# TEORIA CLASSICA DEI LAMINATI (CLT)

**Materiale: Lamine prepreg fibre lunghe unidirezionali di carbonio GY7 in resina epossidica 934**

$E_x$ **= 294.5 GPa** = 2.945e+011 Pa

$E_y$**=6.345 GPa** = 6.345e+009 Pa

$E_s$ **= 4.897 GPa** = 4.897e+009 Pa

$\nu_{xy}$**= 0.230**

| **Sequenza del laminato: [$0_2$/+45/-45/90]s - Laminato SIMMETRICO e BILANCIATO** |
|---|
| Spessore lamina ***t*** **= 0.125 mm= 1.25e-004 m** |
| Numero strati **n = 10** |
| Spessore totale del laminato ***h*** **=** ***t·n*****= 1.25 mm =1.25e-003 m** |

Unità di misura utilizzate nei calcoli: *N*, *m* (S.I.)

## CALCOLO MATRICI DI RIGIDEZZA E CEDEVOLEZZA DEL LAMINATO

### 1. Matrici di cedevolezza $[S]_{xy}$ e di rigidezza $[Q]_{xy}$ della lamina nel sistema locale di ortotropia x-y

$$[S]_{xy} = \begin{bmatrix} 3.396e-012 & -7.810e-013 & 0 \\ -7.810e-013 & 1.576e-010 & 0 \\ 0 & 0 & 2.042e-010 \end{bmatrix} \frac{1}{Pa}$$

$$[Q]_{xy} = \begin{bmatrix} 2.948e+011 & 1.461e+009 & 0 \\ 1.461e+009 & 6.352e+009 & 0 \\ 0 & 0 & 4.897e+009 \end{bmatrix} Pa$$

### 2. Matrici di rigidezza $[Q]_{12}$ delle singole lamine nel sistema globale 1-2

Lamine a 0° (*l* = 1, 2, 9, 10)

$$[Q]_{12} = \begin{bmatrix} 2.948e+011 & 1.461e+009 & 0 \\ 1.461e+009 & 6.352e+009 & 0 \\ 0 & 0 & 4.897e+009 \end{bmatrix} Pa$$

Lamine a 45° (*l* = 3, 8)

$$[Q]_{12} = \begin{bmatrix} 8.092e+010 & 7.113e+010 & 7.212e+010 \\ 7.113e+010 & 8.092e+010 & 7.212e+010 \\ 7.212e+010 & 7.212e+010 & 7.457e+010 \end{bmatrix} Pa$$

Lamine a -45° ($l$ = 4, 7)

$$[Q]_{12} = \begin{bmatrix} 8.092e+010 & 7.113e+010 & -7.212e+010 \\ 7.113e+010 & 8.092e+010 & -7.212e+010 \\ -7.212e+010 & -7.212e+010 & 7.457e+010 \end{bmatrix} Pa$$

Lamine a 90° ($l$ = 5, 6)

$$[Q]_{12} = \begin{bmatrix} 6.352e+009 & 1.461e+009 & 0 \\ 1.461e+009 & 2.948e+011 & 0 \\ 0 & 0 & 4.897e+009 \end{bmatrix} Pa$$

**3. Matrici di rigidezza [*ABD*] e di cedevolezza [*abd*] del laminato nel sistema globale 1-2**

$$A_{ij} = \sum_{l=1}^{n} (Q_{ij})_l \cdot (h_l - h_{l-1}) \qquad i,j = 1,2,6$$

$$B_{ij} = \sum_{l=1}^{n} (Q_{ij})_l \cdot \frac{h_l^2 - h_{l-1}^2}{2} \qquad i,j = 1,2,6$$

$$D_{ij} = \sum_{l=1}^{n} (Q_{ij})_l \cdot \frac{h_l^3 - h_{l-1}^3}{3} \qquad i,j = 1,2,6$$

$$[ABD] = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix}$$

$$[abd] = [ABD]^{-1} = \begin{bmatrix} [a] & [b] \\ [b] & [d] \end{bmatrix}$$

Laminato simmetrico → $[B]= [0]$

Laminato bilanciato → $a_{16} = a_{26} = 0$

$$[ABD] = \begin{bmatrix} 1.895e+008 & 3.666e+007 & 0 & 0 & 0 & 0 \\ 3.666e+007 & 1.173e+008 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.096e+007 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.037e+001 & 2.596e+000 & 1.127e+000 \\ 0 & 0 & 0 & 2.596e+000 & 3.934e+000 & 1.127e+000 \\ 0 & 0 & 0 & 1.127e+000 & 1.127e+000 & 3.156e+000 \end{bmatrix}$$

$$[abd] = \begin{bmatrix} 5.618e-009 & -1.755e-009 & 0 & 0 & 0 & 0 \\ -1.755e-009 & 9.070e-009 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.442e-008 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.590e-002 & -1.609e-002 & -3.504e-003 \\ 0 & 0 & 0 & -1.609e-002 & 2.931e-001 & -9.894e-002 \\ 0 & 0 & 0 & -3.504e-003 & -9.894e-002 & 3.535e-001 \end{bmatrix}$$

Dalla matrice [$a \cdot h$] si ricavano le proprietà elastiche <u>medie del laminato</u> nel piano:

$$\bar{E}_1 = \frac{1}{a_{11}} \cdot \frac{1}{h} = \frac{1}{5.618e-009} \cdot \frac{1}{1.25e-003} = 142.4e+009 = 142.4\ GPa$$

$$\bar{E}_2 = \frac{1}{a_{22}} \cdot \frac{1}{h} = \frac{1}{9.070e-009} \cdot \frac{1}{1.25e-003} = 88.20e+009 = 88.2\ GPa$$

$$\bar{E}_6 = \frac{1}{a_{66}} \cdot \frac{1}{h} = \frac{1}{2.442e-008} \cdot \frac{1}{1.25e-003} = 32.76e+009 = 32.76\ GPa$$

$$\bar{\nu}_{12} = -\bar{E}_1 \cdot a_{12} \cdot h = -(142.4e+009) \cdot (-1.755e-009) \cdot (1.25e-003) = 0.312$$

