I FASORI

martedì 19 maggio 2020 15:30

$v(t) = V_M \cos(\omega t + \varphi)$ DOMINIO DEL TEMPO $v(t)$

V_M ampiezza, ω pulsazione, φ fase

$\omega = 2\pi f = \frac{2\pi}{T}$ [rad/s]

f frequenza [Hz] T periodo [s]

$\bar{V} = V_M e^{j\varphi}$ FASORE

DOMINIO
- DELLA FREQUENZA
- DEI FASORI $\bar{V}$

V_M modulo del fasore

φ fase "

è un n° complesso

$\bar{V} = V_M(\cos\varphi + j\,\mathrm{sen}\varphi)$

Nel fasore manca $\boxed{\omega}$.

$v(t) = V_M \cos(\omega t + \varphi)$

Esempi

1) $v(t) = 3\cos\left(4t + \frac{\pi}{3}\right)$ fase

ampiezza

pulsazione ω

Trovare il fasore

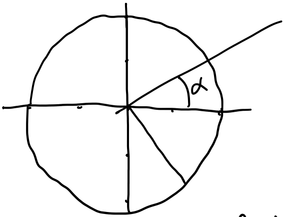
$\bar{V} = 3e^{j\pi/3} = 3\left(\cos\frac{\pi}{3} + j\,\mathrm{sen}\frac{\pi}{3}\right)$

2) $i(t) = 8\cos\left(2t + \frac{\pi}{4}\right)$

Il fasore

$$\bar{I} = 8e^{j\pi/4} = 8\left(\cos\frac{\pi}{4} + j\,\text{sen}\frac{\pi}{4}\right)$$

3) $v(t) = 6\,\text{sen}\left(2t + \frac{\pi}{3}\right) =$



$$\text{sen}\,\alpha = \cos\left(\alpha - \frac{\pi}{2}\right)$$

$$v(t) = 6\cos\left(2t + \overset{60^\circ}{\frac{\pi}{3}} - \overset{90^\circ}{\frac{\pi}{2}}\right) = 6\cos\left(2t - \frac{\pi}{6}\right)$$

$$\bar{V} = 6e^{-j\pi/6}$$

4) $i(t) = 3\cos\left(t - \frac{\pi}{2}\right)$

$$\mathring{I} = 3e^{-j\pi/2} = \cancel{3\cos\left(-\frac{\pi}{2}\right)} + j3\,\text{sen}\left(-\frac{\pi}{2}\right)$$

$$= -3j$$

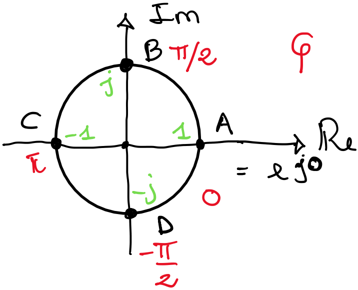
$$e^{-j\pi/2} = -j$$

A ⇒ $1 + j0 = e^{j0}$

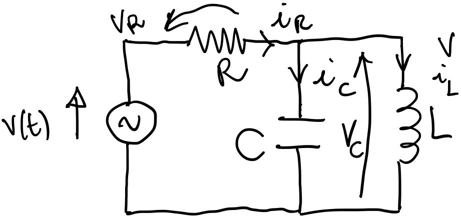
B ⇒ $0 + j1 = e^{j\pi/2}$

C ⇒ $-1 + j0 = e^{j\pi}$

D ⇒ $0 - j = e^{-j\pi/2}$



CIRCUITI A REGIME SINUSOIDALE



$v(t) = V_M \cos(\omega t + \varphi)$

"A regime" Tutte le i e le v "sono sinusoidali".

