M \,\text{sen}(\omega t + \varphi)$

$I_{eff} = ?$ $\quad I_M$ (valore di picco)? $\quad R$?

lampada $\Rightarrow$ Resistore $\begin{cases} Q = 0 \\ P \neq 0 \end{cases}$

$P = \frac{1}{2} V_M I_M = V_{eff} I_{eff}$

$I_{eff} = \frac{P}{V_{eff}} = \frac{100}{230} = 0{,}43\,A$ (A_{eff})

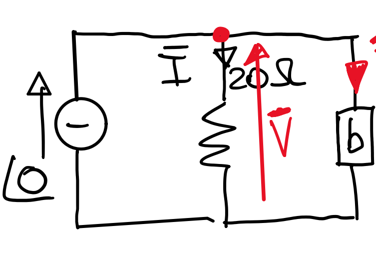
$I_M = I_{eff} \sqrt{2} = 0{,}43 \cdot \sqrt{2} = 0{,}61\,A$

$\bar{V} = R \bar{I} \rightarrow$ calcolo i moduli

$V_M = R I_M \qquad V_{eff} = R I_{eff}$

$R = \frac{V_{eff}}{I_{eff}} = \frac{230}{0{,}43} = 535\,\Omega$

Esercizio



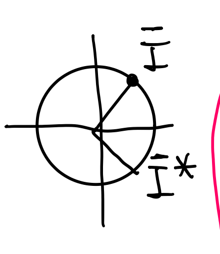
$\bar{I} = 5 \angle 30° \rightarrow \pi/6$

Trovare la potenza media assorbita da b.

potenza media $\rightarrow$ attiva P [W]

$\dot{S} = P + jQ$ RICHIAMO

↓ attiva ↓ reattiva



$\dot{S} = \frac{1}{2} \bar{V} \bar{I}^* = \frac{1}{2} VI \angle \varphi_V - \varphi_I =$

$= \frac{1}{2} VI \angle \varphi = (\varphi = \varphi_V - \varphi_I = \angle \dot{Z})$

$= \frac{1}{2} VI \cos\varphi + j \frac{1}{2} VI \,\text{sen}\,\varphi = P + jQ$

$P = \frac{1}{2} VI \cos\varphi$

- $\bar{V} = R\bar{I} = 20\bar{I} = 20 \cdot 5\, e^{j\pi/6} = 100\, e^{j\pi/6}$
- LKC al nodo

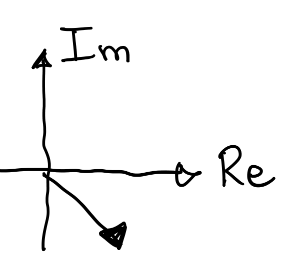
$\dot{I}_b + \dot{I} - 10 e^{j0} = 0 \qquad \bar{I}_b = 10 - \dot{I} =$

$= 10 - 5 e^{j\pi/6} = 10 - 5\left(\cos\frac{\pi}{6} + j \,\text{sen}\frac{\pi}{6}\right) =$

$= 10 - 5\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = 10 - 2{,}5\sqrt{3} - j\,2{,}5 =$

$= 5{,}67 - j\,2{,}5 = \sqrt{5{,}67^2 + 2{,}5^2}$ $\text{atan}\frac{-2{,}5}{5{,}67} =$

$= 6{,}2 \angle -0{,}42$



$-23{,}79°$

$-0{,}42$

OK

$P = \frac{1}{2} V_{Mb} I_{Mb} \cos\varphi =$

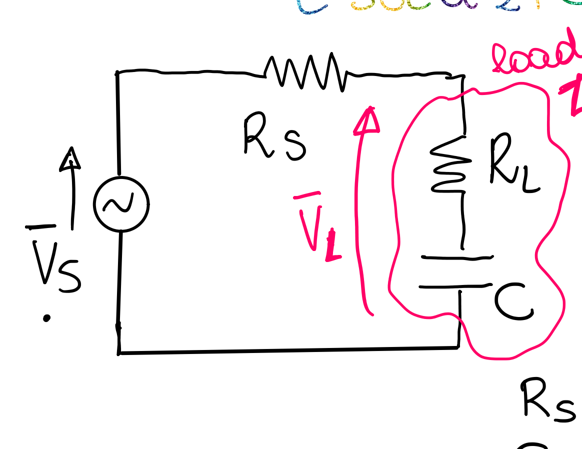
$= \frac{1}{2}\, 100 \cdot 6{,}2 \cdot \cos\varphi$

$\varphi = \varphi_V - \varphi_I = \frac{\pi}{6} - (-0{,}42) = \frac{\pi}{6} + 0{,}42 =$