## CALCOLO SFORZI E DEFORMAZIONI NELLE SINGOLE LAMINE

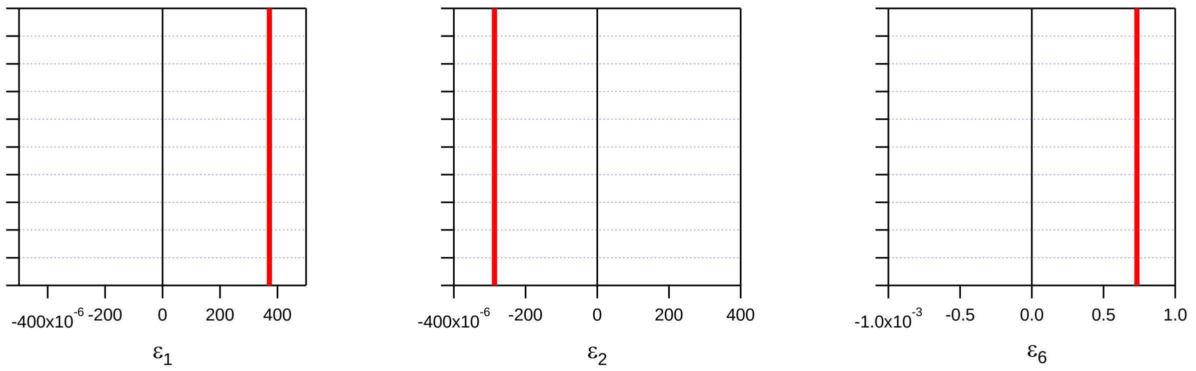
### Caso I) Sollecitazioni sul piano (sole $N_1$, $N_2$, $N_6$)

### 4. Deformazioni $\{\varepsilon\}_{12}$ nel sistema di riferimento globale 1-2

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{Bmatrix} +60 \\ -20 \\ +30 \end{Bmatrix} \frac{kN}{m} = \begin{Bmatrix} 60000 \\ -20000 \\ 30000 \end{Bmatrix} \frac{N}{m}$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \frac{N}{m} \cdot m$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} = \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} = [a] \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{bmatrix} 5.618e-009 & -1.755e-009 & 0 \\ -1.755e-009 & 9.070e-009 & 0 \\ 0 & 0 & 2.442e-008 \end{bmatrix} \begin{Bmatrix} 60000 \\ -20000 \\ 30000 \end{Bmatrix} = \begin{Bmatrix} 3.722e-004 \\ -2.867e-004 \\ 7.325e-004 \end{Bmatrix}$$



## 5. Deformazioni $\{\varepsilon\}_{xy}$ nelle singole lamine (sistemi di riferimento locali x-y)

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_s\end{Bmatrix} = [T_\varepsilon]\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_6\end{Bmatrix}$$

Lamine a 0° ($l$ = 1, 2, 9, 10)

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_s\end{Bmatrix} = \begin{Bmatrix}3.722\text{e}-004\\ -2.867\text{e}-004\\ 7.325\text{e}-004\end{Bmatrix}$$

Lamine a 45° ($l$ = 3, 8)

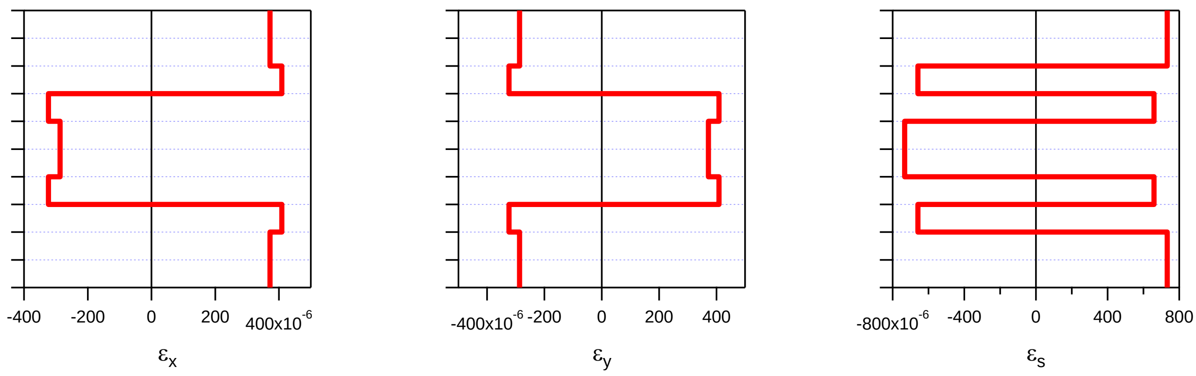
$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_s\end{Bmatrix} = \begin{Bmatrix}4.090\text{e}-004\\ -3.235\text{e}-004\\ -6.588\text{e}-004\end{Bmatrix}$$

Lamine a -45° ($l$ = 4, 7)

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_s\end{Bmatrix} = \begin{Bmatrix}-3.235\text{e}-004\\ 4.090\text{e}-004\\ 6.588\text{e}-004\end{Bmatrix}$$

Lamine a 90° ($l$ = 5, 6)

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_s\end{Bmatrix} = \begin{Bmatrix}-2.867\text{e}-004\\ 3.722\text{e}-004\\ -7.325\text{e}-004\end{Bmatrix}$$



## 6. Sforzi $\{\sigma\}_{xy}$ nelle singole lamine (sistema di riferimento locale x-y)

$$\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_s\end{Bmatrix} = [Q]_{xy}\begin{Bmatrix}\varepsilon_x\\\varepsilon_y\\\varepsilon_s\end{Bmatrix}$$