① Passare al circuito nel dominio dei fasori

$v(t) \rightarrow \bar{V}$ $\qquad i_R \rightarrow \bar{I}_R$

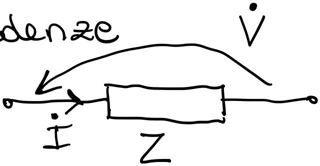
$v_R(t) \rightarrow \bar{V}_R$ $\qquad i_C \rightarrow \bar{I}_C$

$v_C(t) \rightarrow \bar{V}_C$ $\qquad i_L \rightarrow \bar{I}_L$

$R \rightarrow \dot{Z}_R$ $\qquad L \rightarrow \dot{Z}_L$ $\qquad C \rightarrow \dot{Z}_C$

$\dot{Z}$ impedenze

$$\dot{Z} = \frac{\dot{V}}{\dot{I}}$$



rapporto tra i fasori di tensione e corrente

$\bar{V} = \dot{Z}\,\bar{I}$ legge di Ohm nel dominio dei fasori

② Applico i metodi di analisi già visti. Infatti anche nel dominio dei fasori valgono le LK, LKI e LKV

LKC $\bar{I}_R = \bar{I}_C + \bar{I}_L$

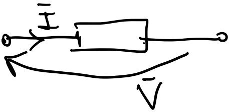
LKT $\bar{V} = \bar{V}_R + \bar{V}_C$ etc.

Trovate i fasori delle grandezze elettriche

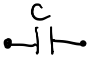
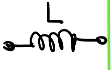
③ Torno nel dominio del tempo
i_R, i_C, i_L ....

IMPEDENZA DI UN BIPOLO

$\mathring{Z} = \bar{V} / \bar{I}$



C.U.

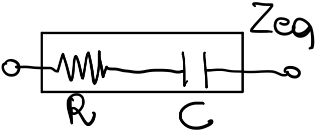
	Z	Re(Z) R	Im(Z) X	|Z|	φ
R	R	R	0	R	0
 C	$\frac{1}{j\omega C}$	0	$-\frac{1}{\omega C}$ (<0)	$\frac{1}{\omega C}$	$-\frac{\pi}{2}$
 L	$j\omega L$	0	ωL (>0)	ωL	$\frac{\pi}{2}$

$$\begin{cases} Z_R = R \\ Z_C = \dfrac{1}{j\omega C} = -\dfrac{j}{\omega C} \\ Z_L = j\omega L \end{cases}$$

$$\frac{1}{j} = -j$$

(Rifate la tabella con le ammettenze

$Y = \frac{1}{Z}$ ammettenze)



$$Z_{eq} = Z_R + Z_C = R + \frac{1}{j\omega C}$$

In generale

$\mathrm{Re}\{Z\}$ resistenza

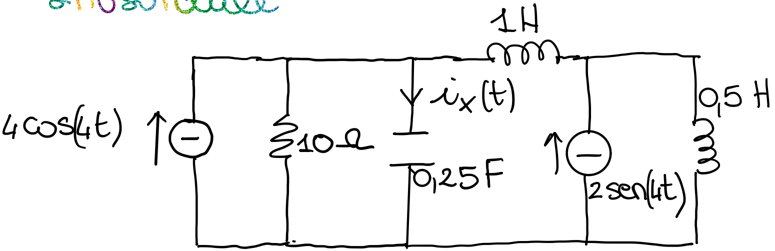
$\mathrm{Im}\{Z\}$ = reattanza (X)

$$Z = R + jX \qquad X$$

Valgono le regole di serie, parallelo, partitori, stella-triangolo, teoremi ...

Esercizio sul metodo dei

potenziali nodali a regime sinusoidale



Il circuito é a regime sinusoidale. Trovare $i_x(t)$ col metodo dei potenziali nodali.

$\omega = 4$ rad/s

Trasformiamo il circuito e passiamo al dominio dei fasori.

$4\cos(4t) \longrightarrow 4e^{j0} = 4$ fasore

$2\,\text{sen}(4t) = 2\cos\left(4t-\frac{\pi}{2}\right) \longrightarrow 2e^{-j\frac{\pi}{2}} = -2j$

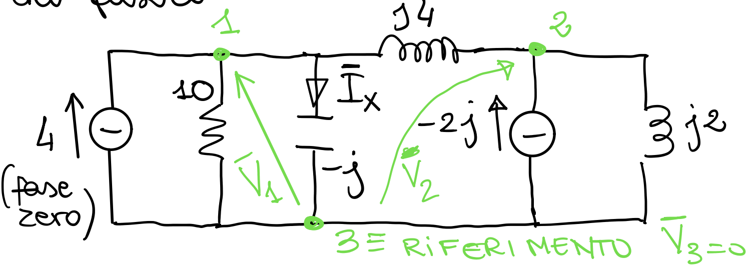
$R = 10\,\Omega$ $\qquad \dot{Z}_R = 10\,\Omega$