$= 0{,}94$

$P = \frac{1}{2} \cdot 100 \cdot 6{,}2 \cdot \cos(0{,}94) = 310\,W$

Esercizio



S SOURCE

L LOAD

$\bar{V}_S = 110 \angle 0$ (V_{eff})

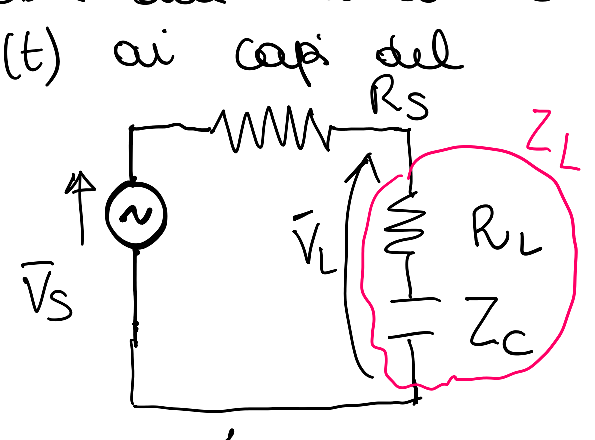
$f = 60\,Hz$

$R_S = 2\,\Omega, \quad R_L = 5\,\Omega$

$C = 2000\,\mu F$

Trovare le potenze attive, reattive, complesse assorbite dal carico e la tensione $v_L(t)$ ai capi del carico.

$Z_L = R_L + Z_C$



$\dot{Z}_C = \frac{1}{j\omega C} = \frac{1}{j 2\pi f C} = \frac{1}{j 2\pi \cdot 60 \cdot 2000 \cdot 10^{-6}} =$

$-j\,1{,}33\,\Omega$; $\quad Z_L = R_L + Z_C = 5 - j\,1{,}33$

Partitore di tensione: R_S e Z_L sono in serie

$\bar{V}_L = \bar{V}_S \frac{Z_L}{R_S + Z_L} = 110 \frac{5 - j\,1{,}33}{\underbrace{2 + 5}_{7} - j\,1{,}33} =$

$= 110 \frac{\sqrt{5^2 + 1{,}33^2} \angle \text{atan}(-1{,}33/5)}{\sqrt{7^2 + 1{,}33^2} \angle \text{atan}(-1{,}33/7)} =$

$= \frac{110 \cdot 5{,}17 \angle -0{,}27}{7{,}1 \angle -0{,}19} = 80 \angle -0{,}08$

$\bar{V}_L = 80 \angle -0{,}08$

$v_L(t) = 80\sqrt{2} \cos(\overbrace{2\pi \cdot 60}^{\omega} t - 0{,}08)\ V$

$\dot{S} = \bar{V}_L \bar{I}_L^* =$

$= \bar{V}_L \left(\frac{\bar{V}_L^*}{Z_L^*}\right) = \frac{V_L^2}{Z_L^*} =$ modulo

ometto $\frac{1}{2}$ perché lavoro con i valori efficaci!

$Z_L = 5 - j\,1{,}33$

$Z_L^* = 5 + j\,1{,}33$

$= \frac{80^2}{5 + j\,1{,}33} = P + jQ$

$= \frac{80^2 (5 - j\,1{,}33)}{25 + 1{,}33^2} = \frac{80^2}{26{,}77} (5 - j\,1{,}33) =$

$= 1195 - j\,318$ VA

$\text{Re}\{\dot{S}\} = P = 1195\,W$

$\text{Im}\{\dot{S}\} = Q = -318$ VAR < 0

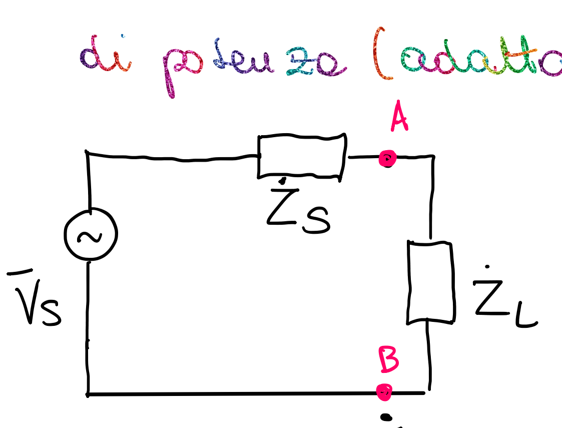
$Q < 0$ infatti è di tipo ohmico-capacitivo.