Lamine a 0° (*l* = 1, 2, 9, 10)

$$\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_s\end{Bmatrix} = \begin{Bmatrix}1.093e+008\\-1.277e+006\\3.587e+006\end{Bmatrix}Pa = \begin{Bmatrix}109.3\\-1.277\\3.587\end{Bmatrix}MPa$$

Lamine a 45° (*l* = 3, 8)

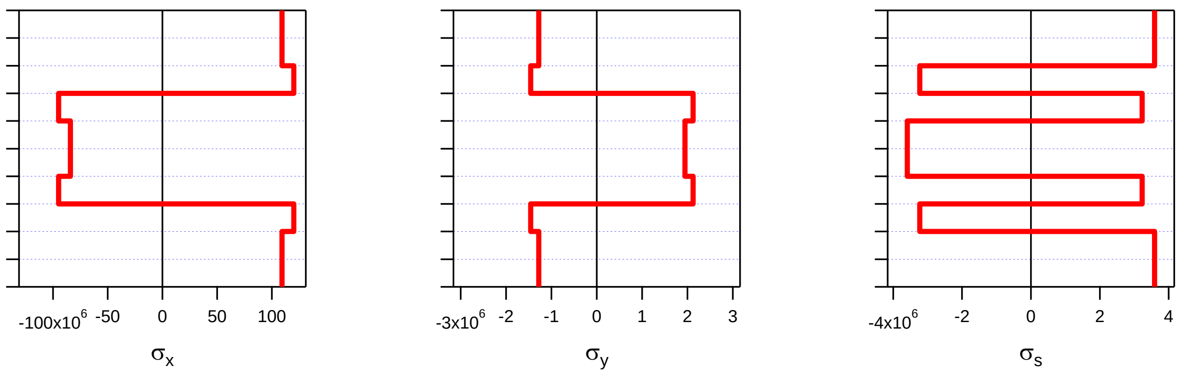
$$\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_s\end{Bmatrix} = \begin{Bmatrix}1.201e+008\\-1.458e+006\\-3.226e+006\end{Bmatrix}Pa = \begin{Bmatrix}120.1\\-1.458\\-3.226\end{Bmatrix}MPa$$

Lamine a -45° (*l* = 4, 7)

$$\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_s\end{Bmatrix} = \begin{Bmatrix}-9.479e+007\\2.125e+006\\3.226e+006\end{Bmatrix}Pa = \begin{Bmatrix}-94.79\\2.125\\3.226\end{Bmatrix}MPa$$

Lamine a 90° (*l* = 5, 6)

$$\begin{Bmatrix}\sigma_x\\\sigma_y\\\sigma_s\end{Bmatrix} = \begin{Bmatrix}-8.399e+007\\1.945e+006\\-3.587e+006\end{Bmatrix}Pa = \begin{Bmatrix}-83.99\\1.945\\-3.587\end{Bmatrix}MPa$$



## Caso II) Sollecitazioni fuori dal piano (soli $M_1$, $M_2$, $M_6$)

### 4. Deformazioni $\{\varepsilon\}_{12}$ nel sistema di riferimento globale 1-2

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{Bmatrix} +16 \\ -6 \\ +8 \end{Bmatrix} \frac{N}{m} \cdot m$$

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \frac{N}{m}$$

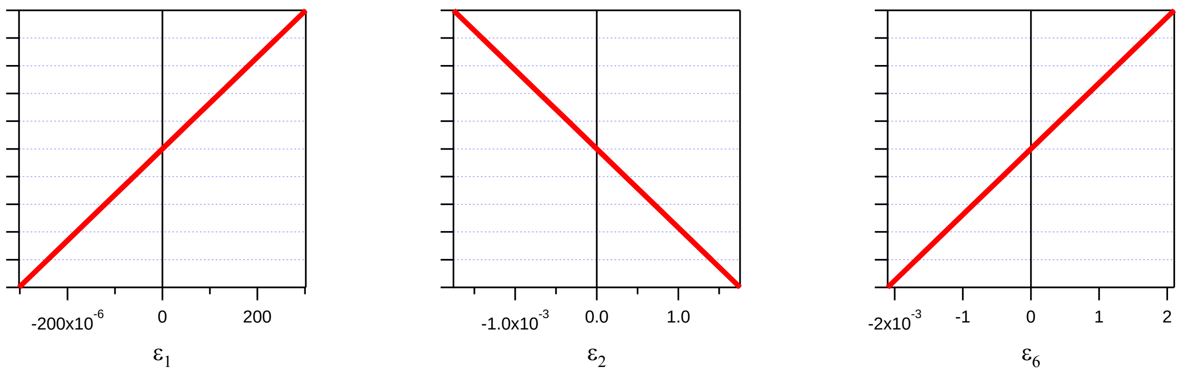
$$\begin{Bmatrix} \kappa_1^0 \\ \kappa_2^0 \\ \kappa_6^0 \end{Bmatrix} = [d] \begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{bmatrix} 2.590\text{e}-002 & -1.609\text{e}-002 & -3.504\text{e}-003 \\ -1.609\text{e}-002 & 2.931\text{e}-001 & -9.894\text{e}-002 \\ -3.504\text{e}-003 & -9.894\text{e}-002 & 3.535\text{e}-001 \end{bmatrix} \begin{Bmatrix} 16 \\ -6 \\ 8 \end{Bmatrix} = \begin{Bmatrix} 4.830\text{e}-001 \\ -2.808\text{e}+000 \\ 3.365\text{e}+000 \end{Bmatrix}$$

da cui, poichè

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} = z \cdot \begin{Bmatrix} \kappa_1^0 \\ \kappa_2^0 \\ \kappa_6^0 \end{Bmatrix} = z \begin{Bmatrix} 4.830\text{e}-001 \\ -2.808\text{e}+000 \\ 3.365\text{e}+000 \end{Bmatrix}$$

si ottengono le seguenti deformazioni nel sistema globale alle superfici bottom e top delle diversi lamine:

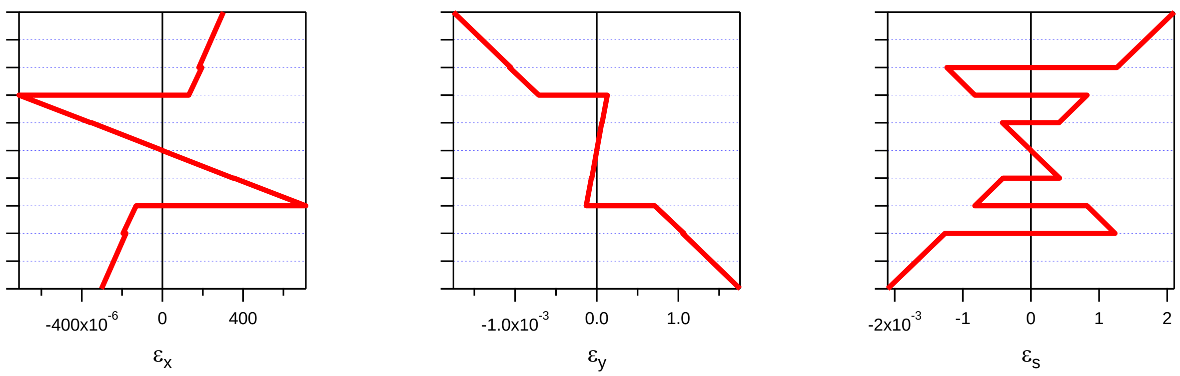
| | BOTTOM | | | TOP | | |
|---|---|---|---|---|---|---|
| Lamina | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_6$ | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_6$ |
| **1** | -3.019e-004 | 1.755e-003 | -2.103e-003 | -2.415e-004 | 1.404e-003 | -1.683e-003 |
| **2** | -2.415e-004 | 1.404e-003 | -1.683e-003 | -1.811e-004 | 1.053e-003 | -1.262e-003 |
| **3** | -1.811e-004 | 1.053e-003 | -1.262e-003 | -1.207e-004 | 7.020e-004 | -8.413e-004 |
| **4** | -1.207e-004 | 7.020e-004 | -8.413e-004 | -6.037e-005 | 3.510e-004 | -4.207e-004 |
| **5** | -6.037e-005 | 3.510e-004 | -4.207e-004 | 5.915e-021 | -2.666e-020 | -3.000e-022 |
| **6** | 5.915e-021 | -2.666e-020 | -3.000e-022 | 6.037e-005 | -3.510e-004 | 4.207e-004 |
| **7** | 6.037e-005 | -3.510e-004 | 4.207e-004 | 1.207e-004 | -7.020e-004 | 8.413e-004 |
| **8** | 1.207e-004 | -7.020e-004 | 8.413e-004 | 1.811e-004 | -1.053e-003 | 1.262e-003 |
| **9** | 1.811e-004 | -1.053e-003 | 1.262e-003 | 2.415e-004 | -1.404e-003 | 1.683e-003 |
| **10** | 2.415e-004 | -1.404e-003 | 1.683e-003 | 3.019e-004 | -1.755e-003 | 2.103e-003 |




## 5. Deformazioni $\{\varepsilon\}_{xy}$ nelle singole lamine (sistemi di riferimento locali x-y)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{Bmatrix} = [T]_\varepsilon \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix}$$

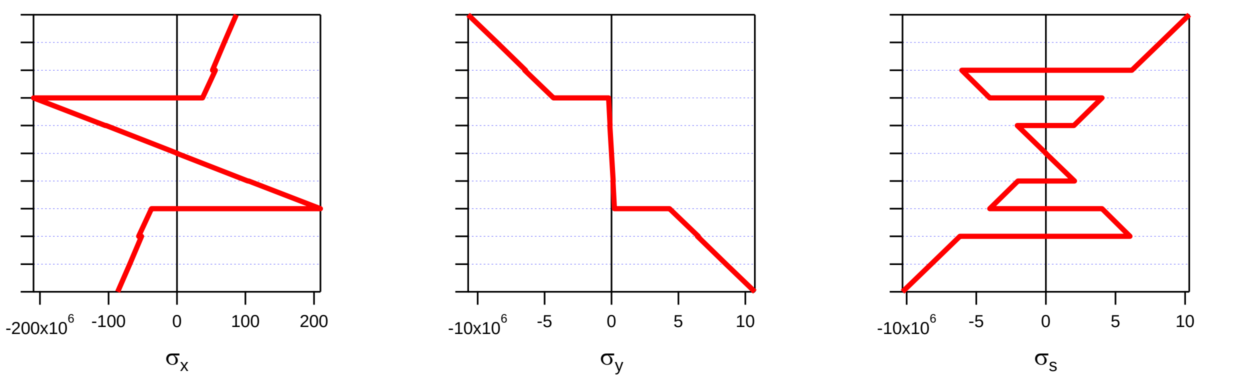
| | BOTTOM | | | TOP | | |
|---|---|---|---|---|---|---|
| Lamina | $\varepsilon_x$ | $\varepsilon_y$ | $\varepsilon_s$ | $\varepsilon_x$ | $\varepsilon_y$ | $\varepsilon_s$ |
| 1 | -3.019e-004 | 1.755e-003 | -2.103e-003 | -2.415e-004 | 1.404e-003 | -1.683e-003 |
| **2** | -2.415e-004 | 1.404e-003 | -1.683e-003 | -1.811e-004 | 1.053e-003 | -1.262e-003 |
| **3** | -1.951e-004 | 1.067e-003 | 1.234e-003 | -1.301e-004 | 7.113e-004 | 8.227e-004 |
| **4** | 7.113e-004 | -1.301e-004 | -8.227e-004 | 3.556e-004 | -6.503e-005 | -4.114e-004 |
| **5** | 3.510e-004 | -6.037e-005 | 4.207e-004 | 0.000e-000 | 0.000e-000 | 0.000e-000 |
| **6** | 0.000e-000 | 0.000e-000 | 0.000e-000 | -3.510e-004 | 6.037e-005 | -4.207e-004 |
| **7** | -3.556e-004 | 6.503e-005 | 4.114e-004 | -7.113e-004 | 1.301e-004 | 8.227e-004 |
| **8** | 1.301e-004 | -7.113e-004 | -8.227e-004 | 1.951e-004 | -1.067e-003 | -1.234e-003 |
| **9** | 1.811e-004 | -1.053e-003 | 1.262e-003 | 2.415e-004 | -1.404e-003 | 1.683e-003 |
| **10** | 2.415e-004 | -1.404e-003 | 1.683e-003 | 3.019e-004 | -1.755e-003 | 2.103e-003 |