$C = 0{,}25\,F$ $\qquad \dot{Z}_C = \frac{1}{j\omega C} = \frac{1}{j4\cdot 0{,}25} = \frac{1}{j} = -j\,\Omega$

$L_1 = 1H$ $\qquad \dot{Z}_{L1} = j\omega L_1 = j4\cdot 1 = j4\,\Omega$

$L_2 = 0{,}5H$ $\qquad \dot{Z}_{L2} = j\omega L_2 = j4\cdot 0{,}5 = j2\,\Omega$

Ridisegno il circuito nel dominio dei fasori



nodo 1 $\quad \bar V_1\left(\frac{1}{10}+\frac{1}{j4}+\frac{1}{-j}\right)-\bar V_2\,\frac{1}{j4} = 4$

nodo 2 $\quad -\bar V_1\,\frac{1}{j4} + \bar V_2\left(\frac{1}{j4}+\frac{1}{j2}\right) = -2j$

$$\begin{cases} \bar V_1\left(0{,}1+0{,}75j\right)+\bar V_2\, j\,0{,}25 = 4 & \times 3 \\ \bar V_1\, j\,0{,}25 - j\,0{,}75\,\bar V_2 = -2j \end{cases}$$

$$3\bar V_1\left(0{,}1+0{,}75j\right) + 0{,}75j\,\bar V_2 = 12$$

$$\bar V_1\, j\,0{,}25 \qquad - 0{,}75j\,\bar V_2 = -2j$$

$$\bar V_1\left(0{,}3 + j\,0{,}75\cdot 3 + j\,0{,}25\right) = 12-2j$$

$$\bar{V}_1(0{,}3 + j\,2{,}5) = 12 - 2j$$

$$\bar{V}_1 = \frac{12-2j}{0{,}3 + j\,2{,}5}$$

$$\bar{V}_1 = \dot{Z}_C \bar{I}_x \qquad \bar{I}_x = \frac{\bar{V}_1}{\dot{Z}_C} \qquad \dot{Z}_C = -j$$

$$\bar{I}_x = -\frac{1}{j}\,\frac{12-2j}{0{,}3 + j\,2{,}5} = \frac{12j + 2}{0{,}3 + j\,2{,}5} =$$

$i_x(t) = ?$

Mi serve il modulo e la fase di $\bar{I}_x \Rightarrow$ in forma polare/esp.

modulo di $\bar{I}_x$

$$|\bar{I}_x| = \frac{\sqrt{2^2 + 12^2}}{\sqrt{0{,}3^2 + 2{,}5^2}} = \frac{12{,}17}{2{,}51} = \boxed{4{,}85}$$

fase di $\bar{I}_x$ $\quad \angle \bar{I}_x$

$$\angle \bar{I}_x = \angle \mathrm{NUM} - \angle \mathrm{DEN} =$$

$$= \operatorname{atan}\frac{12}{2} - \operatorname{atan}\frac{2{,}5}{0{,}3} =$$

$$= 80{,}54^\circ - 83^\circ{,}16 \simeq -2^\circ{,}62 =$$

$$= -\frac{2{,}62^\circ}{180^\circ}\cdot\pi = \boxed{-0{,}046 \text{ rad}}$$

$$i_x(t) = 4{,}85 \cos\underbrace{(4t - 0{,}046)}_{\uparrow \text{ radianti}} \quad A$$

N.B.

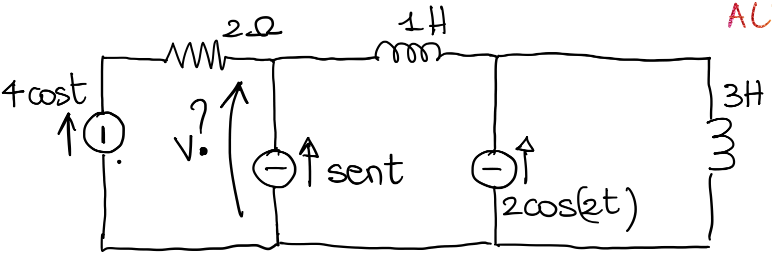
$$(4t - \varphi)$$

$\downarrow$

$\frac{rad}{\cancel{s}} \cdot \cancel{s}$ $\rightarrow$ deve essere in radianti

ESERCIZIO SUL METODO DEI FASORI.