Calcolo anche la potenza apparente $A = |\dot{S}| = S$

$A = |\dot{S}| = S = \sqrt{P^2 + Q^2} = \sqrt{1195^2 + 318^2} =$

$= 1237$ VA

Esercizio sul Massimo trasferimento di potenza (adattamento del carico)



S source

L load

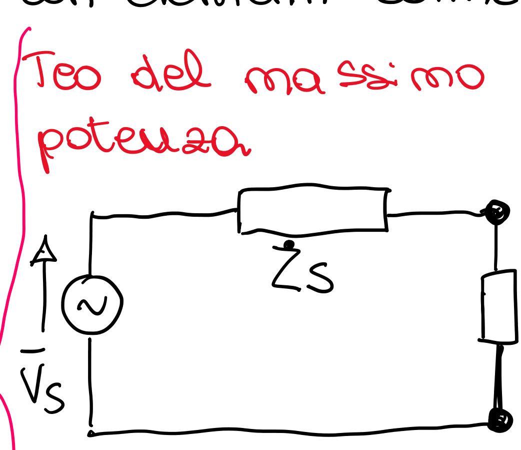
$\dot{Z}_S = 10 - 10j\ K\Omega$

$\omega = 6000$ rad/s

Determinare $\dot{Z}_L$ affinché assorba la massima potenza e realizzarla con elementi connessi in parallelo

Teo del massimo trasferimento di potenza

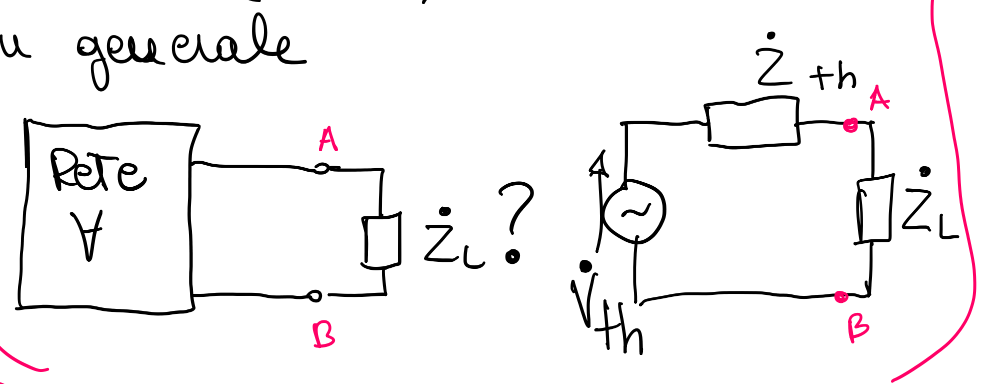
Richiamo



$\boxed{\dot{Z}_L = \dot{Z}_S^*}$

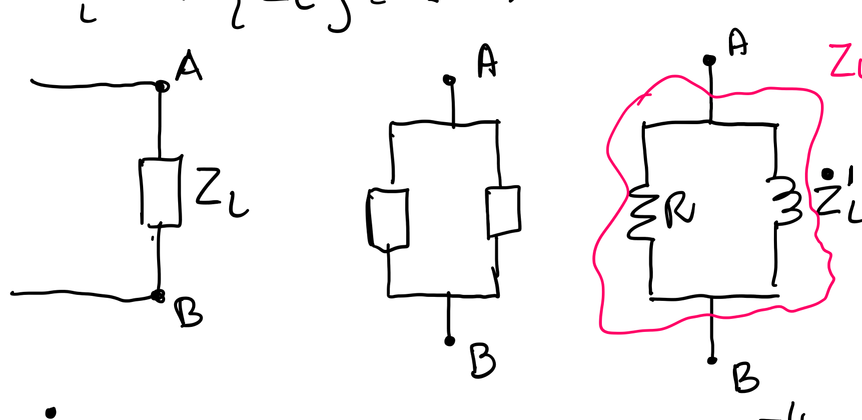
$\dot{Z}_L = \dot{Z}_S^* = (10 + j10) \cdot 10^3\ \Omega$

In generale



$Z_L = (10 + j10)10^3$ carico ohmico-induttivo

$X_L = \text{Im}\{Z_L\} = 10^1 > 0$



$\dot{Y}_L = \frac{1}{\dot{Z}_L} = \frac{1}{10^3(10 + 10j)} = \frac{10^{-4}}{1 + j} = \frac{10^{-4}(1 - j)}{2}$

$= \frac{10^{-4}}{2} - j \frac{10^{-4}}{2}$ Uguaglio le ammettenze

$\frac{1}{R} + \frac{1}{j\omega L} = \frac{1}{R} - \frac{j}{\omega L}$

$\begin{cases} \frac{1}{R} = \frac{10^{-4}}{2} \\ \frac{1}{\omega L} = \frac{10^{-4}}{2} \end{cases}$

$R = \frac{2}{10^{-4}} = 2 \cdot 10^4 = 20\,K\Omega$

$L = \frac{1}{\omega} \cdot \frac{2}{10^{-4}} = \frac{2}{6000 \cdot 10^{-4}} =$

6000

$= 3{,}33\,H$