## 6. Sforzi $\{\sigma\}_{xy}$ nelle singole lamine (sistema di riferimento locale x-y)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = [Q]_{xy} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{Bmatrix}$$

| | BOTTOM | | | TOP | | |
|---|---|---|---|---|---|---|
| Lamina | $\sigma_x$ (Pa) | $\sigma_y$ (Pa) | $\sigma_s$ (Pa) | $\sigma_x$ (Pa) | $\sigma_y$ (Pa) | $\sigma_s$ (Pa) |
| **1** | -8.644e+007 | 1.071e+007 | -1.030e+007 | -6.915e+007 | 8.565e+006 | -8.240e+006 |
| **2** | -6.915e+007 | 8.565e+006 | -8.240e+006 | -5.186e+007 | 6.424e+006 | -6.180e+006 |
| **3** | -5.596e+007 | 6.492e+006 | 6.043e+006 | -3.731e+007 | 4.328e+006 | 4.029e+006 |
| **4** | 2.095e+008 | 2.130e+005 | -4.029e+006 | 1.048e+008 | 1.065e+005 | -2.014e+006 |
| **5** | 1.034e+008 | 1.293e+005 | 2.060e+006 | -0.000e-000 | 0.000e-000 | 0.000e-000 |
| **6** | -0.000e-000 | 0.000e-000 | 0.000e-000 | -1.034e+008 | -1.293e+005 | -2.060e+006 |
| **7** | -1.048e+008 | -1.065e+005 | 2.014e+006 | -2.095e+008 | -2.130e+005 | 4.029e+006 |
| **8** | 3.731e+007 | -4.328e+006 | -4.029e+006 | 5.596e+007 | -6.492e+006 | -6.043e+006 |
| **9** | 5.186e+007 | -6.424e+006 | 6.180e+006 | 6.915e+007 | -8.565e+006 | 8.240e+006 |
| **10** | 6.915e+007 | -8.565e+006 | 8.240e+006 | 8.644e+007 | -1.071e+007 | 1.030e+007 |



| |
|---|
| **Sequenza del laminato: $[0_2/+45/90_2]_s$ - Laminato SIMMETRICO (NON BILANCIATO)** |
| Spessore lamina ***t* = 0.125 mm** |
| Numero strati ***n* = 10** |
| Spessore totale del laminato ***h* = *t·n* = 1.25 mm = 1.25e-3 m** |

Unità di misura utilizzate nei calcoli: *N*, *m* (S.I.)

## CALCOLO MATRICI DI RIGIDEZZA E CEDEVOLEZZA DEL LAMINATO

### 1. Matrici di cedevolezza $[S]_{xy}$ e di rigidezza $[Q]_{xy}$ della lamina nel sistema locale di otrotropia x-y

$$[S]_{xy} = \begin{bmatrix} 3.396e-012 & -7.810e-013 & 0 \\ -7.810e-013 & 1.576e-010 & 0 \\ 0 & 0 & 2.042e-010 \end{bmatrix} \frac{1}{Pa}$$

$$[Q]_{xy} = \begin{bmatrix} 2.948e+011 & 1.461e+009 & 0 \\ 1.461e+009 & 6.352e+009 & 0 \\ 0 & 0 & 4.897e+009 \end{bmatrix} Pa$$

### 2. Matrici di rigidezza $[Q]_{12}$ delle singole lamina nel sistema globale 1-2

Lamine a 0° (*l* = 1, 2, 9, 10)

$$[Q]_{12} = \begin{bmatrix} 2.948e+011 & 1.461e+009 & 0 \\ 1.461e+009 & 6.352e+009 & 0 \\ 0 & 0 & 4.897e+009 \end{bmatrix} Pa$$

Lamine a 45° (*l* = 3, 8)

$$[Q]_{12} = \begin{bmatrix} 8.092e+010 & 7.113e+010 & 7.212e+010 \\ 7.113e+010 & 8.092e+010 & 7.212e+010 \\ 7.212e+010 & 7.212e+010 & 7.457e+010 \end{bmatrix} Pa$$

Lamine a 90° (*l* = 4, 5, 6, 7)

$$[Q]_{12} = \begin{bmatrix} 6.352e+009 & 1.461e+009 & 0 \\ 1.461e+009 & 2.948e+011 & 0 \\ 0 & 0 & 4.897e+009 \end{bmatrix} Pa$$