GENERATORI NON ISOFREQUENZIALI



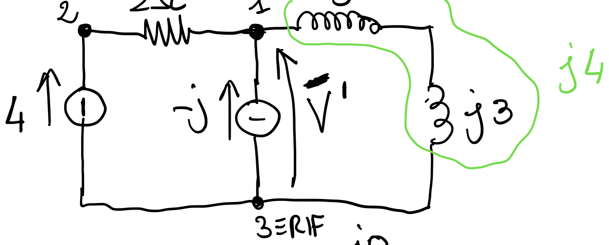
Devo applicare il principio di sovrapposizione degli effetti

$Z_L = j\omega L$ $\quad\quad \omega?$ $\quad \omega=1?$ $\quad \omega=2?$

Due circuiti

1) $\omega = 1$ rad/s $\quad\quad$ 2) $\omega = 2$ rad/s

1) $\omega = 1\ \text{rad/s}$



$$4\cos t \longrightarrow 4e^{j0} = 4$$

$$\operatorname{sen} t = \cos\left(t - \frac{\pi}{2}\right) = e^{-j\frac{\pi}{2}} = -j$$

$$2\,\Omega \longrightarrow 2\,\Omega$$

$$\left.\begin{array}{l} 1\,H \longrightarrow j\omega \cdot 1 = j1 = j \\ 3\,H \longrightarrow j\omega 3 = j1\cdot 3 = j3 \end{array}\right\} Z_L = j\omega L$$

$\bar{V}_2 = 4$ $\qquad\qquad$ $\bar{V}_1 = \bar{V}'$

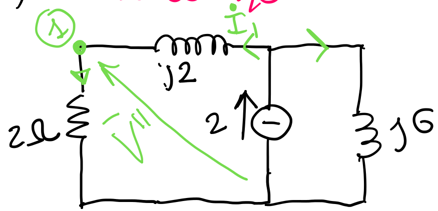
nodo 1 $\quad \bar{V}_1\left(\frac{1}{2} + \frac{1}{4j}\right) - \overset{4}{\bar{V}_2}\,\frac{1}{2} = -j$

$$\bar{V}_1\left(\frac{2j+1}{4j}\right) = 2 - j \qquad \bar{V}_1 = \frac{(2-j)4j}{1+2j} =$$

$$= \frac{4(2-j)j}{1+2j} = 4\,\frac{1+2j}{1+2j} = 4$$

$\bar{V}_1 = 4 = \bar{V}'$ $\qquad$ $v'(t) = 4\cos t$ ←

2) $\omega = 2$ rad/s



i 2 rami in cui si divide le corrente del generatore sono in // ⇒ partitore di corrente

$2\cos 2t \rightarrow 2$

$R = 2\Omega \rightarrow 2\Omega$

$1H \rightarrow j\omega 1 = j2\cdot 1 = j2$ $\qquad Z_L = j\omega L$

$3H \rightarrow j\omega 3 = j2\cdot 3 = j6$

$$\bar{I}_1 = \frac{j6}{(2+j2)+j6}\cdot 2 = \frac{j12}{2+j8} = \frac{j6}{1+j4}$$

$$\bar{V}'' = 2\bar{I}_1 = 2\cdot\frac{j6}{1+j4} = \frac{j12}{1+j4}$$

ATTENZIONE
NON SI SOMMANO I FASORI
MA LE GRANDEZZE NEL DOMINIO DEL TEMPO

v, i

$v' = 4\cos t$

$v''(t)$ scrive il modulo e la fase di $\bar{V}''$

$$\bar{V}'' = \frac{j12}{1+j4} = \frac{12}{\sqrt{1^2+4^2}} \angle \pi/2 - \operatorname{atan}\frac{4}{1} =$$

$$\left(\frac{14}{180}\cdot\pi\right.$$

$$= 2{,}9 \angle 90° - 75{,}96 = 2{,}9 \angle 14°$$

$$= 2{,}9 \angle 0{,}24$$

$$v''(t) = 2{,}9\cos(2t + 0{,}24)$$

Sovrappongo gli effetti NEL TEMPO !

$$v(t) = V' + V'' = 4\cos t + 2{,}9\cos(2t + 0{,}24)\,V \qquad 2$$