## 3. Matrici di rigidezza [*ABD*] e di cedevolezza [*abd*] del laminato nel sistema globale 1-2

$$A_{ij} = \sum_{l=1}^{n} \left(Q_{ij}\right)_l \cdot (h_l - h_{l-1})$$

$$B_{ij} = \sum_{l=1}^{n} \left(Q_{ij}\right)_l \cdot \frac{h_l^2 - h_{l-1}^2}{2}$$

$$D_{ij} = \sum_{l=1}^{n} \left(Q_{ij}\right)_l \cdot \frac{h_l^3 - h_{l-1}^3}{3}$$

$$[ABD] = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix}$$

$$[abd] = [ABD]^{-1} = \begin{bmatrix} [a] & [b] \\ [b] & [d] \end{bmatrix}$$

Laminato simmetrico → [$B$]= [0]

$$[ABD] = \begin{bmatrix} 1.708e+008 & 1.924e+007 & 1.803e+007 & 0 & 0 & 0 \\ 1.924e+007 & 1.708e+008 & 1.803e+007 & 0 & 0 & 0 \\ 1.803e+007 & 1.803e+007 & 2.354e+007 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.969e+001 & 1.961e+000 & 1.784e+000 \\ 0 & 0 & 0 & 1.961e+000 & 5.884e+000 & 1.784e+000 \\ 0 & 0 & 0 & 1.784e+000 & 1.784e+000 & 2.521e+000 \end{bmatrix}$$

$$[abd] = \begin{bmatrix} 6.376e-009 & -2.206e-010 & -4.715e-009 & 0 & 0 & 0 \\ -2.206e-010 & 6.376e-009 & -4.715e-009 & 0 & 0 & 0 \\ -4.715e-009 & -4.715e-009 & 4.971e-008 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.609e-002 & -3.944e-003 & -1.568e-002 \\ 0 & 0 & 0 & -3.944e-003 & 2.170e-001 & -1.508e-001 \\ 0 & 0 & 0 & -1.568e-002 & -1.508e-001 & 5.146e-001 \end{bmatrix}$$

Dalla matrice [$a \cdot h$] si ricavano le proprietà elastiche medie del laminato nel piano:

$$\bar{E}_1 = \frac{1}{a_{11}} \cdot \frac{1}{h} = \frac{1}{6.376e-009} \cdot \frac{1}{1.25e-003} = 125.5e+009 = 125.5\ GPa$$

$$\bar{E}_2 = \frac{1}{a_{22}} \cdot \frac{1}{h} = \frac{1}{6.376e-009} \cdot \frac{1}{1.25e-003} = 125.5e+009 = 125.5\ GPa$$

$$\bar{E}_6 = \frac{1}{a_{66}} \cdot \frac{1}{h} = \frac{1}{4.971e-008} \cdot \frac{1}{1.25e-003} = 16.09e+009 = 16.09\ GPa$$

$$\bar{\nu}_{12} = -\bar{E}_1 \cdot a_{12} \cdot h = -(125.5e+009) \cdot (-2.206e-010) \cdot (1.25e-003) = 0.035$$

| |
|---|
| **Sequenza del laminato: $[0_2/+45/90_2/0_2/+45/90_2]$ -** <br> **Laminato NON SIMMETRICO e NON BILANCIATO** |
| Spessore lamina ***t* = 0.125 mm** |
| Numero strati **n = 10** |
| Spessore totale del laminato ***h* = *t*·*n*= 1.25 mm = 1.25e-3 m** |

Unità di misura utilizzate nei calcoli: *N*, *m* (S.I.)

## CALCOLO MATRICI DI RIGIDEZZA E CEDEVOLEZZA DEL LAMINATO

### 1. Matrici di cedevolezza $[S]_{xy}$ e di rigidezza $[Q]_{xy}$ della lamina nel sistema locale di otrotropia x-y

$$[S]_{xy} = \begin{bmatrix} 3.396e-012 & -7.810e-013 & 0 \\ -7.810e-013 & 1.576e-010 & 0 \\ 0 & 0 & 2.042e-010 \end{bmatrix} \frac{1}{Pa}$$

$$[Q]_{xy} = \begin{bmatrix} 2.948e+011 & 1.461e+009 & 0 \\ 1.461e+009 & 6.352e+009 & 0 \\ 0 & 0 & 4.897e+009 \end{bmatrix} Pa$$

### 2. Matrici di rigidezza $[Q]_{12}$ delle singole lamine nel sistema globale 1-2

Lamine a 0° (*l* = 1, 2, 6, 7)

$$[Q]_{12} = \begin{bmatrix} 2.948e+011 & 1.461e+009 & 0 \\ 1.461e+009 & 6.352e+009 & 0 \\ 0 & 0 & 4.897e+009 \end{bmatrix} Pa$$

Lamine a 45° (*l* = 3, 8)

$$[Q]_{12} = \begin{bmatrix} 8.092e+010 & 7.113e+010 & 7.212e+010 \\ 7.113e+010 & 8.092e+010 & 7.212e+010 \\ 7.212e+010 & 7.212e+010 & 7.457e+010 \end{bmatrix} Pa$$

Lamine a 90° (*l* = 4, 5, 9, 10)

$$[Q]_{12} = \begin{bmatrix} 6.352e+009 & 1.461e+009 & 0 \\ 1.461e+009 & 2.948e+011 & 0 \\ 0 & 0 & 4.897e+009 \end{bmatrix} Pa$$

## 3. Matrici di rigidezza [*ABD*] e di cedevolezza [*abd*] del laminato nel sistema globale 1-2

$$A_{ij} = \sum_{l=1}^{n} (Q_{ij})_l \cdot (h_l - h_{l-1})$$

$$B_{ij} = \sum_{l=1}^{n} (Q_{ij})_l \cdot \frac{h_l^2 - h_{l-1}^2}{2}$$

$$D_{ij} = \sum_{l=1}^{n} (Q_{ij})_l \cdot \frac{h_l^3 - h_{l-1}^3}{3}$$

$$[ABD] = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix}$$

$$[abd] = [ABD]^{-1} = \begin{bmatrix} [a] & [b] \\ [b] & [d] \end{bmatrix}$$

$$[ABD] = \begin{bmatrix} 1.708e+008 & 1.924e+007 & 1.803e+007 & -2.705e+004 & 0.000e+000 & 0.000e+000 \\ 1.924e+007 & 1.708e+008 & 1.803e+007 & 0.000e+000 & 2.705e+004 & 0.000e+000 \\ 1.803e+007 & 1.803e+007\ 0 & 2.354e+007 & 0.000e+000 & 0.000e+001 & 0.000e+000 \\ -2.705e+004 & 0.000e+000 & 0.000e+000 & 2.279e+001 & 1.961e+000 & 1.784e+000 \\ 0.000e+000 & 2.705e+004 & 0.000e+000 & 1.961e+000 & 2.279e+001 & 1.784e+000 \\ 0.000e+000 & 0.000e+000 & 0.000e+000 & 1.784e+000 & 1.784e+000 & 2.521e+000 \end{bmatrix}$$

$$[abd] = \begin{bmatrix} 8.140e-009 & -2.865e-010 & -6.016e-009 & 1.023e-005 & 2.814e-008 & -7.260e-006 \\ -2.865e-010 & 8.140e-009 & -6.016e-009 & -2.814e-008 & -1.023e-005 & 7.260e-006 \\ -6.016e-009 & -6.016e-009 & 5.170e-008 & -7.813e-006 & 7.813e-006 & 0.000e+000 \\ 1.023e-005 & -2.814e-008 & -7.813e-006 & 5.937e-002 & -1.891e-003 & -4.069e-002 \\ 2.814e-008 & -1.023e-005 & 7.813e-006 & -1.891e-003 & 5.937e-002 & -4.069e-002 \\ -7.260e-0060 & 7.260e-006 & 0.000e+000 & -4.069e-002 & -4.069e-002 & 4.543e-001 \end{bmatrix}$$

Dalla matrice $[a{\cdot}h]$ si ricavano le proprietà elastiche medie del laminato nel piano:

$$\bar{E}_1 = \frac{1}{a_{11}} \cdot \frac{1}{h} = \frac{1}{8.140e-009} \cdot \frac{1}{1.25e-003} = 98.3\ GPa$$

$$\bar{E}_2 = \frac{1}{a_{22}} \cdot \frac{1}{h} = \frac{1}{8.140e-009} \cdot \frac{1}{1.25e-003} = 98.3\ GPa$$

$$\bar{E}_6 = \frac{1}{a_{66}} \cdot \frac{1}{h} = \frac{1}{5.170e-008} \cdot \frac{1}{1.25e-003} = 15.5\ GPa$$

$$\bar{\nu}_{12} = -\bar{E}_1 \cdot a_{12} \cdot h = -(98.3e+009) \cdot (-2.865e-010) \cdot (1.25e-003) = 0.035$$

## Sollecitazioni sul piano (sola azione $N_1$)

### 4. Deformazioni $\{\varepsilon\}_{12}$ nel sistema di riferimento globale 1-2

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_6 \end{Bmatrix} = \begin{Bmatrix} +60 \\ 0 \\ 0 \end{Bmatrix} \frac{kN}{m} = \begin{Bmatrix} 60000 \\ 0 \\ 0 \end{Bmatrix} \frac{N}{m}$$

$$\begin{Bmatrix} M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \frac{N}{m} \cdot m$$

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{Bmatrix} = \begin{Bmatrix} 60 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \begin{matrix} \frac{kN}{m} \\ \\ \frac{N}{m} m \end{matrix} = \begin{Bmatrix} 60000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \begin{matrix} \frac{N}{m} \\ \\ \frac{N}{m} m \end{matrix}$$

Deformazioni e curvature al piano medio ⇨ $\begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \\ \kappa_1^0 \\ \kappa_2^0 \\ \kappa_6^0 \end{Bmatrix} = [abd] \begin{Bmatrix} N_1 \\ N_2 \\ N_6 \\ M_1 \\ M_2 \\ M_6 \end{Bmatrix} =$

$$= \begin{bmatrix} 8.140e-009 & -2.865e-010 & -6.016e-009 & 1.023e-005 & 2.814e-008 & -7.260e-006 \\ -2.865e-010 & 8.140e-009 & -6.016e-009 & -2.814e-008 & -1.023e-005 & 7.260e-006 \\ -6.016e-009 & -6.016e-009 & 5.170e-008 & -7.813e-006 & 7.813e-006 & 0.000e+000 \\ 1.023e-005 & -2.814e-008 & -7.813e-006 & 5.937e-002 & -1.891e-003 & -4.069e-002 \\ 2.814e-008 & -1.023e-005 & 7.813e-006 & -1.891e-003 & 5.937e-002 & -4.069e-002 \\ -7.260e-0060 & 7.260e-006 & 0.000e+000 & -4.069e-002 & -4.069e-002 & 4.543e-001 \end{bmatrix} \begin{Bmatrix} 60000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \\ \kappa_1^0 \\ \kappa_2^0 \\ \kappa_6^0 \end{Bmatrix} = \begin{Bmatrix} 4.884e-004 \\ -1.719e-005 \\ -3.610e-004 \\ 6.137e-001 \\ 1.689e-003 \\ -4.356e-001 \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_1(z) \\ \varepsilon_2(z) \\ \varepsilon_6(z) \end{Bmatrix} = \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_1^0 \\ \kappa_2^0 \\ \kappa_6^0 \end{Bmatrix} = \begin{Bmatrix} 4.884e-004 \\ -1.719e-005 \\ -3.610e-004 \end{Bmatrix} + z \begin{Bmatrix} 6.137e-001 \\ 1.689e-003 \\ -4.356e-001 \end{Bmatrix}$$

da cui si ottengono le seguenti deformazioni nel sistema globale alle superfici bottom e top delle diverse lamine:

| | BOTTOM | | |
|---|---|---|---|
| Lamina | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_6$ |
| **1** | 1.049e-004 | -1.825e-005 | -8.872e-005 |
| **2** | 1.816e-004 | -1.803e-005 | -1.432e-004 |
| **3** | 2.583e-004 | -1.782e-005 | -1.976e-004 |
| **4** | 3.350e-004 | -1.761e-005 | -2.521e-004 |
| **5** | 4.117e-004 | -1.740e-005 | -3.065e-004 |
| **6** | 4.884e-004 | -1.719e-005 | -3.610e-004 |
| **7** | 5.651e-004 | -1.698e-005 | -4.154e-004 |
| **8** | 6.418e-004 | -1.677e-005 | -4.699e-004 |
| **9** | 7.185e-004 | -1.656e-005 | -5.243e-004 |
| **10** | 7.953e-004 | -1.635e-005 | -5.787e-004 |

| | TOP | |
|---|---|---|
| $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_6$ |
| 1.816e-004 | -1.803e-005 | -1.432e-004 |
| 2.583e-004 | -1.782e-005 | -1.976e-004 |
| 3.350e-004 | -1.761e-005 | -2.521e-004 |
| 4.117e-004 | -1.740e-005 | -3.065e-004 |
| 4.884e-004 | -1.719e-005 | -3.610e-004 |
| 5.651e-004 | -1.698e-005 | -4.154e-004 |
| 6.418e-004 | -1.677e-005 | -4.699e-004 |
| 7.185e-004 | -1.656e-005 | -5.243e-004 |
| 7.953e-004 | -1.635e-005 | -5.787e-004 |
| 8.720e-004 | -1.614e-005 | -6.332e-004 |

## 5. Deformazioni $\{\varepsilon\}_{xy}$ nelle singole lamine (sistemi di riferimento locali x-y)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{Bmatrix} = [T_\varepsilon] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix}$$

| | BOTTOM | | |
|---|---|---|---|
| Lamina | $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_6$ |
| **1** | 1.049e-004 | -1.825e-005 | -8.872e-005 |
| **2** | 1.816e-004 | -1.803e-005 | -1.432e-004 |
| **3** | 2.143e-005 | 2.190e-004 | -2.761e-004 |
| **4** | -1.761e-005 | 3.350e-004 | 2.521e-004 |
| **5** | -1.740e-005 | 4.117e-004 | 3.065e-004 |
| **6** | 4.884e-004 | -1.719e-005 | -3.610e-004 |
| **7** | 5.651e-004 | -1.698e-005 | -4.154e-004 |
| **8** | 7.761e-005 | 5.475e-004 | -6.586e-004 |
| **9** | -1.656e-005 | 7.185e-004 | 5.243e-004 |
| **10** | -1.635e-005 | 7.953e-004 | 5.787e-004 |

| | TOP | |
|---|---|---|
| $\varepsilon_1$ | $\varepsilon_2$ | $\varepsilon_6$ |
| 1.816e-004 | -1.803e-005 | -1.432e-004 |
| 2.583e-004 | -1.782e-005 | -1.976e-004 |
| 3.267e-005 | 2.847e-004 | -3.526e-004 |
| -1.740e-005 | 4.117e-004 | 3.065e-004 |
| -1.719e-005 | 4.884e-004 | 3.610e-004 |
| 5.651e-004 | -1.698e-005 | -4.154e-004 |
| 6.418e-004 | -1.677e-005 | -4.699e-004 |
| 8.885e-005 | 6.131e-004 | -7.351e-004 |
| -1.635e-005 | 7.953e-004 | 5.787e-004 |
| -1.614e-005 | 8.720e-004 | 6.332e-004 |

## 6. Sforzi $\{\sigma\}_{xy}$ nelle singole lamine (sistema di riferimento locale x-y)

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{Bmatrix} = [Q]_{xy} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{Bmatrix}$$

| | BOTTOM | | | TOP | | |
|---|---|---|---|---|---|---|
| Lamina | $\sigma_x$ (Pa) | $\sigma_y$ (Pa) | $\sigma_s$ (Pa) | $\sigma_x$ (Pa) | $\sigma_y$ (Pa) | $\sigma_s$ (Pa) |
| **1** | 3.090e+007 | 3.734e+004 | -4.345e+005 | 5.352e+007 | 1.508e+005 | -7.011e+005 |
| **2** | 5.352e+007 | 1.508e+005 | -7.011e+005 | 7.613e+007 | 2.642e+005 | -9.677e+005 |
| **3** | 6.639e+006 | 1.423e+006 | -1.352e+006 | 1.005e+007 | 1.856e+006 | -1.727e+006 |
| **4** | -4.703e+006 | 2.102e+006 | 1.234e+006 | -4.529e+006 | 2.590e+006 | 1.501e+006 |
| **5** | -4.529e+006 | 2.590e+006 | 1.501e+006 | -4.355e+006 | 3.077e+006 | 1.768e+006 |
| **6** | 1.440e+008 | 6.044e+005 | -1.768e+006 | 1.666e+008 | 7.178e+005 | -2.034e+006 |
| **7** | 1.666e+008 | 7.178e+005 | -2.034e+006 | 1.892e+008 | 8.312e+005 | -2.301e+006 |
| **8** | 2.368e+007 | 3.591e+006 | -3.225e+006 | 2.709e+007 | 4.025e+006 | -3.600e+006 |
| **9** | -3.832e+006 | 4.540e+006 | 2.567e+006 | -3.658e+006 | 5.028e+006 | 2.834e+006 |
| **10** | -3.658e+006 | 5.028e+006 | 2.834e+006 | -3.483e+006 | 5.515e+006 | 3.101e+006 |